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Automated non-monotonic reasoning in System P

Tatjana Stojanović · Nebojša Ikodinović · Tatjana Davidović · Zoran Ognjanović

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Abstract This paper presents a novel approach to automated reasoning in System **P**. System **P** axiomatizes a set of core properties that describe reasoning with defeasible assertions (defaults) of the form: if α then normally (usually or typically) β. A logic with approximate conditional probabilities is used for modeling default rules. That representation enables reducing the satisfiability problem for default reasoning to the (non)linear programming problem. The complexity of the obtained instances requires the application of optimization approaches. The main heuristic that we use is the Bee Colony Optimization (BCO). As an alternative to BCO, we use Simplex method and Fourier-Motzkin Elimination method to solve linear programming problems. All approaches are tested on a set of default reasoning examples that can be found in literature. The general impression is that Fourier-Motzkin Elimination procedure is not suitable for practical use due to substantially high memory usage and time consuming execution, the Simplex method is able to provide useful results for some of the tested examples, while heuristic approach turns out to be the most appropriate in terms of both success rate and time needed for reaching conclusions. In addition, the BCO method was tested on a set of randomly generated examples of larger dimensions, illustrating its practical usability.

Keywords Non-monotonic reasoning, Probabilistic satisfiability, Conditional probability, Approximate probability, Metaheuristics, Swarm intelligence

T. Stojanović
Faculty of Science, University of Kragujevac, Serbia
E-mail: tanjat@kg.ac.rs

N. Ikodinović
Faculty of Mathematics, University of Belgrade, Serbia
E-mail: ikodinovic@matf.bg.ac.rs

T. Davidović · Z. Ognjanović
Mathematical Institute, Serbian Academy of Science and Arts, Serbia
E-mail: tanjad@mi.sanu.ac.rs
E-mail: zorano@mi.sanu.ac.rs

All our implementations are executed on a cluster that consists of 22 working nodes, each configured with two Intel 2.6GHz 8-core processors, and 64GB of memory capacity (4GB RAM per core) under Scientific Linux 6.5 64-bit with gcc version 4.4.

A Tables and figures

Table 1 Comparison of different strategies

	Average percent of successful runs (%)	Average time (s)
NM	22.22	3978.00
GG	23.97	6718.00
GG, P = 1	23.19	4001.00
GG, P = 5	24.12	6715.00
GG, P = 10	22.50	6830.00
WUP	67.64	592.00
WUP, P = 1	34.31	4861.00
WUP, P = 5	67.78	609.00
WUP, P = 10	70.28	583.00
WUP, GG	90.00	2524.00
WUP, GG, P = 1	34.44	5258.00
WUP, GG, P = 5	90.00	2397.00
WUP, GG, P = 10	90.00	2296.00

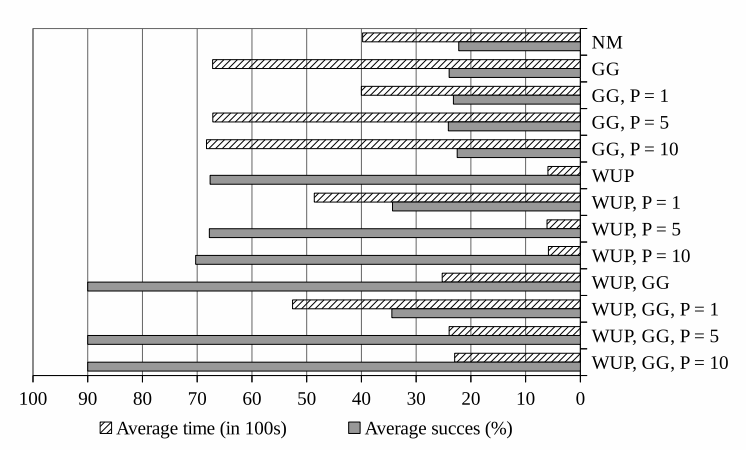

Fig. 1 Comparison of different strategies

Table 2 The test results for varied parameters B and NC when increasing of objective function is not allowed

B	NC	Generating new initial solutions		Hold some solutions from previous iteration	
		Average success (%)	Average time (s)	Average success (%)	Average time (s)
1	10	26.39	25.08	26.39	25.08
	20	41.94	41.10	41.94	41.10
	30	50.14	52.31	50.14	52.31
	40	54.31	64.34	54.31	64.34
	50	62.08	72.94	62.08	72.94
	60	66.11	82.52	66.11	82.52
	70	72.78	85.17	72.78	85.17
5	10	31.25	121.70	62.64	69.56
	20	49.31	187.27	89.44	43.48
	30	60.28	234.77	94.86	41.48
	40	65.83	291.63	99.03	18.60
	50	74.03	306.42	98.75	42.61
	60	80.56	333.61	99.72	30.58
	70	89.03	314.68	99.86	36.95
10	10	35.00	230.35	74.44	94.01
	20	55.07	335.70	98.61	22.45
	30	63.33	445.47	99.17	26.39
	40	74.53	485.30	98.61	51.22
	50	81.53	504.76	99.86	33.21
	60	89.17	484.83	100.00	43.08
	70	94.20	445.85	99.86	55.31
15	10	35.83	335.88	86.39	83.83
	20	52.53	528.45	99.31	26.69
	30	68.33	631.93	99.58	34.02
	40	77.08	685.42	99.44	50.97
	50	88.19	631.23	99.86	45.69
	60	90.93	677.28	100.00	45.78
	70	97.36	526.23	100.00	54.10
20	10	37.50	432.92	82.22	134.46
	20	54.72	659.20	95.28	90.23
	30	71.11	748.89	100.00	27.81
	40	79.58	833.66	99.86	38.49
	50	90.42	701.83	100.00	44.33
	60	95.69	632.13	100.00	45.14
	70	97.92	547.45	100.00	55.93

Table 3 The test results for varied parameters B and NC when degradation of the solutions is allowed

B	NC	Generating new initial solutions		Hold some solutions from previous iteration	
		Average success (%)	Average time (s)	Average success (%)	Average time (s)
1	10	57.64	45.68	57.64	45.68
	20	80.97	53.47	80.97	53.47
	30	91.81	48.53	91.81	48.53
	40	99.03	29.38	99.03	29.38
	50	100.00	21.65	100.00	21.65
	60	100.00	20.15	100.00	20.15
	70	100.00	20.67	100.00	20.67
5	10	73.75	164.55	99.58	9.64
	20	95.14	125.73	100.00	10.49
	30	100.00	69.96	100.00	12.07
	40	100.00	48.44	100.00	19.83
	50	100.00	42.19	100.00	29.88
	60	100.00	40.99	100.00	38.62
	70	100.00	47.97	100.00	45.24
10	10	81.39	282.05	99.86	10.06
	20	98.47	147.70	100.00	13.23
	30	100.00	78.59	100.00	19.53
	40	100.00	42.87	100.00	22.76
	50	100.00	40.50	100.00	31.39
	60	100.00	41.85	100.00	47.06
	70	100.00	42.42	100.00	45.62
15	10	86.11	347.80	99.86	15.02
	20	100.00	142.71	100.00	18.24
	30	100.00	83.20	100.00	20.70
	40	100.00	56.84	100.00	25.29
	50	100.00	48.19	100.00	36.94
	60	100.00	45.79	100.00	42.34
	70	100.00	47.05	100.00	47.01
20	10	90.28	407.93	100.00	18.33
	20	100.00	162.07	100.00	19.79
	30	100.00	89.53	100.00	23.05
	40	100.00	60.40	100.00	33.76
	50	100.00	51.61	100.00	40.43
	60	100.00	53.88	100.00	53.34
	70	100.00	52.56	100.00	51.51

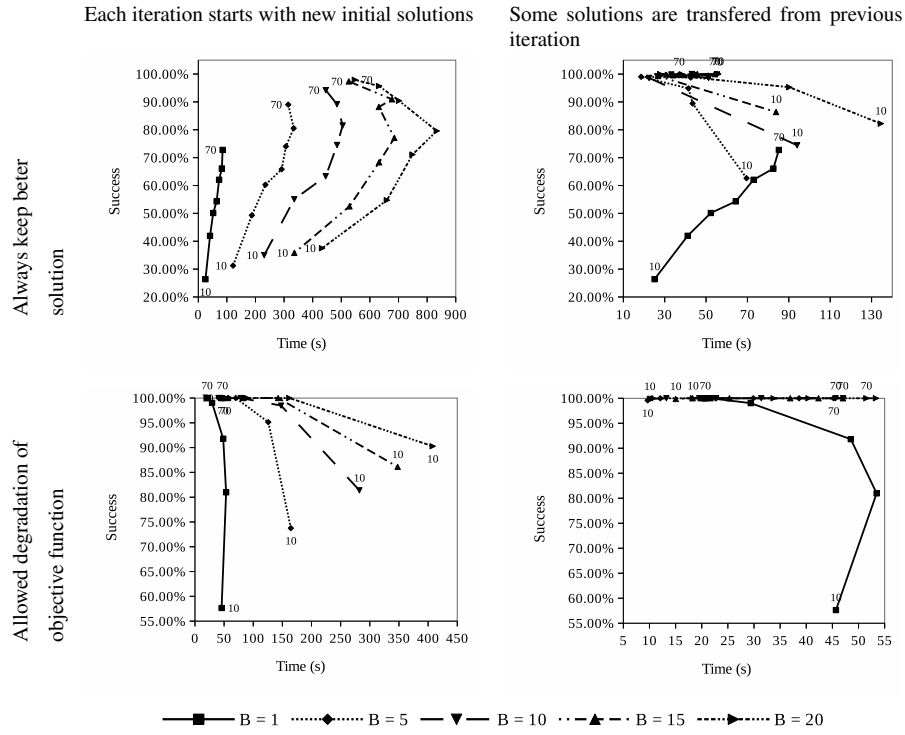

Fig. 2 The ratio of success of the execution time depending on parameters *B* and *NC*

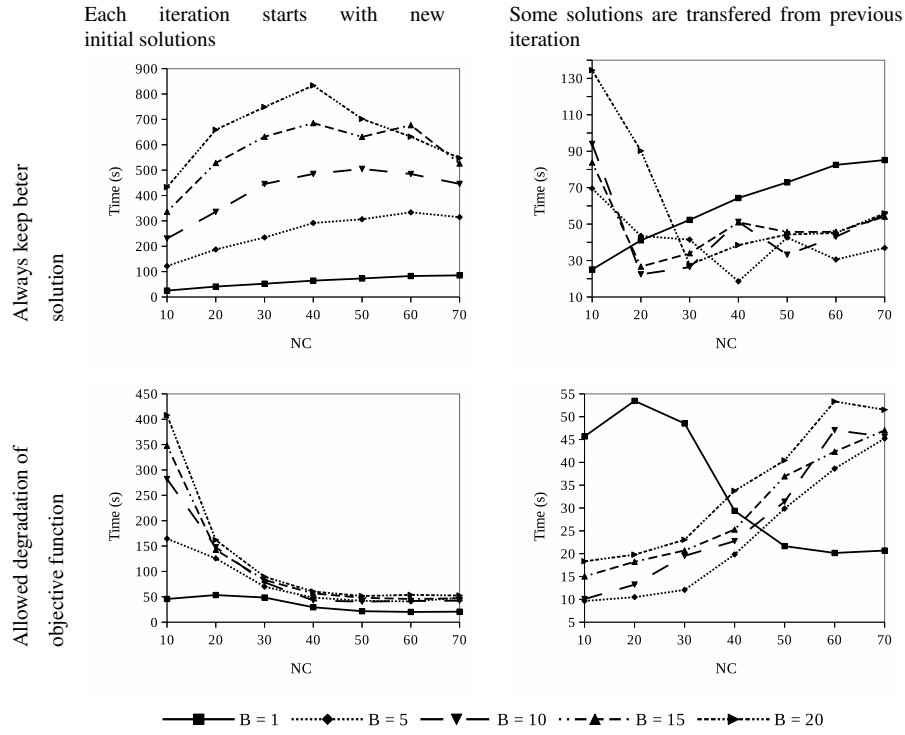

Fig. 3 Execution time depending on parameter *NC* for various values of the parameter *B*

Table 4 Numbers of formulas proven as satisfied, unsatisfied and too demanding on memory resources depending on e and D parameters, using Simplex method

e	4			5			6			7			8			9			10		
D	S	U	M	S	U	M	S	U	M	S	U	M	S	U	M	S	U	M	S	U	M
ε	23	11	3																		
ε/K	23	11	2																		
ε^2	23	15	4	23	12	6															
ε^2/K	24	12	6	23	13	5															
ε^3	28	13	1	22	13	7	24	16	2												
ε^3/K	29	10	3	23	12	7	24	16	2												
ε^4	32	7	3	29	7	6	23	16	3	21	18	4									
ε^4/K	**35**	**6**	**1** *	30	5	7	24	17	1	22	17	4									
ε^5				30	8	4	26	15	1	24	18	1	21	15	6						
ε^5/K				33	5	4	24	17	1	23	19	1	22	16	4						
ε^6							32	8	2	23	13	6	23	16	3	25	9	8			
ε^6/K							**36**	**6**	**0** †	25	15	2	24	16	2	26	10	6			
ε^7										31	7	5	26	13	3	26	10	6	25	12	5
ε^7/K										32	9	2	24	17	1	27	11	4	26	13	3
ε^8													23	9	9	22	12	8	23	16	3
ε^8/K													24	8	10	22	15	5	23	18	1
ε^9																28	10	3	30	7	5
ε^9/K																30	10	2	30	10	2
ε^{10}																			25	7	9
ε^{10}/K																			25	8	9

* Values used for our implementation of Simplex method
† Best obtained results

Table 5 Percentage of successfully proven formulas and average execution times depending on e and D parameters, using Simplex method

e	4		5		6		7		8		9		10	
D	succ [%]	time [s]	succ [%]	time [s]	succ [%]	time [s]	succ [%]	time [s]	succ [%]	time [s]	succ [%]	time [s]	succ [%]	time [s]
ε	47.92	7.55												
ε/K	47.92	1.22												
ε^2	47.92	0.27	47.92	0.32										
ε^2/K	50.00	0.18	47.92	0.30										
ε^3	58.33	0.25	45.83	0.27	50.00	0.98								
ε^3/K	60.42	0.21	47.92	0.20	50.00	0.43								
ε^4	66.67	0.25	60.42	0.26	47.92	0.48	43.75	1.05						
ε^4/K	**72.92**	**0.24** *	62.50	0.19	50.00	0.48	45.83	1.05						
ε^5			62.50	0.38	54.17	0.39	50.00	0.78	43.75	0.76				
ε^5/K			68.75	0.25	50.00	0.35	47.92	0.82	45.83	0.78				
ε^6					66.67	0.39	47.92	0.71	47.92	0.53	52.08	0.69		
ε^6/K					**75.00**	**0.37** †	52.08	0.75	50.00	0.63	54.17	0.74		
ε^7							64.58	1.06	54.17	0.70	54.17	0.42	52.08	0.99
ε^7/K							66.67	1.09	50.00	0.65	56.25	0.52	54.17	1.20
ε^8									47.92	0.54	45.83	0.39	47.92	0.83
ε^8/K									50.00	0.56	45.83	0.38	47.92	1.47
ε^9											58.33	0.67	62.50	0.71
ε^9/K											62.50	0.74	62.50	0.63
ε^{10}													52.08	0.69
ε^{10}/K													52.08	0.73

* Values used for our implementation of Simplex method

† Best obtained results

Table 6 The test results for examples when $\Delta|\sim \alpha \rightarrowtail \beta$

Example	Method	Δ		Φ_1		Φ_2	
		Status	Time	Status	Time	Status	Time
$bird \rightarrowtail fly$ $penguin \rightarrowtail bird$ $penguin \rightarrowtail \neg fly$ $\overline{bird \wedge penguin \rightarrowtail \neg fly}$	BCOi	30/30	6.34	30/30	4.44	0/30	301.16
	SMP	sat	0.15	sat	0.09	MEM	–
	FME	sat	79957.29	sat	124076.15	unsat	626.75
$bird \rightarrowtail fly$ $penguin \rightarrowtail bird$ $penguin \rightarrowtail \neg fly$ $\overline{fly \rightarrowtail \neg penguin}$	BCOi	30/30	6.34	30/30	16.89	0/30	333.40
	SMP	sat	0.15	sat	0.26	MEM	–
	FME	sat	79957.29	MEM	–	MEM	–
$bird \rightarrowtail fly$ $penguin \rightarrowtail bird$ $penguin \rightarrowtail \neg fly$ $\overline{bird \rightarrowtail \neg penguin}$	BCOi	30/30	6.34	30/30	6.08	0/30	356.27
	SMP	sat	0.15	sat	0.14	MEM	–
	FME	sat	79957.29	MEM	–	MEM	–
$bird \rightarrowtail fly$ $penguin \rightarrowtail bird$ $penguin \rightarrowtail \neg fly$ $\overline{bird \vee penguin \rightarrowtail \neg penguin}$	BCOi	30/30	6.34	30/30	12.89	0/30	378.22
	SMP	sat	0.15	sat	0.13	MEM	–
	FME	sat	79957.29	MEM	–	MEM	–
$bird \rightarrowtail fly$ $penguin \rightarrowtail bird$ $penguin \rightarrowtail \neg fly$ $\overline{bird \vee penguin \rightarrowtail fly}$	BCOi	30/30	6.34	30/30	10.74	0/30	371.69
	SMP	sat	0.15	MEM	–	MEM	–
	FME	sat	79957.29	MEM	–	MEM	–
$\theta \rightarrowtail \phi$ $\theta \rightarrowtail \psi$ $\overline{\theta \rightarrowtail \phi \wedge \psi}$	BCOi	30/30	0.80	30/30	0.46	0/30	291.12
	SMP	sat	0.04	sat	0.03	unsat	11.07
	FME	MEM	–	MEM	–	MEM	–
$\theta \rightarrowtail \phi$ $\phi \rightarrowtail \psi$ $\overline{\theta \vee \phi \rightarrowtail \psi}$	BCOi	30/30	0.48	30/30	0.32	0/30	303.80
	SMP	sat	0.03	sat	0.04	MEM	–
	FME	MEM	–	MEM	–	MEM	–
$\theta \rightarrowtail \psi$ $\theta \rightarrowtail \phi$ $\overline{\theta \wedge \phi \rightarrowtail \psi}$	BCOi	30/30	0.80	30/30	0.26	0/30	259.93
	SMP	sat	0.04	sat	0.02	unsat	34.09
	FME	MEM	–	MEM	–	MEM	–
$\theta \rightarrowtail \phi$ $\theta \wedge \phi \rightarrowtail \psi$ $\overline{\theta \rightarrowtail \psi}$	BCOi	30/30	0.18	30/30	0.61	0/30	261.02
	SMP	sat	0.02	sat	0.03	unsat	88.52
	FME	sat	115.77	MEM	–	MEM	–
$\theta \rightarrowtail \phi$ $\theta \wedge \phi \rightarrowtail \psi$ $\overline{\theta \rightarrowtail \psi}$	BCOi	30/30	0.18	30/30	0.83	0/30	257.37
	SMP	sat	0.02	sat	0.03	unsat	88.54
	FME	sat	115.77	MEM	–	MEM	–
$\theta \rightarrowtail \phi$ $\phi \rightarrowtail \theta$ $\theta \rightarrowtail \psi$ $\overline{\phi \rightarrowtail \psi}$	BCOi	30/30	1.85	30/30	1.95	0/30	339.45
	SMP	sat	0.03	sat	0.04	MEM	–
	FME	MEM	–	MEM	–	MEM	–
$\theta \rightarrowtail \phi \rightarrow \psi$ $\theta \rightarrowtail \phi$ $\overline{\theta \rightarrowtail \psi}$	BCOi	30/30	0.22	30/30	0.76	0/30	269.01
	SMP	sat	0.03	sat	0.05	unsat	13.06
	FME	MEM	–	MEM	–	MEM	–

Table 7 The test results for examples where $\Delta \mid\!\!\sim \alpha \rightarrowtail \beta$ and $\Delta \mid\!\!\sim \alpha \rightarrowtail \neg\beta$

Example	Method	Δ		Φ_1		Φ_2	
		Status	Time	Status	Time	Status	Time
$bird \rightarrowtail fly$ $penguin \rightarrowtail bird$ $penguin \rightarrowtail \neg fly$ $metalWings \rightarrowtail fly$ $yogi \rightarrowtail fly$ $bird \wedge penguin \wedge metalWings \rightarrowtail \neg yogi \vee fly$	BCOi	30/30	29.52	28/30	148.14	30/30	26.91
	SMP	sat	0.80	unsat	1.13	MEM	–
	FME	MEM	–	MEM	–	MEM	–
$bird \rightarrowtail fly$ $penguin \rightarrowtail bird$ $penguin \rightarrowtail \neg fly$ $metalWings \rightarrowtail fly$ $yogi \rightarrowtail fly$ $bird \wedge penguin \wedge metalWings \rightarrowtail yogi \wedge \neg fly$	BCOi	30/30	29.52	29/30	91.01	30/30	43.58
	SMP	sat	0.80	sat	0.71	unsat	1.08
	FME	MEM	–	MEM	–	MEM	–
$penguin \rightarrowtail featherAnim \vee bird$ $featherAnim \rightarrowtail bird$ $bird \rightarrowtail featherAnim$ $penguin \rightarrowtail \neg fly$ $bird \rightarrowtail fly$ $penguin \wedge featherAnim \wedge bird \rightarrowtail \neg fly$	BCOi	30/30	16.76	30/30	32.67	30/30	8.83
	SMP	unsat	0.64	sat	0.28	MEM	–
	FME	MEM	–	MEM	–	MEM	–
$penguin \rightarrowtail featherAnim \vee bird$ $featherAnim \rightarrowtail bird$ $bird \rightarrowtail featherAnim$ $penguin \rightarrowtail \neg fly$ $bird \rightarrowtail fly$ $penguin \wedge featherAnim \wedge bird \rightarrowtail fly$	BCOi	30/30	16.76	30/30	43.73	30/30	16.27
	SMP	unsat	0.64	unsat	0.61	sat	0.38
	FME	MEM	–	MEM	–	MEM	–

Table 8 The test results for examples where $\Delta \not\sim \alpha \rightarrowtail \beta$ and $\Delta \not\sim \alpha \rightarrowtail \neg\beta$

Example	Method	Δ		Φ_1		Φ_2	
		Status	Time	Status	Time	Status	Time
$quaker \rightarrowtail pacifist$ $republican \rightarrowtail \neg pacifist$ $quaker \wedge republican \rightarrowtail pacifist$	BCOi	30/30	2.24	30/30	0.83	30/30	1.99
	SMP	sat	0.03	sat	0.06	unsat	0.24
	FME	sat	9866.65	sat	10620.05	sat	25394.76
$quaker \rightarrowtail pacifist$ $republican \rightarrowtail \neg pacifist$ $quaker \wedge republican \rightarrowtail \neg pacifist$	BCOi	30/30	2.24	30/30	0.62	30/30	1.50
	SMP	sat	0.03	sat	0.07	sat	0.08
	FME	sat	9866.65	sat	29637.52	sat	10675.40
$quaker \rightarrowtail pacifist$ $republican \rightarrowtail \neg pacifist$ $ecologist \rightarrowtail pacifist$ $quaker \wedge ecologist \wedge republican \rightarrowtail pacifist$	BCOi	30/30	12.02	30/30	11.92	30/30	9.08
	SMP	sat	0.11	sat	0.11	sat	0.23
	FME	MEM	–	MEM	–	MEM	–
$quaker \rightarrowtail pacifist$ $republican \rightarrowtail \neg pacifist$ $ecologist \rightarrowtail pacifist$ $quaker \wedge ecologist \wedge republican \rightarrowtail \neg pacifist$	BCOi	30/30	12.02	30/30	11.13	30/30	9.46
	SMP	sat	0.11	sat	0.18	MEM	–
	FME	MEM	–	MEM	–	MEM	–
$teenAger \rightarrowtail poor$ $teenAger \rightarrowtail student$ $poor \rightarrowtail employed$ $student \rightarrowtail \neg employed$ $teenAger \rightarrowtail employed$	BCOi	30/30	18.04	30/30	23.64	30/30	11.98
	SMP	sat	0.48	sat	0.29	MEM	–
	FME	MEM	–	MEM	–	MEM	–
$teenAger \rightarrowtail poor$ $teenAger \rightarrowtail student$ $poor \rightarrowtail employed$ $student \rightarrowtail \neg employed$ $teenAger \rightarrowtail \neg employed$	BCOi	30/30	18.04	30/30	18.23	30/30	20.24
	SMP	sat	0.48	sat	0.43	MEM	–
	FME	MEM	–	MEM	–	MEM	–
$young \rightarrowtail lawAbiding$ $joyRiders \rightarrowtail \neg lawAbiding$ $joyRiders \rightarrowtail young$ $young \wedge joyRiders \rightarrowtail lawAbiding$	BCOi	30/30	12.77	30/30	14.62	30/30	4.02
	SMP	sat	0.15	MEM	–	sat	0.28
	FME	sat	84695.65	MEM	–	MEM	–
$young \rightarrowtail lawAbiding$ $joyRiders \rightarrowtail \neg lawAbiding$ $joyRiders \rightarrowtail young$ $young \wedge joyRiders \rightarrowtail \neg lawAbiding$	BCOi	30/30	12.77	30/30	17.47	30/30	6.56
	SMP	sat	0.15	sat	0.29	MEM	–
	FME	sat	84695.65	MEM	–	MEM	–

Table 9 Summarized values from tables 6, 7 and 8

Method	Successfully proven formula satisfiability	Average time for completed executions [s]
BCOi	99.79%	15.25
SMP	72.92%	0.25
FME	18.75%	41671.03

Table 10 Test results of large dimension examples

n	ℓ	Inst. no	$B = 5, NC = 20$			$B = 20, NC = 60$		
			Succ	Iter. avg	CPU Time	Succ	Iter. avg	CPU Time
10	10	1	10/10	1.1	3.48	10/10	1.0	6.38
10	10	2	1/10	34.0	1282.77	10/10	3.4	1164.86
10	10	3	10/10	2.9	67.35	10/10	1.5	225.05
10	10	4	9/10	21.2	811.23	10/10	2.4	714.83
20	20	1	4/10	12.0	1528.01	10/10	2.5	2433.87
20	20	2	0/10	–	–	8/10	3.0	3162.54
20	20	3	10/10	1.8	110.32	10/10	1.2	348.51
20	20	4	0/10	–	–	10/10	8.8	12027.23
20	50	1	10/10	1.2	128.37	10/10	1.2	1019.38
20	50	2	3/10	10.3	7851.76	10/10	1.5	4875.90
20	50	3	10/10	1.5	395.46	10/10	1.2	1496.90
20	50	4	10/10	3.5	2132.46	10/10	1.2	1957.91
50	100	1	10/10	1.1	346.42	10/10	1.0	698.44
50	100	2	10/10	1.2	351.08	10/10	1.0	638.26
50	100	3	10/10	1.6	2069.07	10/10	1.2	6055.86
50	100	4	10/10	1.2	502.74	10/10	1.0	698.09
100	100	1	0/10	–	–			
100	100	2	0/10	–	–	Time for one		
100	100	3	0/10	–	–	iteration $\approx$ 12h		
100	100	4	0/10	–	–			