University of Kragujevac
Faculty of Science

The First Conference on
Mathematics and Computer Science Teaching
August • 29–30th • 2024 • Kragujevac • Serbia

THE CONFERENCE PROCEEDING
- School Practice Section -

TEMATCOM

University of Kragujevac
Faculty of Science

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– School Practice Section –

The First Conference on Mathematics and Computer Science Teaching
August, 29–30, 2024

Kragujevac, 2024

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Preface

The Faculty of Science has a tradition of 52 years of training teachers of mathematics, computer science, and natural sciences (biology, physics, and chemistry). Over the years, we have enhanced our study programs to address the educational system's evolving demands and foster essential competencies in teachers, including scientific-professional, psychological-pedagogical, and subject-methodological competencies. In terms of the education of mathematics and computer science teachers in the Department of Mathematics and Informatics, this process has been significantly accelerated and intensified over the past 25 years, especially in the area of innovations in the methodology of teaching mathematics and computer science, as well as in increasing the scope and share of these subjects in the curriculum for those students who choose the teaching modules. At the same time, more teachers and associates from the Department of Mathematics and Informatics have decided to focus part of their research interests on the topics of subject didactics, thus actively contributing at a scientific level to a better understanding of problems in the teaching of mathematics and computer science at all educational levels, as well as offering concrete solutions for implementation in teaching practice. As a natural course of action, the Faculty of Science has organized a scientific conference, **The First Conference on Mathematics and Computer Science Teaching, TEMATCOM 2024**, held on August 29-30, 2024, in Kragujevac (https://imi.pmf.kg.ac.rs/TEMATCOM2024/index-en.php). The organizer of the conference is the Faculty of Science in Kragujevac, and the co-organizer is the Mathematical Society of Serbia – Branch Kragujevac.

The organization of the conference was financially supported by the Ministry of Science, Technological Development and Innovation.

Република Србија
МИНИСТАРСТВО НАУКЕ,
ТЕХНОЛОШКОГ РАЗВОЈА И ИНОВАЦИЈА

The organizers' desire is to organize the same conference periodically. The immediate reason for the first in the series is to celebrate the 70th birthday of Prof. Branislav Popović, who has had a crucial impact on the reforms of study programs in mathematics (and informatics) in the field of methodology of teaching mathematics and related subjects focused on acquiring teaching competencies.

At the TEMATCOM 2024 conference, nearly 100 participants from Serbia, region, and beyond gathered in two sections: *Research in teaching mathematics and computer science* and *School practice – yesterday, today, and tomorrow*. They had the opportunity to exchange scientific and professional results, seeking answers to numerous questions about modern mathematics and computer science teaching. The actual theme of the conference has been illuminated from various angles through numerous presentations, and based on the research presented, specific recommendations for improving the teaching process have been proposed. The attendees supported the idea that the TEMATCOM conference should become a tradition, thus serving as a continuous exchange of scientific and professional knowledge about teaching mathematics and computer science. During the discussion, it was concluded that providing more space, opportunities, and channels for communication among all stakeholders in education, especially concerning significant reforms in the educational system and strategic issues, is necessary. Although each level of education has its specificities and local problems and opportunities, it was emphasized that strengthening their cooperation in everyday matters is essential. The organizers are particularly pleased that many participants highlighted that one of the conference's dominant impressions was the warm, relaxed, yet productive and effective atmosphere created during the event.

The quality of the presented papers and the accumulated positive energy from the conference further solidified and strengthened our efforts to establish a new serial publication (an open-access scientific journal without publication fees) of the Faculty of Science: **The Journal of Educational Studies in Mathematics and Computer Science (JESMAC)**. After the review process, the selected scientific papers from section *Research in teaching mathematics and computer science* were published in the inaugural issue of this newly established journal of the Faculty (JESMAC, https://imi.pmf.kg.ac.rs/jesmac), and selected papers from the section *School practice* were published in **The Conference Proceedings**.

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PARISmatic eTwinning

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Abstract

Mathematics is not only a tool for counting, analyzing, comparing, etc. It is also a subject through which students discover "new worlds", explore new spaces, broaden their horizons, and can collaborate with peers from different meridians. At the same time, it offers professional orientation, as project activities can place students in various roles and positions from which they will search for the best solutions. The eTwinning project "Discovering Mechanics: Louis Renault" was implemented by students from the primary school "Svetislav Golubović Mitraljeta" in Batajnica, Serbia, and students from Collège Jean Renoir, Boulogne-Billancourt, France. The aim of the project was for students to understand better the principles of mechanics, and the cultural and industrial heritage of France (especially regarding Renault), as well as to improve their language skills in a foreign language. In addition to these objectives, the goal was also for students to improve their mathematical skills. The goal of mathematics education is, among other things, to prepare students to: Solve problems and tasks in new and unfamiliar situations; Express and justify their opinions and discuss with others; Develop motivation for learning and interest in the subject content; Prepare students to apply acquired mathematical knowledge in solving various real-life tasks. Richard Skemp, who wrote the popular book "The Psychology of Learning Mathematics" (1971), stated: "The problems of teaching and learning are psychological problems, and before we make many improvements in teaching mathematics, we need to know more about how learning occurs." This perspective emphasizes that the process of learning mathematics is not just about mastering concepts, but also about understanding how students process and internalize those concepts. The challenge of engaging students with different types of problems highlights the need for a comprehensive approach to teaching, one that recognizes the varying levels of difficulty and the diverse ways in which students learn. The success of students in solving mathematical tasks using symbolic, graphical, or verbal representations in problem formulations varies significantly depending on the complexity level of the tasks.

Keywords: eTwinning, PBL, skills, functional knowledge.

1. eTwinning

The main motives for writing this paper are, on the one hand, the misalignment of physics and mathematics curricula at almost all levels of education, both in Serbia and in most other countries, and on the other hand, the strong insistence on linking the content of different subjects in a formal and bureaucratic manner within our educational system. The most significant consequences of this misalignment are difficulties in mastering physics and related disciplines during the first year of studies at faculties in the fields of technical and natural sciences, as well as poor integration and application of differential and integral calculus in the description of immediate natural phenomena. The mentioned bureaucratization of the teaching process leads to a lack of motivation among teachers and inadequate organization of the educational process in schools in terms of introducing interdisciplinary approaches.

The eTwinning is the largest European network for educators and teachers. It is part of the Erasmus+ program, aimed at staff in preschools, primary schools, and secondary schools. It was launched in 2005 by the Council of Europe.

The eTwinning platform became available to teachers, educators, professional associates, and school directors in Serbia in early 2015. The main idea of the portal was collaboration between teachers and students in virtual projects using ICT tools.

eTwinning projects are virtual projects that can help meet the standards and requirements set by curricula for teachers and students:

- Project-based learning,
- Use of ICT tools and media literacy,
- Interdisciplinary connections,
- Skills for the 21st century,
- Independent learning, group work, and peer learning.

eTwinning provides opportunities for collaboration, exchange, and professional development.

In 2022, two platforms (eTwinning and School Education Gateway) merged into a new educational platform called the *European School Education Platform* (ESEP), so eTwinning is now part of this platform.

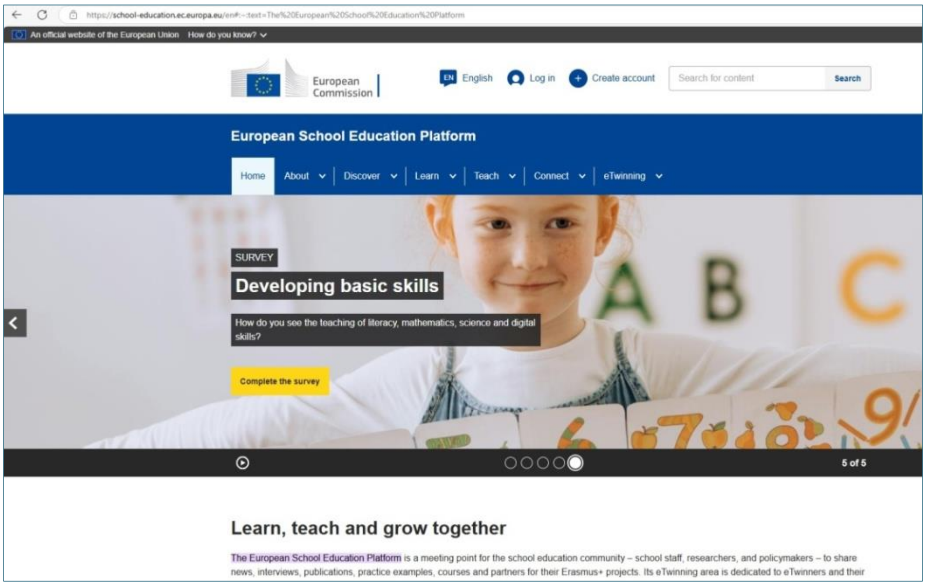


Fig 1. European School Education Platform (ESEP)

In contemporary education, developing digital skills becomes a key factor in improving teaching and the educational system. Introducing new technologies and digital tools enables students and teachers to enhance the quality of their work, create more efficient educational processes, and develop the competencies needed to navigate the digital world. In this context, several key principles lay the foundation for a modern approach to education:

- Raising the quality of educational work and knowledge.
- Developing digital competencies for students and teachers.
- Developing media, digital culture, and literacy (internet safety, intellectual and personal rights, and digital technology as a tool for acquiring and expanding knowledge).
- Students/learners are active creators, not passive consumers of ready-made multimedia products.
- Emphasis is on processes, not on digital tools.
- Digital technology is not an end in itself; it is a tool for achieving educational goals.

By working on eTwinning projects, students gain and improve their skills in recognition, analysis, interpretation, photography and recording, discussion, critical thinking, problem-solving, justifying, and presentation.

These projects allow students to connect knowledge not only across different subjects but also with various real-life situations, breaking down the barrier between what they learn in the classroom and outside of it. Learning happens everywhere and at any time, not just in the classroom. Students become traders, painters, architects, journalists, writers, researchers, archaeologists, guides, organizers ...

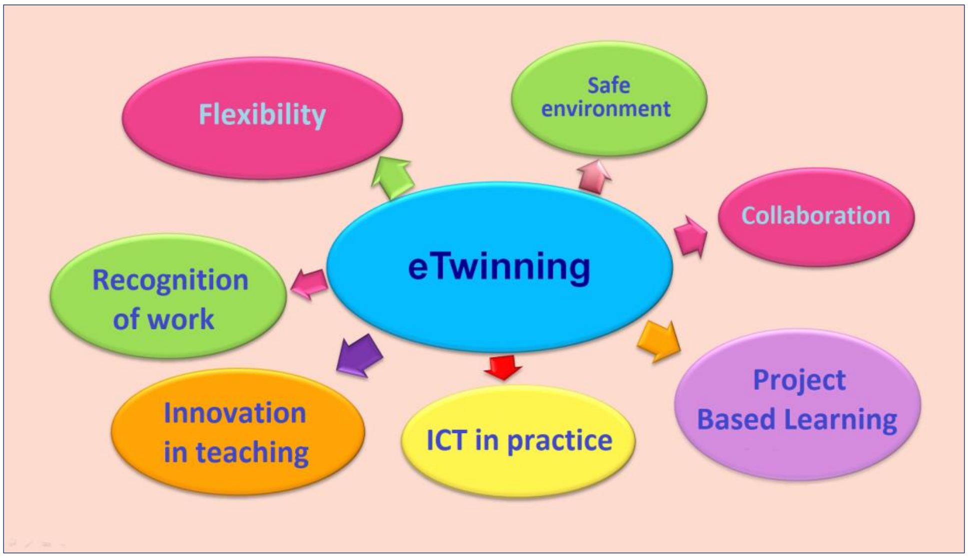


Fig. 2. Advantages of eTwinning projects

eTwinning creates an inclusive environment that caters to the diverse needs of students, including those with learning difficulties and gifted individuals. It provides a safe space where all students can express themselves and grow at their own pace.

2. Educational and Developmental Goals

The section headings are numbered in boldface capital and lowercase letters. Second-level headings are typed as subsection headings of this paragraph.

eTwinning projects, due to their flexibility, offer teachers and educators the opportunity to create diverse activities that enable students to achieve several educational and developmental goals:

- Full intellectual, emotional, social, moral, and physical development of each child.
- Acquisition of quality knowledge, skills, and attitudes necessary for personal fulfillment, development, inclusion, and employment.
- Development of creative abilities, aesthetic perception, and taste.
- Development of the ability to find, analyze, apply, and communicate information using ICT effectively and efficiently.
- Preparation to solve problems, connect and apply knowledge and skills in further education, professional work, and daily life.
- Development of motivation for learning, independent study, and lifelong education.
- Development of self-awareness, initiative, self-evaluation, and expression of one’s opinion.
- Ability to make decisions about career choice, personal development, and future life.
- Ability to work and acquire professional competence.
- Development and practice of healthy lifestyles.
- Awareness of sustainable development, environmental protection, and nature conservation.
- Communication skills, dialogue, solidarity, and teamwork, fostering effective cooperation and friendship.

- Role as a responsible citizen in a democratic and humane society, respecting human rights, diversity, and core values such as justice, truth, freedom, and personal responsibility.
- Formation of attitudes, values, and personal identity, including respect for national culture, multiculturalism, and global heritage.
- Respect for racial, national, cultural, linguistic, religious, gender, and age equality, and promoting tolerance and diversity.

About the role of the teacher in forming mathematical concepts while working with students, Gusić (2016) discussed. In modern education, the teacher's role is to guide the learning process toward greater student engagement, particularly through interactive experiences. The use of modern ICT tools is crucial for enhancing education. Collaboration and integration of these tools expand teaching topics and their visualization, helping students focus better and assimilate material more easily. eTwinning projects create a safe environment, supporting all students, including those with learning difficulties and gifted students who can express their talents creatively.

By providing opportunities for students to solve problems, connect, and apply knowledge in both academic and real-life contexts, these projects contribute to lifelong learning, independent study, and career development. Furthermore, eTwinning helps nurture self-awareness, initiative, and the ability to make informed decisions about personal growth and career choices, while fostering teamwork, cooperation, and respect for diversity, human rights, and democratic values.

Richard Skemp's insights from *The Psychology of Learning Mathematics* (1971) stress the importance of understanding the psychological processes in learning. He argues that teaching isn't just about transferring knowledge but guiding students through cognitive processes that enable them to internalize and apply concepts effectively. This aligns with eTwinning's goals of not only acquiring knowledge but also fostering critical thinking and problem-solving skills. His theory advocates for students' deep engagement with mathematical concepts, fostering an active understanding of why mathematical procedures work. The goal is to encourage problem-solving and the meaningful application of concepts, similar to eTwinning's approach to hands-on, contextual learning. By offering an inclusive learning environment, eTwinning projects allow students to express creativity, collaborate, and engage meaningfully with content. Skemp's approach suggests that by considering how students process learning, teachers can better support students' development, ensuring they are appropriately challenged while receiving the necessary support. This makes eTwinning an ideal tool for enhancing education and preparing students for the challenges of the modern world.

3. Discovering Paris

"If you are lucky enough to have lived in Paris
as a young man, then wherever you go for the rest of your life,
it stays with you, for Paris is a movable feast."
— Ernest Hemingway

During the 2018/19 school year, we implemented the eTwinning project *Discovering Mechanics: Louis Renault* in collaboration with a school in France (Collège Jean Renoir, Boulogne-Billancourt). Since the partner school is located near the Renault car factory, one of the reasons for choosing this project was to introduce students to mechanics as well as the life and work of Louis Renault, the founder of the Renault car factory. This was also an opportunity for students to become more familiar with Paris and its landmarks within the context of mathematics classes. Some tasks were solved during the preparatory period, while certain tasks were meant for collaborative problem-solving by students in international teams after returning from France.

Through various tasks, students had to master skills such as comparison, searching, conversion, calculation, and selection, based on real-life problems in order to gain functional knowledge. It should be noted that word problems are important for students while acquiring mathematical knowledge (Verschaffel & Greer, 2000).

Task 1. When Tamara tried to research the best way to travel on a two-day trip to Paris, she received the results shown below. If she travels only with carry-on luggage, the round-trip ticket costs £171, while the ticket with checked luggage costs £237.

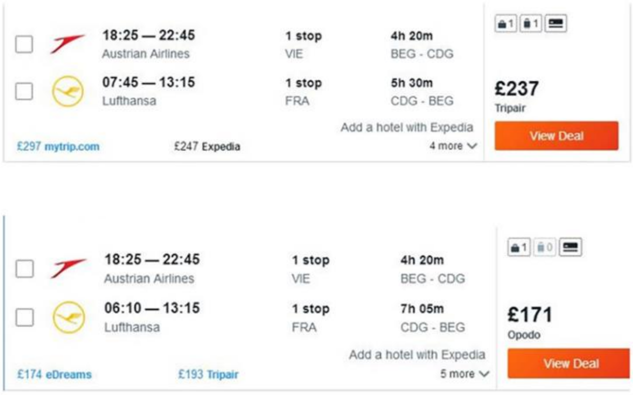


Fig. 3. Flight tickets, student task

Note: For currency conversion (changing currency), use the current exchange rate from the National Bank of Serbia. Take a photo with your mobile phone and upload it as part of your solution on Padlet.

a) By what percentage is the price of the round-trip ticket with checked luggage higher than the price of the carry-on luggage ticket?
b) The price of the more expensive ticket in dinars is ___________, and the price of the cheaper ticket converted to dinars is ___________.
c) The price of the more expensive ticket in euros is ___________, and the price of the cheaper ticket converted to euros is ___________.

Task 2. Nikola, Jana, and Nemanja are traveling from Belgrade to Paris. Nikola is traveling with Air Serbia, while Jana and Nemanja have decided to fly with Wizz Air. The return ticket prices for both airlines can be found online (https://www.airserbia.com/ and https://wizzair.com/en-gb). Answer the following questions and show your calculation method:

a) By what percentage is the return ticket from Air Serbia cheaper/more expensive than the one offered by Wizz Air?
b) Charles de Gaulle (CDG) Airport is 34.7 km from the Eiffel Tower, while Beauvais-Paris (BP) Airport is 67 km away. The return bus ticket from CDG to the Eiffel Tower costs €30, while the return ticket from BP to the Eiffel Tower costs €15. If the current exchange rate for 1€ is 117.5 dinars, calculate how much the total return tickets for the flight and bus will cost in dinars.
c) If the price of the Wizz Air ticket increases by 15%, which airline would it be cheaper to travel with?

Task 3. Lighting of the Eiffel Tower

The Eiffel Tower is illuminated with 20,000 light bulbs of 45 W. How many 75 W light bulbs would be needed to provide the same amount of lighting?

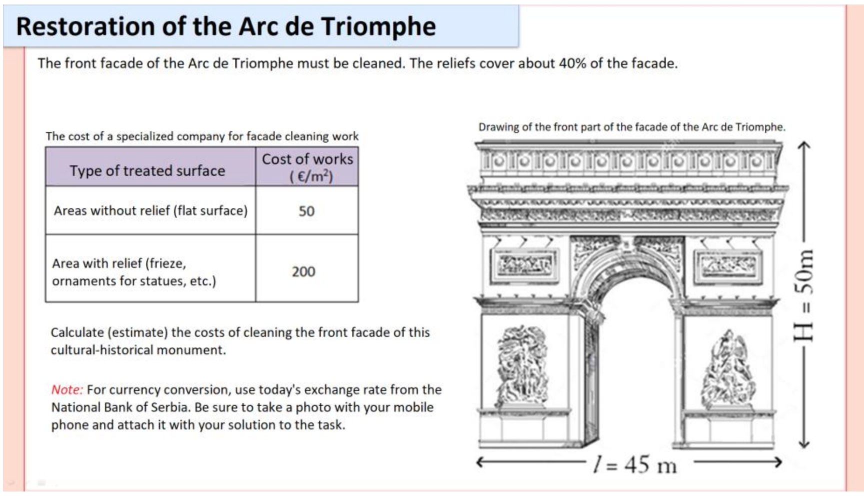
Task 4. Models of the Eiffel Tower and Avala Tower

The Eiffel Tower is ____ m tall. Max has a model of the Eiffel Tower that is 8.1 cm high. What will be the height of the model of the Avala Tower he wants to create in the same scale, if the height of the Avala Tower is ____ m?

Fig. 4. Eiffel Tower and Avala Tower

Task 5. Restoration of the Arc de Triomphe

Based on the data in the picture, you need to estimate the costs of restoring the Arc de Triomphe, one of the most famous cultural and historical symbols of Paris, the capital of France. (For more about the Arc de Triomphe, visit the following link: https://bookaweb.com/sr/ad/2223.)



Restoration of the Arc de Triomphe

The front facade of the Arc de Triomphe must be cleaned. The reliefs cover about 40% of the facade.

The cost of a specialized company for facade cleaning work

Type of treated surface	Cost of works (€/m^2)
Areas without relief (flat surface)	50
Area with relief (frieze, ornaments for statues, etc.)	200

Calculate (estimate) the costs of cleaning the front facade of this cultural-historical monument.

Note: For currency conversion, use today's exchange rate from the National Bank of Serbia. Be sure to take a photo with your mobile phone and attach it with your solution to the task.

Fig. 5. Restoration of the Arc de Triomphe

The use of technology in modern teaching allows students and teachers not only to more easily master the curriculum but also provides the opportunity for students to deepen their knowledge of regions and phenomena from any place at any time. This is also the case with the use of the Google Earth tool, which enabled my students to “walk” through Paris and mark points (landmarks) that some of them had the chance to visit in Paris.

Task 6. Suzana found a picture of the orangery palace at Versailles on the internet and immediately liked the image of the fountain located in that part of the park. Help Suzana estimate the length of the radius of the circular-shaped fountain, and then calculate the circumference and area of that circle ($\pi \approx 3.14$). What portion of the area of the square part of the park is occupied by the fountain (expressed as a percentage)?

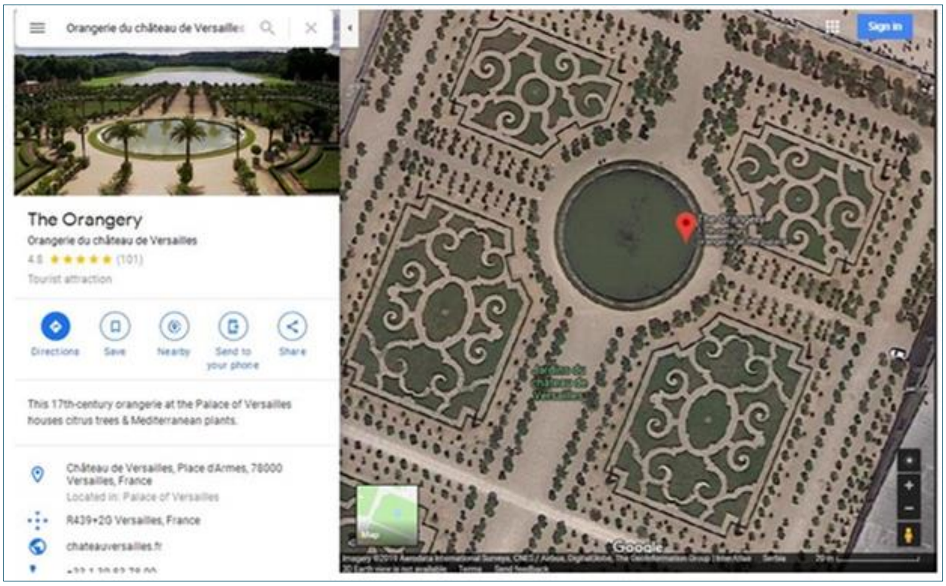


Fig. 6. The Orangery of the Palace of Versailles

Task 7. Cruising on the Seine

During the cruise, you were able to see many landmarks shown in the image:

a) Approximately measure how many kilometers you traveled cruising along the Seine River using Google Earth.
b) How many bridges did you pass under (counting only once)? ____ What percentage is that concerning the 37 bridges built over the Seine River in Paris?

Interesting fact: Out of the 37 bridges over the Seine, 31 are for cars, 4 are pedestrian bridges, 2 are railway bridges, and of the total number, 32 are illuminated.

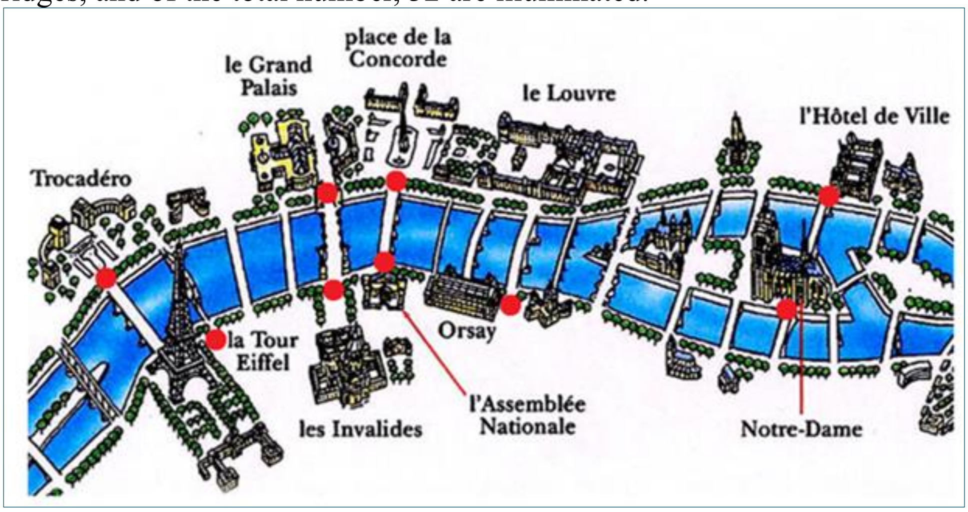


Fig. 7. Picture of landmarks in Paris along the banks of the Seine River.

c) Express in percentage how many bridges are not for cars.
d) Express in percentage by how much the number of illuminated bridges exceeds the number of bridges for cars.

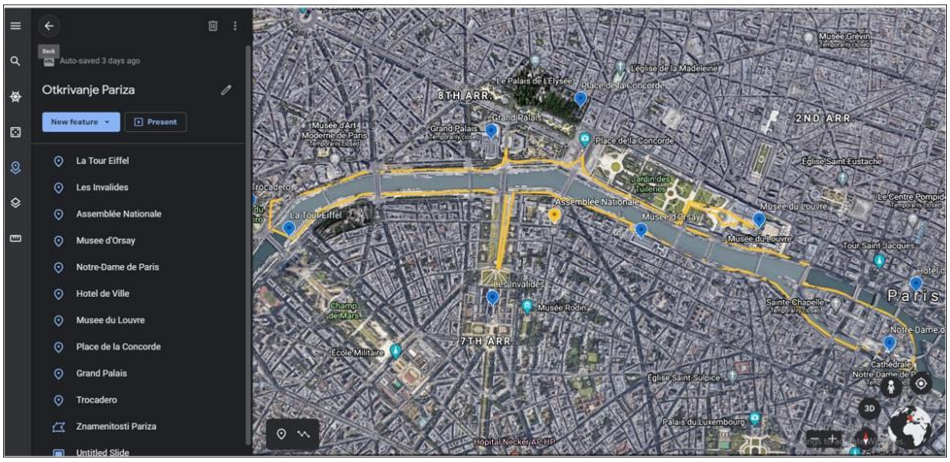


Fig. 8. Map with marked landmarks in Paris.

e) If you had visited these landmarks by land, how many kilometers would you have traveled?
f) What percentage of the land route is represented by the river route traveled by passengers who go on a Seine River cruise to see these landmarks of Paris?

Task 8. Rowing on the Seine

Using the internet, gather data about the Seine River necessary for solving the following tasks:

a) Which part of its course does the Seine River flow through the French capital? (Express in %)
b) To strengthen teamwork, students went rowing on the Seine. There were 6 students in the boat, 5 of whom were rowing, and one was steering. They completed the planned route in 1.5 hours. How long would it take to row the same distance if there were 8 students in the boat (7 rowers + one who steers)?
c) Five students rowed 2 km in 36 minutes. How long would it take them to row 1.5 kilometers under the same conditions?
d) Five students rowed 2 km in 36 minutes. How much time would be needed to cover the same distance if, after 9 minutes, 3 students jumped into the river and continued swimming?

The students posted their solutions on Padlet (collaborative board), and then their solutions were analyzed, and the lesson was evaluated. In the evaluation, the majority of students selected the following statements:

- I have gained new skills. (73.9%)
- I had fun while solving the tasks. (73.9%)
- I practiced mathematics. (60.9%)
- The tasks were interesting to me. (56.6%)

In the study conducted by Russo & Minas (2020), the authors emphasize that students, although faced with challenges and difficulties when solving problems, often have a positive experience and develop a stronger sense of achievement. This effect is similar to what was recorded in the evaluation of the eTwinning project, where 73.9% of students stated that they gained new skills while solving the tasks. This result confirms the finding in the research that challenging tasks help students develop new skills and boost their self-confidence, which is crucial for their further development.

Additionally, 73.9% of students stated that they had fun while solving the tasks, which aligns with the research's claim that students can develop a positive attitude toward learning when challenges are presented in a way that allows them to recognize their progress and feel successful. Students are not afraid of difficulty when they can overcome it, strengthening their motivation and interest in the subject. The study also emphasizes that students participating in challenging mathematics tasks develop a deeper understanding of mathematical concepts and are often more motivated to continue learning. Similarly, in the evaluation, 60.9% of students stated that they "practiced mathematics" during the eTwinning project. This result indicates that the tasks effectively contributed to their mathematical literacy and understanding of core mathematical skills. Although the tasks were challenging, students saw them as an opportunity to practice and learn from these experiences, which aligns with the study's findings that students who face challenges are more likely to develop a deeper understanding (Russo & Minas, 2020). One of the study's key findings is that challenging tasks can increase students' interest in mathematics, even though these tasks are demanding. The study highlights that students who face mathematical problems and then successfully solve them are motivated to continue learning and develop a deeper interest in the subject. In the evaluation of the eTwinning project, 56.6% of students stated that the tasks were interesting to them. This connects with the idea in the research that, despite difficulties, challenging tasks can maintain or even increase interest in mathematics because students recognize the value of the learning process and effort. This result also underscores the importance of providing challenging tasks that are stimulating enough to motivate students to continue exploring, discovering, and connecting information to real-life situations. Through eTwinning projects, students had the chance to participate in tasks that not only allowed them to practice mathematical skills but also engaged them in active learning, which is directly related to positive attitudes toward learning. Connecting these two situations, it can be concluded that both the study on students' attitudes toward learning mathematics through challenging tasks and the evaluation of the eTwinning project confirm that learning challenges, especially when well-structured and supported, can significantly contribute to the development of positive attitudes toward learning, as well as a deeper understanding of the material. Although the tasks in both cases were demanding, students felt motivated and engaged because they recognized the importance of these challenges for their intellectual and social development. Through these projects, students learned not only how to solve mathematical problems but also how to engage in real-world problems, developing skills that will be valuable in the future.

Both in the study and in the evaluation of the eTwinning project, the significance of student engagement in challenging tasks is highlighted. When combined with collaboration, these tasks help students develop critical thinking, problem-solving, and positive acceptance of challenges, which are key to their success in education and life.

4. eTwinning/ERASMUS+ Project *Let's Live The Olympic Dream Together*!

The tasks from the previous chapter served as preparation for the ERASMUS+ student mobility to Collège Les Colliberts in Saint-Michel-en-l'Herm, France. In the 2023/24 school year, students from the "Svetislav Golubović Mitraljeta" Primary School in Batajnica participated in the eTwinning/ERASMUS+ project "Let's Live The Olympic Dream Together!" The goal of the eTwinning project was for students to understand better the history and values of the Olympic and Paralympic movements, to explore and try out different sports disciplines, and to learn about the cultural heritage of partner countries (Spain, Italy, Hungary, Denmark, France, and Serbia). The project lasted for two school years, during which students worked in international teams, collaborating on various tasks related to the project theme. There were also tasks in line with the project activities (dedicated to Olympic sports, records, arenas, etc.), promoting healthy lifestyle habits and getting to know the cultural, sports, and industrial heritage of each partner country. In this paper, I will focus on France, which was the last country the students traveled to for mobility. This was not by chance, as the final event took place in France, just like the last 33rd Summer Olympics, which were held in Paris in the summer of 2024. Once again, some primary school students "discovered" Paris not only as the cultural capital of Europe but also as a sports hub through various problem-solving tasks, always incorporating mathematics.

Task 1. European Long-Distance Hiking Trails – Managing the Given Budget

The first long-distance hiking trail in Europe was Hungary's National Blue Trail in 1938. With the formation of the European Union, transnational connections for long-distance trails became possible. Today, the network consists of 12 trails (E1-E12) and covers more than 65,000 km, creating a network across Europe, and is considered European heritage.

Fig. 9. Map of the European Long-Distance Hiking Trail Network

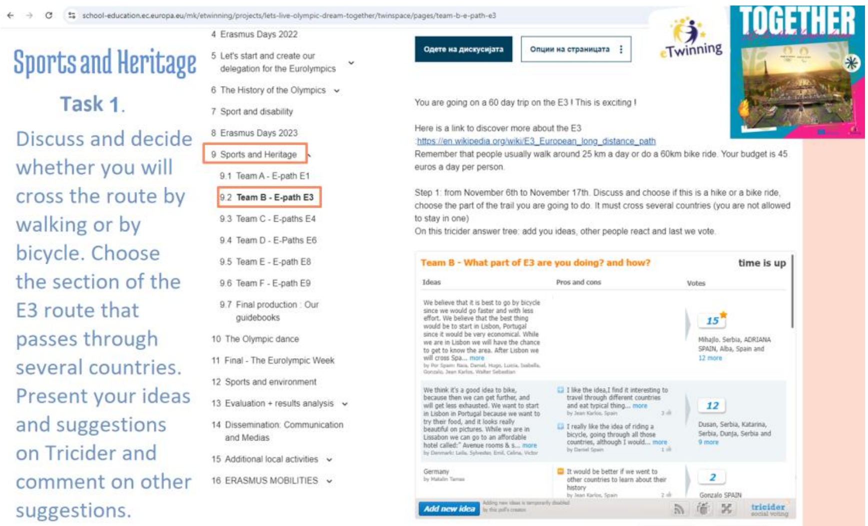


Fig. 9. Tricider, a Web 2.0 tool for idea collection, discussion, and voting

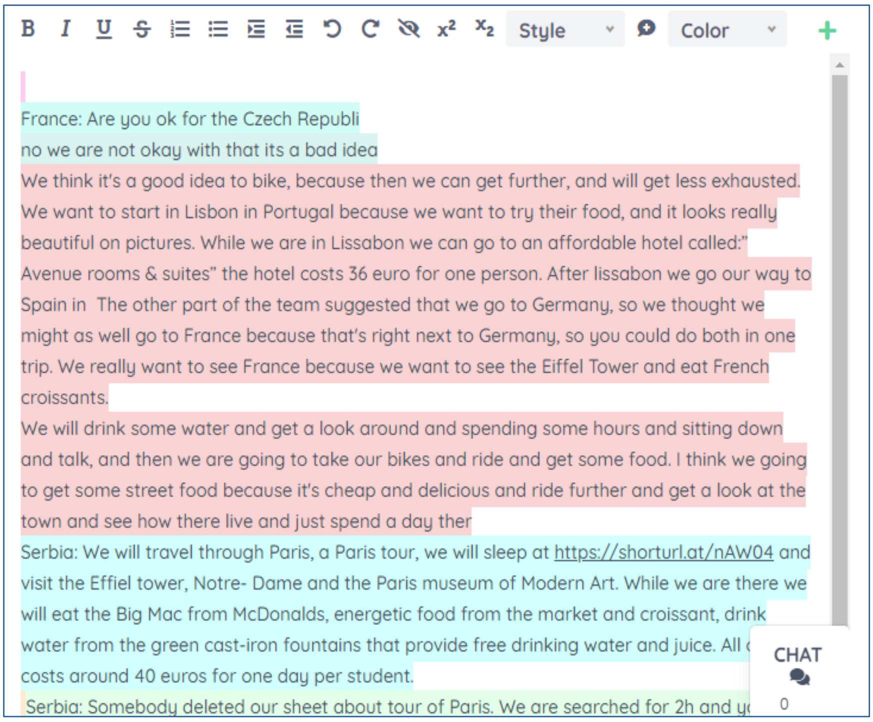


Fig. 10. Web 2.0 tool for collaborative writing

Task 2. Travel Organization Table and Budget Planning.

If you spend less than €45 on one day, you can use the remainder on another day. The total amount per person cannot exceed €2700.

A	B	C	D	E	F	G	H	I	J	K
89	Starting point	finish	Km for that day	place to sleep (add link)	Price	food (add link)	Price	Cultural site to see	Price	TOTAL FOR THE DAY
adèle,france 1	lisbon	igreja nova	31,1 Km		28	Lunch: fries and chiken drumstick dîner:salade of chiken and agua	10,7€	Sea	0e	38,7e
Denmark - Edith	Corullón, Léon, Spain	Ponferreda, Spain	21,9 km	https://admin.google.com/ServiceNotAllowed?hl=da&service=searchandassistant	34,75 euros	grocery store	7 euros	The city	Free	41,75 euros
team Serbia Mihajlo, Katarina, Dunja, Dusan	El gordo Spain	Tavalera de la reina	53 km	https://www.booking.com/hotel/es/pension-la-playa.en-gb.html?aid=2311236&label=en-rs-booking-desktop-ysuYGnJ_UFpsmDsHV0JOxwS65279601760 8	28e (per person)	Caesar salat, fruits and sandwiches (From Lidl)	10e (per person)	https://www.tripadvisor.com/Attraction_Review-g265784-d6018302-Reviews-Iglesia_de_Santa_Maria_la_Mayor-Ronda_Costa_del_Sol_Province_of_Malaga_Andalucia.html	0e	42e (per person)

Fig. 11. Shared Google Document

The challenge of engaging students with different types of problems highlights the need for a comprehensive approach to teaching that recognizes the varying levels of difficulty and the diverse ways in which students learn. This approach must incorporate differentiated instruction, where teachers adjust the complexity of tasks to suit students' individual needs. Additionally, educators should offer varied representations of mathematical concepts - through symbols, graphs, and verbal explanations - so that students can approach problems from different angles and develop a more flexible understanding of the material. This flexibility is key to ensuring that all students, regardless of their learning styles or cognitive abilities, have the opportunity to succeed in mathematics.

By acknowledging the psychological dimensions of learning and the varied levels of task difficulty, educators can design a learning environment that is responsive to students' cognitive and emotional needs. This, in turn, creates a more inclusive and effective approach to teaching mathematics, ensuring that students develop both the knowledge and the critical thinking skills necessary to apply mathematics in real-life situations.

The challenge of engaging students with different types of problems is a key focus of the study. The idea is that while students often feel frustrated or overwhelmed by challenging tasks, they can also experience a sense of achievement and satisfaction when they successfully navigate these difficulties. The phrase "It is so hard – in a good way" encapsulates this mixed emotional response - where students struggle but ultimately gain valuable learning experiences, such as improved problem-solving skills, resilience, and confidence.

This aligns with the broader challenge of engaging students in education. Many students find it difficult to stay engaged with complex tasks, particularly when faced with tasks that require significant effort, persistence, and critical thinking. Halverson's (2019) research suggests that when educators design appropriately challenging tasks, students may feel more engaged, even though they find the work difficult. The sense of accomplishment and competence gained after completing such tasks can be a powerful motivator. This sense of accomplishment also reinforces the importance of persistence, helping students develop a growth mindset - believing that their abilities can improve through effort.

Regarding the different types of problems, it shows that providing students with problems that stretch their thinking, require creative solutions, and involve real-world contexts can encourage deeper engagement. This can help students see the relevance of the material to their lives and foster an intrinsic motivation to learn. The study suggests that the challenge of engaging students with diverse problem types, while difficult, can also help them develop essential life skills like adaptability, critical thinking, and problem-solving abilities, all while fostering an attitude that "it is hard but in a good way".

5. Advantages of eTwinning Projects

eTwinning projects offer numerous benefits for students and teachers, contributing to developing essential skills needed in today's interconnected world. Participants develop key social and communication skills by collaborating on joint projects with students from other countries. Collaborative learning fosters teamwork, builds confidence, and encourages exchanging ideas in a multicultural context. Through the use of ICT tools, students enhance their ability to search, analyze, and communicate information effectively. eTwinning promotes digital literacy by providing students with opportunities to use technology in meaningful ways for research, communication, and presentation. These projects allow students to interact with peers from diverse cultural backgrounds, promoting respect for diversity, global awareness, and intercultural understanding. Exposure to different cultures helps students appreciate various traditions and perspectives. Students are encouraged to express their opinions, make decisions about their learning, and take ownership of their personal development. Some of the key advantages include:

Interdisciplinarity

- Connecting multiple subjects (foreign languages, history, geography, biology-environmental protection, mathematics, art, physics, etc.). eTwinning projects allow students to explore various subjects within a single context, fostering a holistic understanding of different fields. Students can draw connections between subjects by integrating knowledge from different disciplines, making learning more engaging and meaningful.
- Acquiring more lasting and functional knowledge. The interdisciplinary approach in eTwinning projects encourages students to apply their knowledge in real-life situations. This not only enhances the retention of information but also helps students understand

how theoretical concepts are used in practice, leading to a deeper and more functional understanding of the subjects involved.

Collaboration Between Students and Teachers

- Development of teamwork skills. Through collaborative activities, students learn to work together toward a common goal, a crucial skill as in education and future careers. Working in teams helps students improve their ability to cooperate, communicate effectively, and contribute to group discussions.
- Increased motivation for learning and work. The interactive nature of eTwinning projects, combined with the opportunity to collaborate with peers from different countries, motivates students to engage more deeply with the material. Students often feel more connected to the project when they see their work shared with others, leading to greater enthusiasm and commitment to learning.
- Creation of joint content (e-books, posters, route planning). Students actively participate in creating tangible outcomes, such as collaborative e-books or digital posters, shared with peers across different countries. This helps them develop not only technical skills but also creativity and problem-solving abilities.
- Development of social and communication skills that boost students' self-confidence. Working in an international context helps students improve their language skills while interacting with peers from different cultures. Feedback and collaboration build students' self-esteem as they see the value of their contributions in a global context.

Independence and Responsibility

- Fostering student independence in their work. eTwinning projects encourage students to take ownership of their learning. By participating in various stages of project development, from research to creation, students are empowered to make decisions and manage their time effectively, which enhances their ability to work independently.
- Developing personal responsibility for project completion. As students often take part in different aspects of the project, they learn to manage tasks and deadlines. This responsibility helps them understand the importance of completing assignments and following through on commitments - skills essential for academic success and future employment.
- Responsible attitude toward cultural and other heritages. Working on projects with international partners encourages students to appreciate and respect cultural differences, fostering a sense of responsibility for preserving cultural heritage, understanding diverse perspectives, and acting with respect and tolerance in a globalized world.

For educators, eTwinning projects offer the opportunity to collaborate with colleagues across Europe, share best practices, and learn new teaching methods. Teachers can expand their professional network, enhance their skills, and stay updated on innovative educational tools.

6. Conclusion

eTwinning projects significantly benefit students and teachers, helping them develop essential skills for the modern educational system. Students gain a broader understanding of various subjects through interdisciplinarity, while collaboration fosters social, communication, and teamwork skills. These projects encourage creativity, independence, and responsibility, making students active participants in their learning. Beyond academic knowledge, eTwinning projects prepare students to navigate a connected and diverse world. They develop global awareness and essential skills for solving complex problems, positioning them as future leaders. Effective education recognizes diverse learning styles and cognitive processes, especially in mathematics, which requires flexible and adaptive teaching approaches to meet students' needs. Incorporating differentiated instruction and varied representations of mathematical concepts helps students approach problems from multiple angles, improving comprehension and critical thinking. Recognizing cognitive load and task complexity enables educators to create supportive learning environments, equipping students with problem-solving skills for real-life challenges. In conclusion, both eTwinning projects and Richard Skemp's theories place significant value on active, engaged learning that encourages critical thinking, problem-solving, and deep understanding. While eTwinning projects emphasize collaboration, global engagement, and ICT use, Skemp's work focuses more on the cognitive processes in mathematics learning. Together, these approaches highlight the importance of creating inclusive, motivating, and engaging learning environments that not only provide knowledge but also foster the development of life skills necessary for students to thrive in an interconnected world.

Ultimately, both eTwinning projects and understanding how students learn are crucial in shaping education's future. These approaches help students thrive in a globalized world, fostering collaboration, creativity, and adaptability. By guiding students through challenges, both in projects and mathematics, educators nurture resilience and problem-solving abilities, ensuring students succeed and adopt the mindset that "it is hard, but in a good way." eTwinning encourages independent learning and self-directed study, preparing students to become lifelong learners. The skills developed in these projects - such as problem-solving, self-awareness, and adaptability - are essential for personal and professional growth in an ever-evolving world.

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Similarity of Triangles in Integrative Learning

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Abstract

This paper presents a concrete example of integrative learning applied in the special class of talented students in the Makers Lab of the First Grammar School of Kragujevac, Serbia. It was held on May 30th, 2024, as an open school Makers Lab activity titled "Similar Triangles". Students, at the age of sixteen, are from the student class with special abilities in computer science. They were organized into five teams. One of them had the aim to make a geodesic dome model. For this occasion, working together with their peers and teachers, speaking English, they demonstrated the similarity of figures as inspiration for the work of art, but also for fractals, real context examples, and, of course, in math tasks. These activities have been presented on the eTwinning platform Twinspace pages; for the sake of clarity, the eTwinning project "Math without frontiers" is our own project as an innovative approach teaching mathematics, for students and teacher partners.

Keywords: integrative learning, similarity of triangles, art, cross-curricular connections.

1. Introduction

Nowadays, math teaching offers a variety of learning models. We engage students by giving them problems they want to solve because both the question and the answer are relevant to them and their learning program theme. They are especially interested in problems with interdisciplinary learning activities. Teachers, as guides, need to grow student's ability to ask the right questions, formulate math problems with real context, and conduct exploration.

2. Open class of integrative teaching

On May 30th, 2024, the computer science gifted grade I students from the First Grammar School of Kragujevac, Serbia, held an open class, "Similar Triangles", at the school's Makers Lab. Several other teachers and their students also attended the class. Makers Lab of the First Grammar School of Kragujevac is an open laboratory for all creatives looking for a place to realize their ideas. It is a science place for students, teachers and visitors with technical materials, tools, workspace, and computers. They can get professional help and advice from colleagues with whom they can develop their projects. It is a place that can awaken their inspiration and allow them to network, share experiences, and use knowledge.

For this occasion, they have applied integrative learning, as a process of making connections among concepts and experiences from different subjects, where teaching content has been linked from subjects such as Mathematics, English, and Art. Using problem-solving, they have developed skills such as cooperation and communication, inquiry-based learning, creative thinking, using heuristic teaching system, individual discovery, creative work, interactive exemplary teaching, digital competences and practical teamwork skills, making geodesic dome model.

Teaching methods: illustration and demonstration, method of student's written works, game activities, quizzes, project method, games for the purpose of learning, student's presentation, discussion and debate, student's research and method of practical work, experimental work on the construction of a geodesic dome model.

Plan of the activities: For each of the four teams, the topic for the interactive presentation has been chosen in such a way that the team would create it to show their results of applying the similarity of figures inspired by artwork, by fractals, with discussion of real context problems, or using similarity of triangles in math tasks. The fifth team chose to build a model of a geodesic sphere out of newsprint, based on an icosahedron with a selected frequency 2 – the International Mathematical Art Movement Workshop for Experience-Based Mathematical Education (Fig. 10).

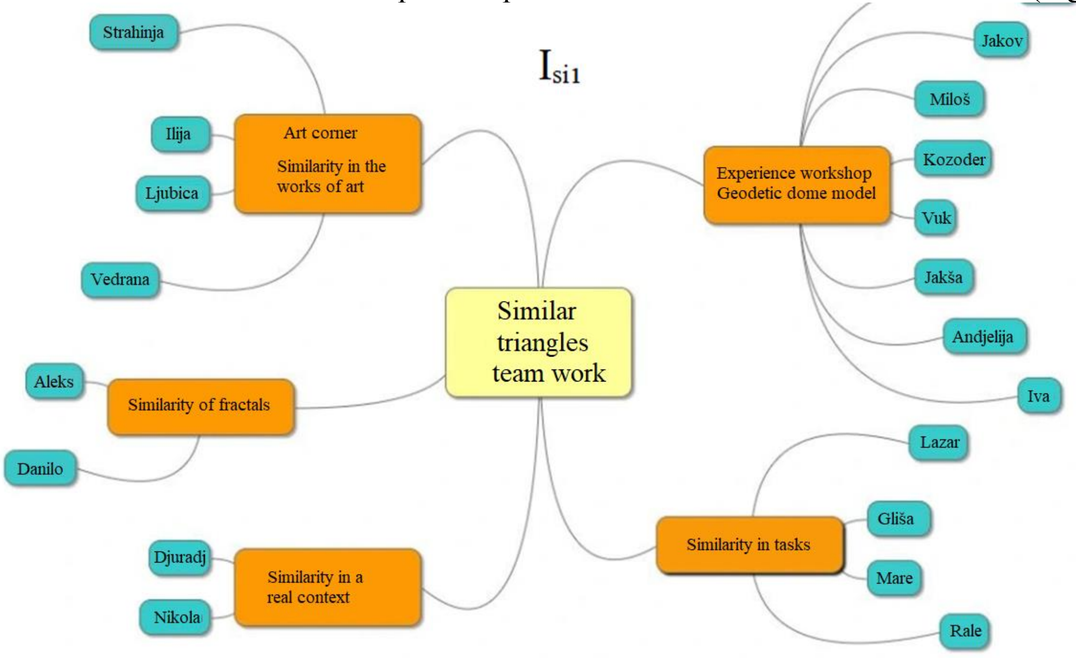

Fig. 1. Team compositions

It is one of the activities within our eTwinning project, “Math without frontiers”, that has connected students and teachers from Serbia, Greece, Croatia, Turkey, Moldova, and Georgia. For participation in this project, we have been awarded the eTwinning National Quality Label and eTwinning European Quality Label.

Realization of the activities: Here are the products of student’s teamwork discussed in English with visitors, as well as the teamwork illustration of the experience workshop and the geodetic dome model.

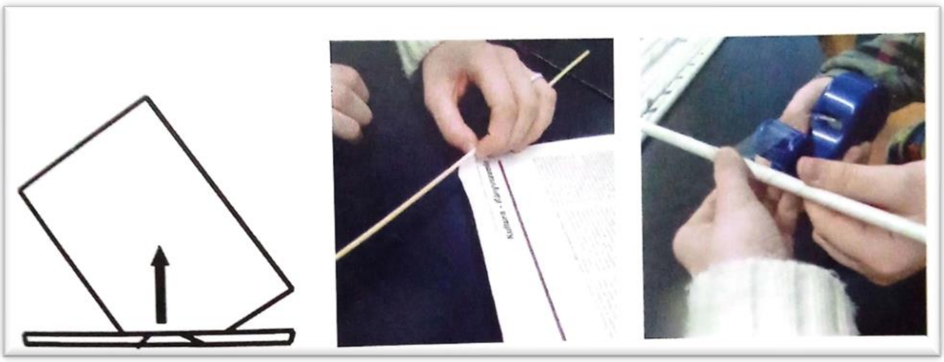

Fig. 2. Preparing materials for the workshop

For a successful lesson start, the fifth team provided workshop materials: newspaper for 35 sticks 65 cm long and 30 sticks 58 cm long, 10 templates for the inner corner of a regular decagon, scotch tape, and a stapler.

3. Presentations of results

In the sequel, the works and results of all five teams are illustrated.

1. The theme for the first team was: SIMILARITIES WITHIN WORKS OF ART. They presented several works of art connected with mathematics and similarities (Fig. 3).

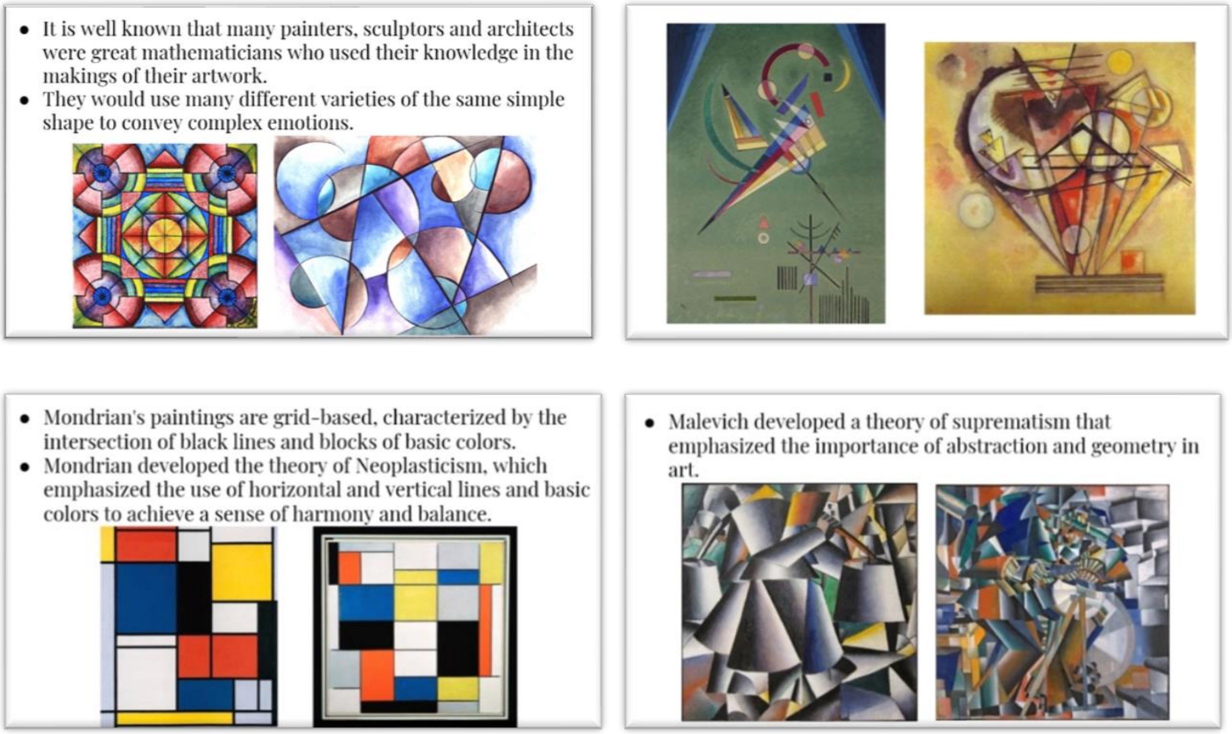

Fig. 3. Art corner teamwork

2. The theme for the second team was: THE SELF-SIMILARITY OF FRACTALS. They presented several famous math fractals and in nature and explained the fractal property of self-similarity (Fig. 4 and 5).

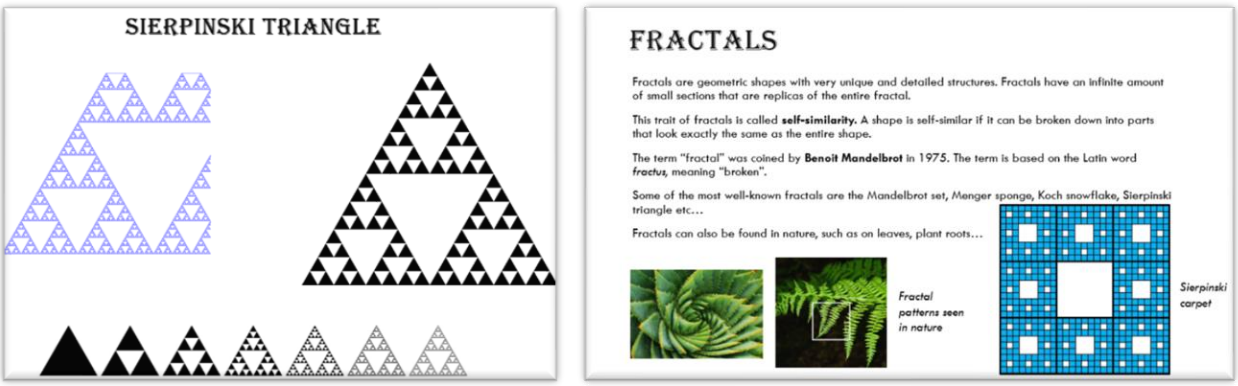

Fig. 4. Self-similarity of fractals team work

Fig. 5. Examples of fractals

3. The theme for the third team was: SIMILARITY IN A REAL CONTEXT. They presented similarity of triangles and application in real life. (Fig. 6 and 7).

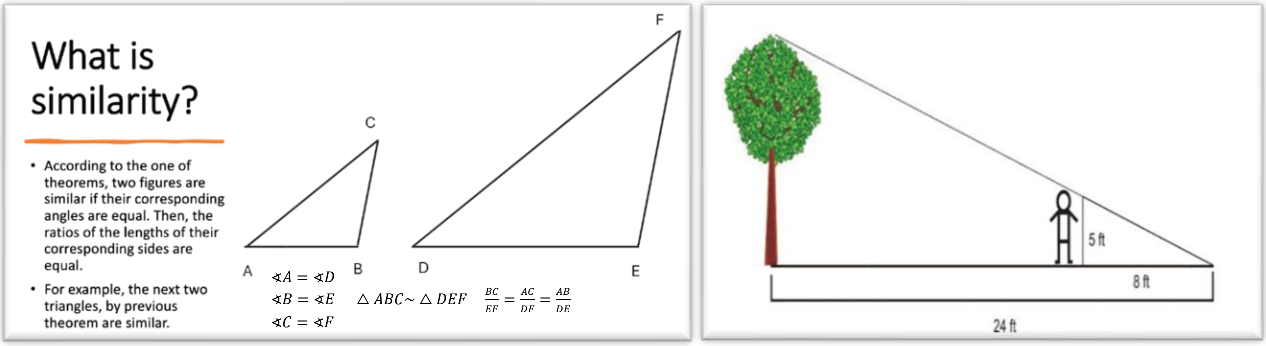

Fig. 6. Similarity of triangles

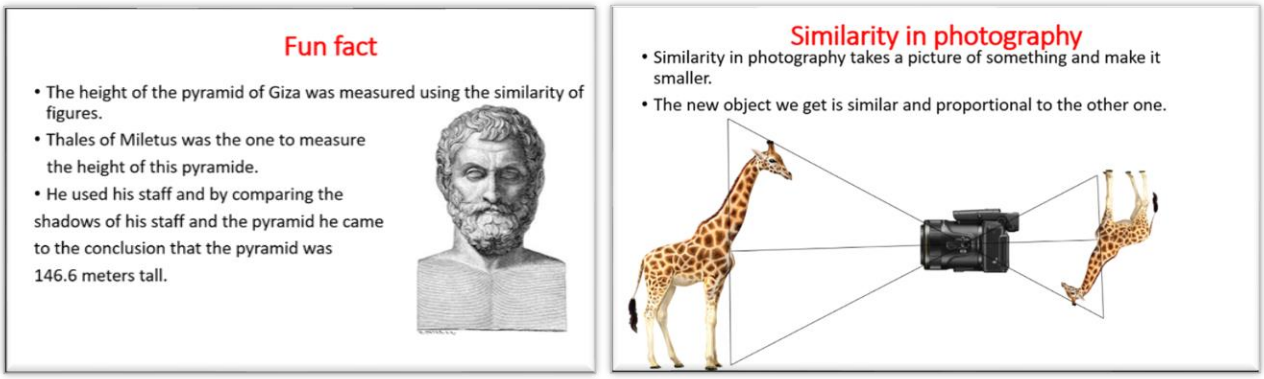

Fig. 7. Application of similarity in real context

4. The theme for the fourth team was SIMILARITY IN MATH TASKS. They presented the similarity of triangles in a typical mathematical context (Fig. 8 and 9).

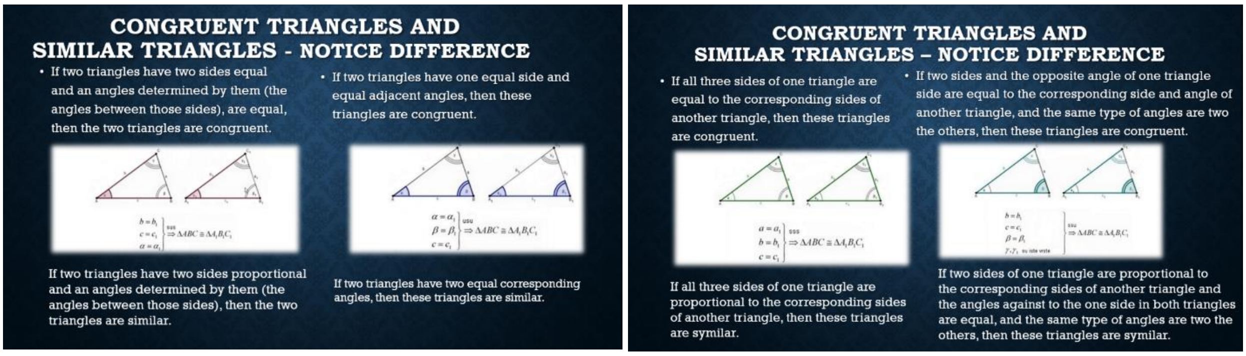

Fig. 8. Congruent and similar triangles

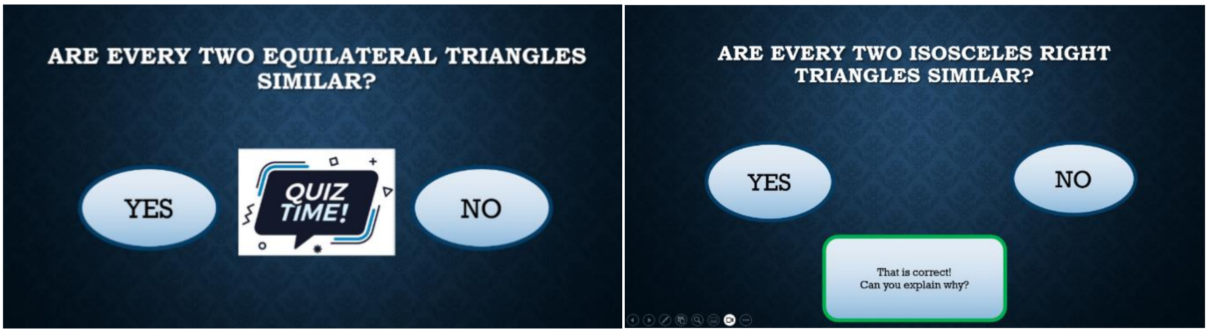

Fig. 9. Quiz about similarity

5. The theme for the fifth team was: EXPERIENCE WORKSHOP GEODETIC DOME MODEL. They presented how using similar triangles with multiple less sides of the equilateral triangle, based on the icosahedron, the difference between a geodetic dome and a geodetic sphere becomes less (Fig. 10). Also, they constructed a geodetic dome model.

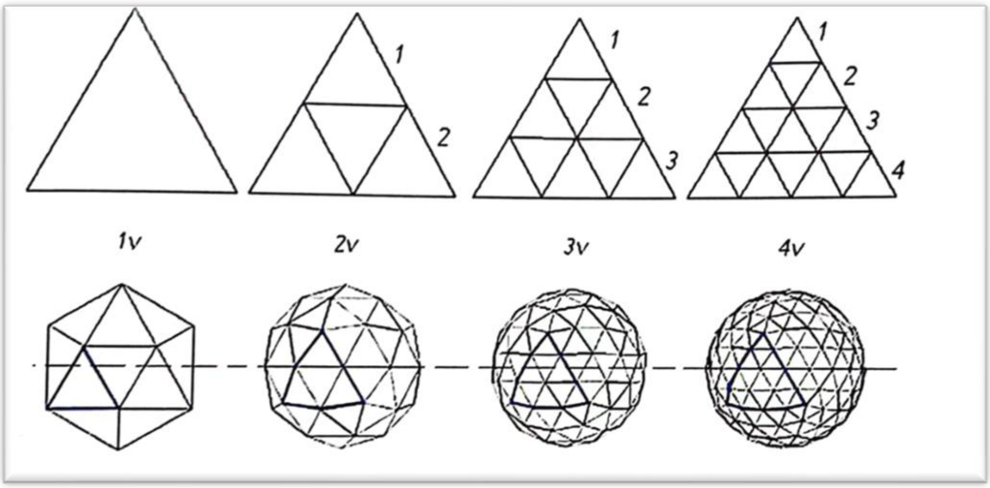

Fig. 10. Geodetic sphere based on icosahedron with frequency numbers 1, 2, 3, and 4 and their construction. (Siladi, 2012, p.129)

Fig. 9. Some steps in construction and the final product in the school Makers Lab

4. Conclusion

The impression of the students and classmates of this exemplary lesson would be described in this way: this meeting of mathematics, English, art, and practical student skills of cooperation and communication was unusual and instructive. Students learn by doing.

Ana Veljković, English teacher who actively supported students in their work and participated in the class, and I are grateful to all colleagues and their students who dedicated their time to accompany these interdisciplinary activities in math class. For the next occasion of applying this class scenario, we would rather plan two school hours and more time for preparing materials for making a model of a geodetic dome. We are proud of the product of our lesson and it still decorates the school Maker lab.

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Application of Differential Calculus in High School Physics Teaching

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Abstract

In physics education at all levels, a distinct lack of correlation between the teaching content of mathematics and physics is evident. In the high school physics curriculum, there is a pronounced misalignment between teaching the content related to differential and integral calculus and teaching the content of various areas of physics covered during high school education. The two most significant consequences of this are: 1) the majority of high school students demonstrate an insufficient level of knowledge and skills needed for studying physics and related disciplines at the university level; 2) the majority of high school students show a low level of competencies required for the application of differential and integral calculus, especially in connection with students' direct experience and various natural phenomena. This paper will present an example that demonstrates the author's attempt to address this problem in a class of students with special abilities in mathematics at the First Grammar School of Kragujevac.

Keywords: differential calculus, interdisciplinary approaches, application of derivatives and integrals in physics.

1. Introduction

The main motives for writing this paper are, on the one hand, the misalignment of physics and mathematics curricula at almost all levels of education, both in Serbia and in most other countries, and on the other hand, the strong insistence on linking the content of different subjects in a formal and bureaucratic manner within our educational system. The most significant consequences of this misalignment are difficulties in mastering physics and related disciplines during the first year of studies at faculties in the fields of technical and natural sciences, as well as poor integration and application of differential and integral calculus in the description of immediate natural phenomena. The mentioned bureaucratization of the teaching process leads to a lack of motivation among teachers and inadequate organization of the educational process in schools in terms of introducing interdisciplinary approaches.

Research on interdisciplinary connections in education, particularly when it involves students with special abilities in specific subjects, tends to be more limited for several key reasons such as lack of large-scale studies and institutional barriers. Evidence supporting this scarcity can be found in various studies and reviews. For instance, a systematic review by Tonnetti and Lentillon-Kaestner (2023) highlighted that while the concept of interdisciplinarity has been widely discussed, actual implementation in secondary schools remains rare. The review analyzed 40 studies and found that few schools achieve a real integration of disciplines. The gap in literature reflects a broader trend in educational research, where more attention is given to general strategies for students with special needs, with less focus on specialized approaches such as interdisciplinary teaching for gifted or talented students.

This paper will present an example of an attempt to overcome these issues in the teaching of physics in a fourth-grade class of students with special abilities in mathematics at the First Grammar School of Kragujevac during the 2023/24 school year.

2. Method of implementation

The starting point is to determine the most important topics in high school physics and mathematics that serve as a solid foundation for the teaching of physics and related disciplines at the university level. Standard first-year university courses typically cover various areas of mechanics and electromagnetism, followed by thermodynamics, and, to a lesser extent, optics and modern physics. Mechanics is the most natural choice, given that the physical meaning of derivatives in mathematics is most easily observed through the description of motion. Differential calculus is the first, most essential, and most important mathematical tool that first-year students at the university level encounter in physics and related disciplines. Therefore, the application of differential calculus in high school physics, especially in mechanics, is the most suitable choice for conducting a model lesson that would demonstrate an example of how to overcome the difficulties faced by first-year students.

In order to conduct such a lesson, there needs to be good cooperation between the math and physics teachers teaching the same class of students. However, the organization of the teaching process also needs to be flexible and adapted to interdisciplinary approaches, which is generally not the case in the Serbian education system. Nevertheless, since collaboration between teachers is at an exceptionally high level at the First Grammar School of Kragujevac, there were no issues regarding the scheduling of classes. It was agreed that this would be a model two-hour lesson held during the analysis with algebra classes, as part of the review and systematization of the material related to the derivative of a function. Many colleagues teaching math and physics, as well as representatives from the Psychological-Pedagogical Service, attended this model two-hour lesson.

The introductory part of the lesson was dedicated to reviewing the basic properties of the derivative of a function of one real variable and some of the fundamental theorems of differential calculus, with special emphasis on Fermat's and Rolle's theorems. Digital teaching materials, such as resources available on GeoGebra and similar platforms, were used. This was followed by a review of the basic concepts of kinematics of a material point — position, displacement, velocity, and acceleration.

The main part of the two-hour lesson began by introducing a different approach to teaching the derivative of a function than the usual one. One-dimensional motion of a material point was considered to avoid vector functions, focusing instead on the dependence of the material point's coordinate on time (it is common to assume that in one-dimensional motion, this occurs along the x-axis of the given reference system). In this way, the continuity of the function is naturally explained, and how, through a limiting process as the time interval approaches zero, the instantaneous velocity is obtained as the derivative of the point's displacement concerning time, and the velocity projection on the given axis is the derivative of the coordinate with respect to time.

This concept was then generalized to the case of motion in a plane, where digital tools were used to visualize the transition from a secant of the path to a tangent as the time interval of motion approaches zero, that is, obtaining instantaneous velocity from the average velocity. Fermat's and Rolle's theorems were then explained using the same example of motion along a straight line at constant and variable speeds. By plotting the graph of the point's coordinate depending on time, when the body moves in both directions along the x-axis, it becomes easy to see when the function has a maximum and when at the maximum point the derivative exists or does not exist, based on whether the velocity only momentarily changes direction. It is evident that there is no unique tangent at the maximum point in the case of the momentarily change of velocity direction, but when the velocity changes in intensity over a time interval before and after stopping, it clearly shows that in this case, the tangent at the maximum point is parallel to the time axis, illustrating the physical meaning of Fermat's theorem. It is also clearly observed that the point must stop to return to its starting position, thus explaining the physical meaning of Rolle's theorem.

The main part of the two-hour lesson was dedicated to solving carefully selected computational problems that illustrate the application of derivatives in various areas of physics, primarily in mechanics. These types of problems are exactly what first-year university students can expect in different physics and related discipline courses. The students were given a sheet with seven problems — three in the field of mechanics, three in thermodynamics (connected to mechanics), and one in electromagnetism. The texts and solutions of the problems will not be provided in this paper; therefore, interested readers are directed to the author or the references. Each of the 14 students in the class was given one problem to try to solve individually. The problems were assigned to students based on their prior knowledge and individual inclinations. After completing that task, they were free to work on the other problems individually, in pairs, or groups, according to their choice. The teacher supervised the students' work, providing suggestions and guidance, and correcting certain steps in the problem-solving process when necessary. Since some students in the class had been attending additional physics lessons since the first year, where they had already had the opportunity to use differential calculus, these students also helped others solve the problems.

Each problem, in addition to determining the first and, in some cases, the second derivative of various functional dependencies that the students arrived at during the problem-solving process, illustrated other applications of differential calculus: determining the maximum or minimum value of a physical quantity in a given motion or process, differentiating the obtained expressions to determine the interdependence of certain physical quantities, writing equilibrium conditions for infinitely thin layers of a substance, using the derivative of a composite function to determine the required dependencies of physical quantities, and so on. The final problem also served as an introduction to the definite integral, as it required the students to divide a physical body into infinitely small parts and, in the final step, perform a simple summation.

At the end of this part of the lesson, three students presented their solutions to the rest of the class on the board, accompanied by an analysis and discussion of the problem-solving process in which all students participated. The problems that students did not solve were given as homework assignments. For homework, they were also asked to solve the same problems using alternative methods, as well as to draw additional conclusions from the solutions beyond those discussed in class.

The final part of the lesson summarized the activities and feedback, and the students and colleagues present were given a Google survey for evaluation, which was later analysed by the subject teachers. The general impression was that the model two-hour lesson was successful — all students were engaged and made efforts to work independently. They successfully connected the content from mathematics and physics and understood why and how differential calculus is used in various ways in different areas of physics. The plan is to continue to integrate the application of derivatives and integrals in physics throughout the school year, as it has been the case in previous years with classes for students with special aptitudes in mathematics, in order to prepare them for further education at faculties in the fields of technical and natural sciences.

Some of the problems that the students solved during the given lessons, including the solutions, were published in the paper by Milenković and Momčilović (2024). Additionally, the mentioned paper also states that some teachers were surveyed after the lessons on how often and in what way they implement interdisciplinary connections in the curriculum. However, the sample of respondents was not representative enough to obtain relevant results; therefore, the obtained results will not be discussed in this paper and will be the topic of future studies.

3. Final Considerations

In addition to demonstrating the application of differential calculus in physics lessons, this two-hour lesson also attempted to highlight the continuity of natural processes, which makes mathematical analysis a necessary tool for describing nature. It also aimed to help students understand how this is done in physics as a fundamental natural science — through simplification and modelling by distinguishing important from less important facts, using the most obvious example from nature: mechanical motion.

The main issues that limit the frequency of such lessons are the heavy administrative burdens imposed on teachers, which are largely unnecessary and non-functional, and the limited weekly hours allocated to physics classes in all high school tracks, except in the first and third years for students with special abilities in mathematics and partially in the fourth year of this and the natural sciences track (this was also the reason why this two-hour lesson was conducted during analysis with algebra classes). Additionally, in all tracks except for students with special mathematical abilities, differential calculus is only introduced in the fourth year. The author hopes that this example will inspire colleagues in other high schools to develop their own ways of implementing differential and integral calculus within physics lessons, as well as methods for connecting mathematics and physics material in math classes.

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Puzzles in mathematics lessons

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Abstract

How can puzzles be used in mathematics teaching in geometric and algebraic content? The use of assembling geometric shapes will be demonstrated in the teaching units on Angles along the transversal, Polygons, and Operations with Polynomials, as well as in various extracurricular activities. Additionally, puzzles have been used in some model lessons. They introduce children to new areas or help systematize already-covered material through play. They also encourage peer learning and teamwork. Both ready-made shapes and those created by students and some digital tools (Tarsia) have been utilized. They are suitable for working with children who follow individual educational plans. The poster will showcase the materials used and some photographs from these lessons.

Keywords: puzzle, geometry, algebra.

1. Introduction

In modern teaching, it is essential to use various methods and forms of work to keep students focused on the content. Different educational games can help with this, and various types of puzzles can also be used.

It has been shown that puzzles support learning through play and contribute to developing mental abilities: perception, memory, problem-solving, concentration, and observation. They can also be used with preschool children because they contribute to early development. Research shows that students who learn using this type of didactic tool have greater potential. The conclusion is that puzzles in mathematics teaching contribute to the development of critical thinking, cooperation, motivation, and engagement in understanding abstract concepts. They can be used in both algebraic and geometric content (Tangram, Tarsia, Pentomino, mosaics, Hatching the Egg …).

2. Lesson presentations

2.1. Angles along the transversal

This paper showcases the use of puzzles in several different lessons. In the Angles with a Transversal lesson, students were tasked with drawing three identical rectangles at home, coloring them with different colors, and cutting them out. They then arranged the pieces one beneath the other and drew a transversal over them (see Fig. 1). They cut out the resulting parts, combined pieces of different colors, and thus concluded that all acute and obtuse angles formed by the transversal are equal.

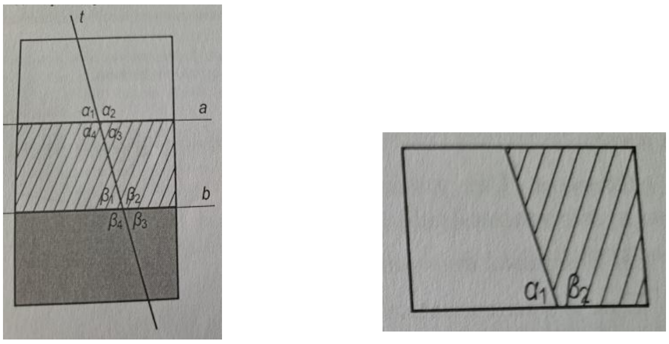

Fig. 1. Angles on a Transversal.

In the following lessons, whenever angles appear along the transversal, the students comment that this is what they cut and assembled and immediately know the rule. What can be a problem is that not everyone comes to class with prepared material, but that can be solved.

2.2. Polygons

In the lesson on Polygons, students were introduced to the basic concepts related to polygons, and then they created different types of polygons from smaller parts (see Fig. 2). In the following lessons, they will connect various properties of polygons with what they observed from those puzzles. Students work in pairs, and the teacher only supervises and occasionally gives instructions. The students' impressions are that they find this way of working interesting, and in subsequent lessons, it was seen that what they observed while arranging polygons can also be recognized in formalized properties. The parts that make up the figures are prepared in advance and can be made on a 3D printer.

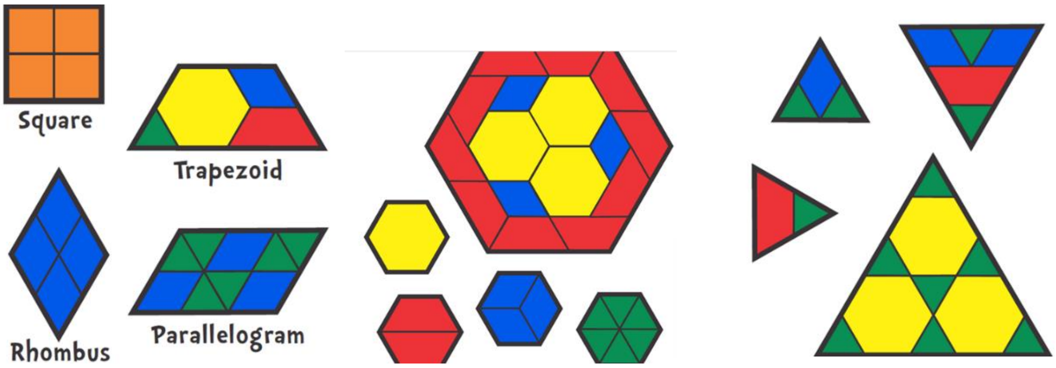

Fig. 2. Images of colored polygons.

This activity can also be used when working with quadrilaterals to establish students' knowledge of the types of quadrilaterals.

2.3. Tarsia

Using the Tarsia tool, students created a puzzle by matching the pieces by their sides to equal lengths. For the puzzle, students needed to match pieces that contained equal polynomials on their sides. By correctly assembling all the pieces, they formed a regular triangle.

The students are divided into heterogeneous groups, but in such a way that each one has one with better achievements in mathematics so that he or she can help the others. Students work on different examples and then try to find a polynomial equal to the given one and put the parts together. By correctly assembling all the pieces, they formed different polygons.

Observing how each group works is interesting because certain students have good organizational skills. It is also important to see if the students notice that some parts have polynomials on only one edge and that such parts are likely to be somewhere on the perimeter of the figure (see Fig. 3). In such a way, Tarsia connects different areas of mathematics, geometry and algebra.

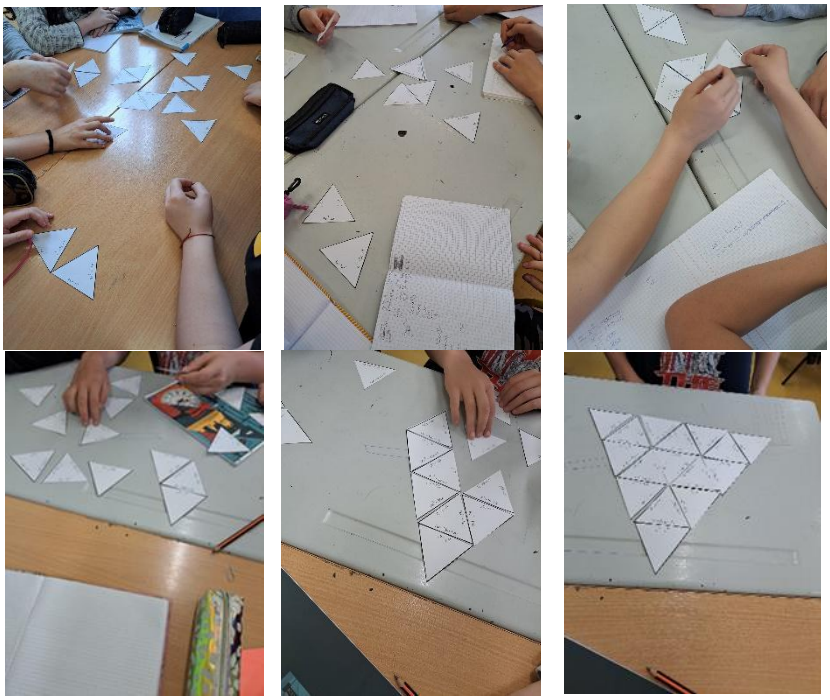

Fig. 3. Assembling triangle with Tarsia models

2.4. Extracurricular activities

Extracurricular activities also play an important role in the overall development of students' competencies. They contribute to learning about the specific area, but also to developing critical thinking, improving cooperation among students, problem-solving skills, motivation, engagement, and creativity**.**

The remaining images (see Fig. 4 and 5) showcase the students' works created in various workshops. In these workshops, students used polygons to create different shapes, thus expressing their creativity. The material is prepared in advance, and the students are free to assemble whatever they want from the parts. Some students have created small works of art. During the work, the teacher can also use short questions to review the content related to the geometric objects that the students are working with.

Fig. 4. Some students' work on extracurricular activities

Fig. 5. Another students' work on extracurricular activities.

Polyominoes are also good tool for encouraging logical thinking and developing problem-solving strategies. The pictures show activities with polyominoes. Students assemble squares of different dimensions from polyominoes. They start with the smaller ones, and when they overcome them, they move to the next level.

Students can work individually or in small groups. It is also suitable for organizing a puzzle competition, and students of different ages can work together. After these workshops, students like to comment on how they solved the problems and what strategies they used.

After the first such workshop with fourth-grade students, I monitored their achievements, and it turned out that those who were more successful then are still achieving excellent results now (that generation is now in their final year of high school).

Fig. 6. Students' work with polyominoes.

3. Conclusion

Mathematics teaching must be innovated to maintain students' interest. It is very important to occasionally change the approach and allow students to explore a bit, thus introducing them to new concepts. Additionally, review lessons should be refreshed with different methods to keep students focused on the content. A successful strategy could also be using games in introductory lessons on the topic and then reminding them of what they observed during those lessons when defining a specific object or explaining concepts.

Extracurricular activities play a very significant role in educational work, so they should be utilized more. They can contribute to greater interest in the subject and foster student creativity. They can also be a good tool to encourage collaboration, communication, and critical thinking.

Both in regular classes and in extracurricular activities, puzzles as a form of play can be useful in engaging the full capacity of students. They are useful as a support for learning through play, which contributes to the development of mental abilities such as analysis, observation, and noticing the relationship between parts and wholes. At a younger age, they can promote creativity, critical thinking, and problem-solving and can also influence a students' cognitive and motor development.

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Application of New Technologies in the Teaching of Informatics

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Abstract

In modern society, which is rapidly developing under the influence of new technologies, it is important to recognize the importance of applying new technologies in teaching computer science. Changes in society and the work environment require students to develop different skills and competencies that are supported by technological tools. The aim of this research is to examine the application of new technologies in the teaching of informatics and to investigate their effects on the learning process. Special emphasis is placed on the analysis of technology's impact on students' results and their motivation for learning. The research was conducted through a mixed methodological approach that includes quantitative and qualitative methods through a survey of 150 participants. The participants were teachers and computer science students from various educational institutions. The results of the research showed that the application of new technologies in the teaching of informatics can have a significant impact on the learning and motivation of students.

Students who had access to new technologies had better test results and higher levels of engagement in the teaching process. Also, students expressed greater interest in the subject and greater willingness to participate in learning activities when the teacher applies new technologies in teaching. Based on the obtained results, it was concluded that applying new technologies in teaching informatics is essential for improving the educational process. Implications for modern educational processes include the need for teacher training for the successful integration of technology into teaching. Also, it is important to promote a culture of innovation and exchange of good practices among educational institutions.

Keywords: educational resources, new technologies, digital literacy, innovations in teaching, teaching computer science.

1. Introduction

The application of new technologies can help teachers by saving time, improving the quality of work, and contributing to the personalization of experiences. The model for improving the teaching process is "understanding the needs of students" (Shyr et al., 2024), more precisely, "identifying areas where new technologies can help improve the learning experience". Applying new technologies in education should enable more purposeful, dynamic, and interesting learning (Kumar, 2019).

New technologies in the teaching of informatics are applied to improve the quality of teaching, that is, to improve educational programs and curricula. Also, it can help improve learning tools, such as games, simulations, and interactive applications, and be useful in distance education, providing personalized support to pupils/students and thus contributing to their greater interest in learning and better understanding of the material (Canbek & Mutlu, 2016). The use of artificial intelligence (AI) tools is very prevalent in teaching, both in primary and secondary education, as well as in higher education (Fig. 1).

Fig. 1. Personalized learning

Many digital tools are used in computer science education, and their goal is to provide students with practical insight into technologies and their applications. Some tools that have proven useful are:

1. Tools for programming and coding: Scratch - a visual programming language, Code.org - an interactive platform with lessons and projects for learning the basics of programming, Replit - an online development environment, Python IDEs - one of the most popular languages in education.
2. Tools for learning algorithms and logical thinking: Blockly - a tool similar to Scratch that allows students to develop algorithms using blocks instead of code, Tynker - a platform for learning programming with interactive lessons and projects, AlgoLab - allows simulation and visualization of algorithms so that students understand their work.
3. Tools for robotics and IoT: Arduino - a development platform for learning electronics and programming, Raspberry Pi - a mini-computer that students can use to learn electronics, coding and IoT, LEGO Mindstorms - a tool for learning robotics through building and programming LEGO robot.
4. Tools for simulation and visualization: Tinkercad online tool for 3D design and modeling, Fritzing - a tool for creating electronic schematics and simulations, Kahoot! and Quizizz - platforms for creating interactive quizzes for students to test their knowledge in a fun way.
5. Platforms for collaboration and project work: Google Workspace for Education - includes tools such as Google Docs, Slides, and Google Classroom for teaching organization and collaborative work, GitHub - a platform for working on code, ideal for group projects and learning version control.
6. Artificial intelligence in education (ChatGPT) - helps students understand complex concepts and solve problems, AI simulations (eg Teachable Machine) - Google’s tool for learning the basic principles of machine learning through practical projects.
7. Platforms for games and simulations: Minecraft: Education Edition enables students to learn programming, design, and collaboration through play, Roblox Studio - a tool for creating games and learning the basics of programming.

These tools can significantly enrich the teaching of computer science and make it more interactive and relevant while giving students the practical skills needed for the modern technological era.

The gradual ascent and evolution of AI were not abrupt occurrences. As artificial intelligence exerts a greater impact on humanity, the urgency to comprehend it becomes more pronounced. AI has the capacity to enhance human learning, skill development, and overall performance, thereby facilitating proficiency and efficacy in various tasks and endeavors (Yang et al., 2024, p20).

2. Methods

Using a combined research design, the study included a sample of N=150 participants from primary and secondary schools and colleges. The questionnaire was constructed by combining an adapted attitude scale with a set of original questions. The instrument is divided into two parts: I - Demographic characteristics of the sample (5 questions) and II - Questions related to teachers' attitudes about applying tools based on artificial intelligence (10 questions). In the second part, an adapted scale for AI was used.

The questions are related to the use of artificial intelligence in teaching, including solving tasks from different subjects, the types of tasks for which artificial intelligence is most often used, as well as the method of assessment. The data was collected through an online survey conducted in May 2024. Students and teachers from the elementary school in Ivanjica, technical secondary school in Ivanjica, and professors and assistants of the Faculty of Technical Sciences in Čačak participated in the research.

The research was conducted through a mixed methodological approach, including literature analysis, review of existing studies, and empirical research in the form of a survey. The questionnaire used in this study was meticulously developed by combining an adapted attitude scale with a set of original questions. The adapted attitude scale was tailored to capture students' perceptions and attitudes toward integrating AI in educational settings (Humble & Mozelius, 2019).

3. Key steps and strategies for teacher preparation

Preparing teachers for new technologies in computer science teaching is crucial to ensure that teachers are able to impart current and relevant knowledge to students. This process requires a comprehensive approach that includes various aspects of professional development (Ashaeryanto et al., 2017).

Some of the key steps and strategies for preparing teachers for teaching, which are represented in the schools and colleges where the research was conducted, are:

1. Identification of needs

- Assessment of current knowledge: Assessment of teachers' current level of technological literacy and competence.
- Needs analysis: Identification of specific areas in which improvements are needed, such as new software tools, programming, cyber security, etc.

2. Creating a customized training program

- Modular Curriculum: Developing a modular program that allows teachers to choose modules according to their needs and interests.
- Interdisciplinary approach: Integrating technology with other subjects to create a holistic approach to education.

3. Teaching and learning methods

- Hybrid learning: A combination of online and traditional learning to allow for flexibility and availability of resources.
- Practical workshops: Organizing workshops and laboratories where teachers can apply the acquired knowledge practically.
- Mentoring programs: Guidance through the mentoring work of experienced teachers or industry experts (Azizovic, 2023).

4. Use of advanced technologies

- Introduction of new tools: Training in the use of new software and hardware tools, such as VR (Virtual Reality), AR (Augmented Reality), AI (Artificial Intelligence), and -IoT (Internet of Things).

- Learning platforms: Training in using digital platforms for learning and teaching management, such as LMS (Learning Management Systems), (Lin et al., 2021).

5. Evaluation and monitoring of progress

- Pre- and post-testing: Measuring the level of knowledge and skills before and after the training.

- Feedback mechanisms: Regular collection of feedback from teachers to identify problems and opportunities for improvement (Shyr et al., 2024).

6. Networking and exchange of experiences

- Professional networks: Active involvement in professional networks and communities for exchanging knowledge and experiences.

- Conferences and seminars: Participation in conferences, seminars and workshops for continuous professional development and networking.

7. Integration with the curriculum

Curriculum updating: School curricula are regularly updated to include the latest technologies and trends in informatics.

- Interdisciplinary teaching: Integrating technology into the teaching of other subjects to create a relevant and practical learning environment (Brown and Johnson, 2019).

8. Support from the administration

- Financial support: Ensuring the necessary financial resources for training and technological infrastructure.

- Strategic plan: Development of a long-term strategic plan for the integration of new technologies into the educational system.

4. Results

The main part of the questionnaire was related to the investigation of the application of new technologies in teaching Informatics in primary and secondary education and on faculty. Although AI tools can bring significant benefits to education, educators face a number of challenges and obstacles:

Technological literacy – Many teachers lack the necessary knowledge and skills to effectively use AI tools, which requires additional training and professional development.

Data privacy and security – Using the AI tool involves collecting and processing a large amount of data about pupils/students, which poses challenges in protecting the privacy and security of that information.

Resistance to change – A number of teachers may be skeptical of new technologies and reluctant to abandon traditional teaching methods, which may hinder the wider application of the AI tool.

Ethical Issues and Bias – AI systems can be subject to biases that exist in the data on which they are trained. Teachers must be aware of these risks and able to recognize and address potential problems (Chen et al., 2020).

Based on the survey results, the most frequently used artificial intelligence tools were selected (Table 1). The most frequently used devices in the teaching of computer science are shown in Fig. 2 and were obtained based on the results of a survey.

Table 1. The most frequently used artificial intelligence tools.

AI tool	Percentage
Intelligent systems for teaching	3%
Natural language processing	7%
Smart content creation	28%
Learning foreign languages	10%
Chatbot	52%

The analysis shows significant differences in applying different AI technologies based on the presented data on the most commonly used artificial intelligence tools. The results are as follows:

Chatbots (52%) – Most commonly used tools More than half of users prefer chatbots, making them the most popular AI tools. This indicates their wide application in education, customer support, and communication automation. Chatbots are popular for providing instant responses, personalizing the learning experience, and reducing the need for human intervention in repetitive tasks.

Smart content creation (28%) The next most used AI tool is related to creating content with the help of artificial intelligence. This includes generating teaching materials, lesson summaries, and creating interactive learning elements. The popularity of these tools can be attributed to their effectiveness in reducing the time required for lesson preparation.

Learning foreign languages (10%) AI tools for learning foreign languages, such as the Duolingo or Rosetta Stone applications, take third place. Their applications enable personalized access to students, use of voice search, and automatic error correction.

Natural Language Processing (7%) NLP technologies, although powerful, are less commonly used. They are most often used in automatic translation, text analysis, and sentiment analysis. Limited use may be due to implementation complexity or specific needs.

Intelligent teaching systems (3%) – The least used These tools have the smallest share of the total use. This may indicate challenges in integrating these systems into educational processes or their specificity, which is intended for narrower user groups.

The data shows that the application of artificial intelligence is largely directed towards tools that offer direct interaction (chatbots) and tools that facilitate the daily tasks of teachers (smart content creation). On the other hand, the lower participation of intelligent systems for teaching and NLP indicates the need for further education of users and development of accessible solutions.

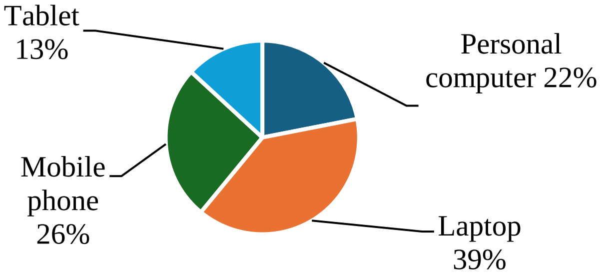

Fig. 2. The most frequently used devices

Laptops dominate because of their versatility, and their popularity could grow as educational programs increasingly rely on digital resources. Cell phones are affordable, but careful consideration should be given to their integration into teaching to avoid problems such as distraction. Desktop computers remain key in specialized environments but may become less prevalent due to the increased portability of laptops. Tablets have the potential to increase usage, especially if specific applications are developed to suit educational needs. For the optimal use of devices in the teaching of informatics, schools, and faculties should adapt the choice of devices to the specific needs of the subject, taking into account the available resources and learning objectives.

Teachers also evaluated their competence in using different devices for applying artificial intelligence tools in teaching. The results were as follows: very low – 19%, low – 32%, weak – 45%, high – 3%, and very high – 1%, (1 – very low, 2 – low, 3 – weak, 4 – high, 5 – very high). These ratings suggest that a significant majority of teachers feel underprepared to effectively use AI tools in their teaching, with almost none feeling highly competent (Fig. 3).

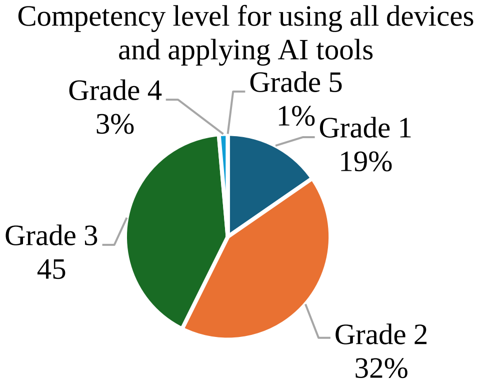

Fig. 3. Competency level for using all devices and applying AI tools

Teachers rated their competence in using computers for applying AI tools in the classroom as follows: very low – 25%, low – 58%, weak – 14%, high – 2%, and very high – 1%. This indicates that most teachers feel inadequately prepared to integrate AI tools into their teaching practices (Fig. 4).

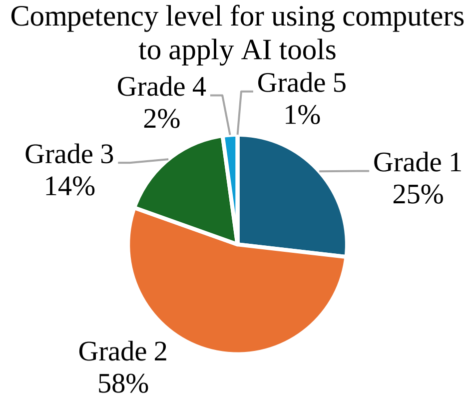

Fig. 4. Competency level for using computers to apply AI tools

The research results showed that the application of new technologies in the teaching of informatics can significantly impact the learning and motivation of pupils and students. Students who had access to new technologies had better test results and higher levels of engagement in the teaching process. Also, students expressed a greater interest in the subject and a greater willingness to participate in learning activities if the teacher applies artificial intelligence tools in class.

The development of information technologies (IT) and the application of AI have significantly changed education and influenced student motivation. These tools enable more interactive, personalized, and engaging forms of learning, which positively affects student motivation and achievement.

The use of IT and AI in education has numerous advantages that directly contribute to increasing student motivation: Personalized learning: With the help of VI, students receive customized teaching content that corresponds to their level of knowledge and learning style, which encourages them to participate more actively and increases their interest in classes. Interactivity: Educational software, applications, and digital platforms provide students with the opportunity to solve tasks in interactive environments where they can receive immediate feedback. Gamification and rewards: Applying gamification (for example, using points, rewards, and rankings) motivates students through fun and competition. Easier access to information: Students quickly access different sources of information, which increases self-confidence and gives them a sense of control over their learning. Flexibility: The possibility of learning at any time and from any place increases motivation, especially for students who have responsibilities outside of school.

Key research results: Improved motivation and engagement of pupils/students:

- Over 70% of those surveyed stated that new technologies, such as interactive platforms, digital simulations, and exercise applications, had a positive impact on their motivation. Most respondents pointed out that interactive tools and gamification made the learning process more interesting, further motivating them to put in more effort. Improvement of learning results and efficiency;
- The survey showed that over 65% of the students believed that new technologies significantly improved their efficiency in mastering the material. Students who used technologies such as virtual labs and programs for automated task checking reported greater progress in understanding complex concepts, which led to better results on tests and assignments. Personalization of the learning process;
- About 60% of the respondents expressed satisfaction with the possibility of personalized learning through technologies based on artificial intelligence. These tools allowed students to focus on their weak points, age.

5. Discussion

Many teachers recognize the numerous advantages that artificial intelligence tools can bring to the educational process:

Personalization of learning – Artificial intelligence enables the creation of customized lesson plans that match the needs and pace of each student, which can significantly improve their involvement and ability to learn material.

Automate Administrative Tasks – AI-based tools can automate routine tasks such as test grading, score analysis, and homework administration, freeing up time for teachers to focus more on working directly with students.

Regarding the devices used for teaching, respondents mainly rely on laptops, followed by mobile phones, personal computers, and tablets.

This suggests that teachers use familiar technology in their teaching practices, albeit with limited integration of newer devices. Improving access to educational resources – Artificial intelligence can help organize and search large amounts of educational material, making it easier for students and teachers to access important information (Abbas et al., 2021).

Preparing teachers for new technologies requires a holistic and well-structured approach. Continuous education, support and resources are key to successfully mastering new technologies and their integration into teaching, which will directly affect the improvement of the quality of education.

6. Conclusion

Based on the obtained results, it was concluded that the application of new technologies in the teaching of informatics is of essential importance for improving the educational process. Traditionally, teachers are the main sources of knowledge, imparting information through lectures and textual materials. However, in the digital age, this model is changing, as pupils/students can now access a large amount of information through the Internet, educational applications, and digital tools. Teachers are no longer the only source of knowledge, but take on the role of guides and mentors who help students navigate a world full of information. Many educators recognize the numerous benefits that AI tools can bring to the educational process: personalization of learning, automation of administrative tasks, and improving access to educational resources.

The results indicate the existence of challenges in the process of integrating AI technologies into the teaching process. Implications for modern educational processes include the need for teacher training for the successful integration of technology into teaching. Also, it is important to promote a culture of innovation and exchange of good practices among educational institutions.

Preparing teachers for applying new technologies in computer science teaching is key to ensuring that students are up-to-date with modern technology trends and skills. Effective preparation includes a series of activities and strategies that ensure teachers can effectively use and integrate new technologies into their teaching.

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Fibonacci Sequence, Golden Ratio, and Golden Spiral – Examples

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Abstract

"Mathematical Fascination" is a project by the teachers and students of the Technical School in Požega on the topic of the Fibonacci sequence, the golden ratio, and the golden spiral. It was implemented through project-based learning using the STEM concept. Over the course of eight weeks, second-year students at the Technical School in Požega worked on a project that combined mathematics and biology and focused on the golden ratio in geometry and architecture. The students were divided into six groups and carried out research and product development according to given instructions/research tasks:

1. Art and Architecture – They analyzed works that use the golden ratio for aesthetic harmony, as well as architectural examples throughout history.
2. Anatomy – They found examples of the golden ratio in the human body, such as facial proportions or the arrangement of fingers.
3. Nature – They discovered examples of the golden ratio and the Fibonacci sequence in the formations of natural objects, such as hurricanes or galaxies.
4. Plants – They identified Fibonacci sequences in the arrangement of leaves, flowers, or seeds.
5. Algebra – They found the algebraic solution to the golden ratio problem (dividing a segment of length a into parts x and $a-x$ such that $a : x = x : (a-x)$).
6. Geometry – They performed a geometric construction of dividing a segment by the golden ratio (dividing the segment by the golden ratio using a compass and a ruler and proving the correctness of the construction using the similarity of triangles).

The students systematized and presented their research results in the form of posters (first, second, third, and fourth groups) and videos (fifth and sixth groups).

All the project outputs, as well as the method of outcome assessment, are presented in an e-book: https://read.bookcreator.com/reM4j2B1GLfwDXmUZMRALyc5RIp2/AEZc-vfqSqKb2mp9c66WHA?authuser=0.

The achievement of outcomes was verified by organizing an Escape Classroom: https://view.genially.com/663de70191e5290014b293f7/interactive-image-escape-classroom-za-matematichka-fascinacia.

Keywords: golden ratio, Fibonacci sequence, anatomy, architecture, morphology, STEM, Project-Based Learning.

1. Introduction

For two months, teachers and students of the Technical School in Požega worked on a project titled Mathematical Fascination focused on Fibonacci sequence, the golden ratio, and the golden spiral. It was carried out through project-based learning, applying the STEM concept, which is based on the integration of science, technology, engineering, and mathematics. For the eight weeks of the project, second-year students of the Technical School in Požega worked on a project that combined subjects like mathematics and biology, exploring the topic of the golden ratio in geometry, anatomy, plant and animal morphology, architecture, and art.

1.1. What Are STEM Skills and Why Are They Important to Us?

Every day at school, you learn certain definitions. Sometimes, they are quite difficult to memorize, sometimes you barely understand them, and sometimes you don't even understand what you're supposed to understand.

In the STEM approach, we don't learn definitions; we learn concepts, phenomena, and scientific truths through experiments. Here, we prove definitions, test, and observe scientific experiments. We play while actually learning serious science.

1.2. What Is STEM?

STEM is a combination of knowledge, skills, and abilities that help you discover scientific concepts and definitions in everyday activities. To achieve this, you need to connect biology, chemistry, physics, technology, and engineering, as well as mathematics. Not JUST programming, not JUST robotics, but also robotics, programming, biology, chemistry, and more.

1.2.1. Project-Based Learning

Project-based learning allows for the integration and interaction of multiple different subjects and serves as a problem-solving method that requires students to engage in independent activities and leave a record of their efforts. It is accompanied by numerous student activities, including:

- Independent research using various resources (written sources, media, local community, etc.),
- Problem-solving skills,
- Independent mastery of content and self-directed learning,
- Acquisition of various skills (collaboration, communication, digital skills, learning strategies, data processing, etc.),
- Group work and cooperation,
- Critical evaluation of one's own and others' work,
- Decision-making and reasoning,
- Adoption of alternative and innovative work methods,
- Planning and adhering to deadlines.

2. Project Progress

2.1. Preparatory Phase

At the beginning, the teachers created a Google Classroom, which served as the main platform for communication among all project participants. Initially, they posted an introductory text about the golden ratio and a link to a lecture on Fibonacci numbers (https://youtu.be/SjSHVDfXHQ4). Students were asked to share their first impressions of the project topic. All communication took place through Google Classroom.

After this task, students received essential information on collaborative learning, rules for group work, and evaluation criteria during the project. Additionally, they were informed about what STEM is and why it is important.

To further motivate students to explore the connection between mathematics, anatomy, and Renaissance art, the next task was to watch the American film The Da Vinci Code from 2006, based on the novel by Dan Brown and directed by Ron Howard. After watching the movie, students filled out a Google questionnaire on the connections between mathematics and the world around them as observed in the film. Following that, students shared their thoughts on a Linoit board about the project topic, what they hoped to learn, the skills they wanted to acquire, and how the knowledge gained could help them in their future work and development. Their responses are shown in the image below.

These are some of the initial student observations about the project, posted on the Linoit board shown in Fig. 1:

- „The Fibonacci sequence, the golden ratio, and the golden spiral are topics that deeply interest me. I know the Fibonacci sequence is a series of numbers where each number is the sum of the two preceding ones in the sequence. However, I want to explore more deeply the connection between the Fibonacci sequence, the golden ratio, and the golden spiral.“
- „I want to learn how they are applied in different fields, as well as their benefits and significance in mathematics, science, and art. I aim to acquire skills in analyzing and interpreting these concepts and applying them in practice, such as designing and graphically representing the golden spiral. This knowledge could assist me in further advancing in fields like computer graphics, design, architecture, and finance.“
- „The material covered in the classroom has taught me that the Fibonacci sequence forms a kind of vortex where everything is interconnected. The golden ratio is most closely tied to mathematics but can also be found all around us in everyday life.“
- „I would like to learn about the purpose and application of the Fibonacci sequence in daily life. A skill I would like to develop is applying these two topics to various tasks or projects. Any new knowledge is valuable and could help me as I continue my education.“

Fig. 1. Initial student reflections on the project

Using the Wheel of Names, students were divided into six groups, and each group selected a representative. Every serious project should have a strong visual identity, so each group presented their version of the project logo on the Linoit board. A vote was then held to choose the most successful logo, which was later used by all students and teachers throughout the project.

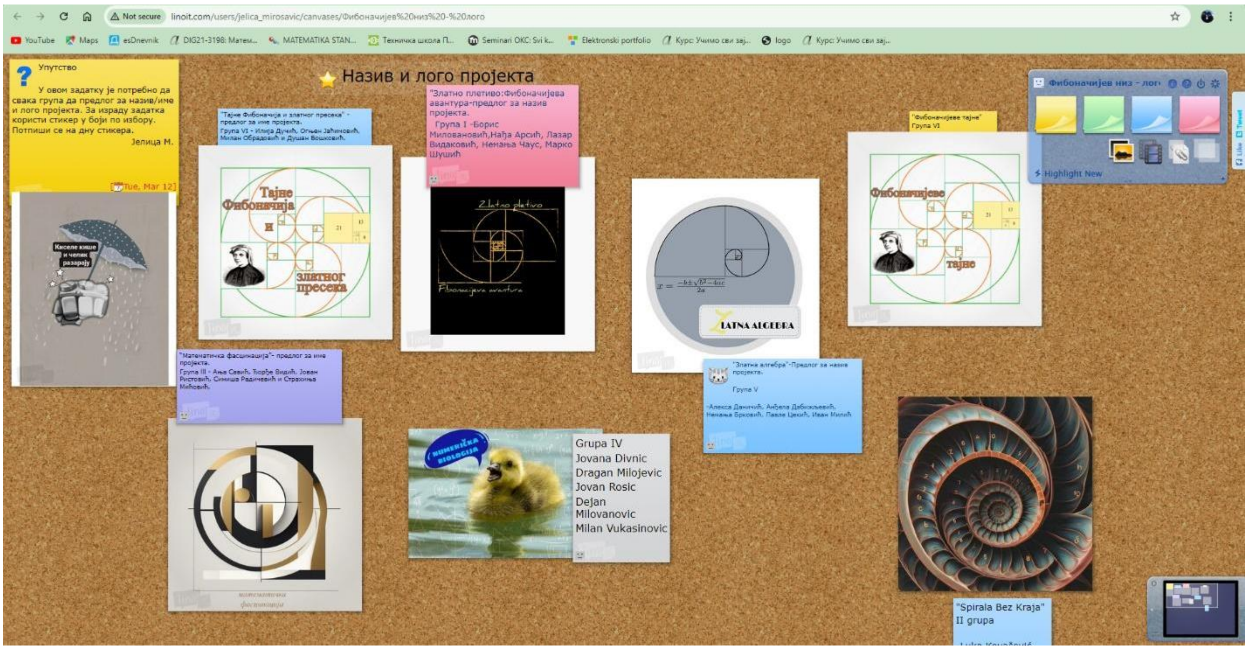

Fig. 2. Student suggestions for the project logo, the adopted logo is in the lower left

2.2. Research

Each of the six student groups received their research task, worked on it, and presented their findings to other students and teachers on a Linoit board. The teachers reviewed the work and provided their observations and feedback to ensure the tasks were completed successfully. The research tasks for each group were as follows:

- **Group I – Art and Architecture**: Analyze works that use the golden ratio for aesthetic harmony, as well as architectural examples throughout history.
- **Group II – Anatomy**: Find examples of the golden ratio in the human body, such as facial proportions or finger anatomy.
- **Group III – Nature**: Discover examples of the golden ratio and Fibonacci sequence in natural formations, such as hurricanes or galaxies.
- **Group IV – Plants**: Identify Fibonacci sequences in the arrangement of leaves, flowers, or seeds.
- **Group V – Algebra**: Find the algebraic solution to the problem of dividing a line segment using the golden ratio, i.e., dividing a line segment of length a into segments of length x and $a - x$ such that $a : x = x : (a - x)$. Write a formula for calculating the value of x for a given length a.
- **Group VI – Geometry**: Perform a geometric construction to divide a line segment using the golden ratio, i.e., divide a segment using only a ruler and compass, and prove the correctness of the geometric construction by applying the similarity of triangles.

The next research task the students worked on was titled *The Human Body Through Measurements*. The goal of this task was to quantitatively determine the body proportions of the students in the class. In collaboration with the physical education teacher, measurements were taken of each student's height, arm span, forearm length, upper arm length, and specified measurements of the hand.

2.3. Project

After conducting their research, the first four groups presented the results of their work using posters created in the Canva program, while Groups V and VI made video presentations using the ActivePresenter program.

The group that worked on the algebraic solution to the golden ratio division created an Excel calculator that calculates the remaining two values when any of the lengths a, x, or a - x is input. The following task, which built upon this, was titled My φ Number. The students expressed the difference between their height and arm span as a percentage. The collective conclusion was that a person's height and arm span generally match, with slight deviations. They also expressed the percentage deviation between the division of the arm into the forearm and upper arm compared to the division of the arm according to the golden ratio. The conclusion was that the division of the arm into the forearm and upper arm largely corresponds to the golden ratio. Similarly, through measurement, the golden ratio was observed in the anatomy of the hand.

As part of the project, each group created its own glossary of terms. These glossaries were then compiled into one common glossary. Based on the terms in the glossary, a quiz was created for independent learning and evaluation of knowledge on the topic. It can be found at the following link: Quizlet - Fibonacci Sequence, Golden Ratio, and Golden Spiral Examples.

In the next phase, the students were introduced to 3D object modeling and printing with a 3D printer. To incorporate an engineering dimension and modern technology into the project, each group was required to select a 3D model of an object related to the topic they were working on and, in collaboration with the mechanical engineering teacher, create it in the school workshop. These 3D objects represent the physical products of their work on the project.

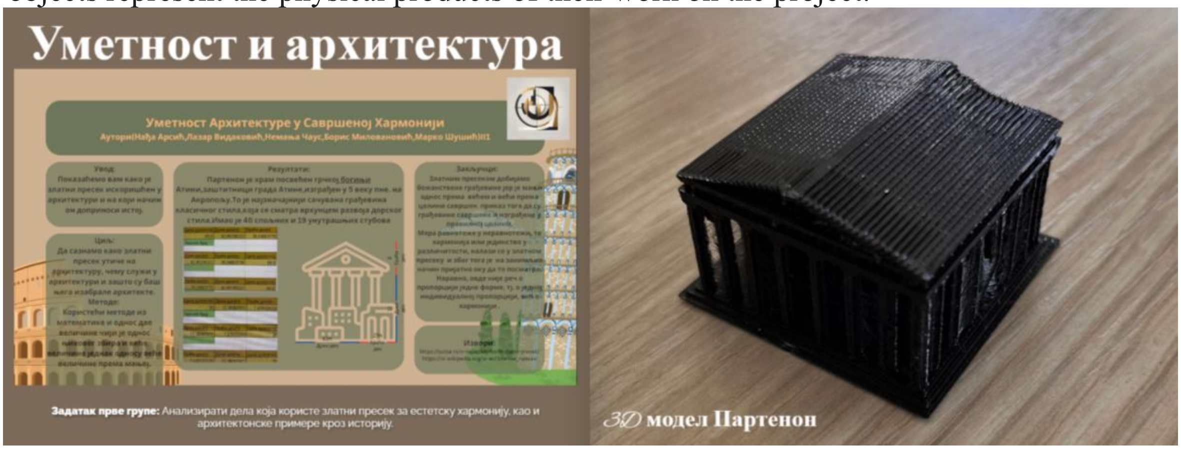

Fig. 3. Poster and 3D object made by group I

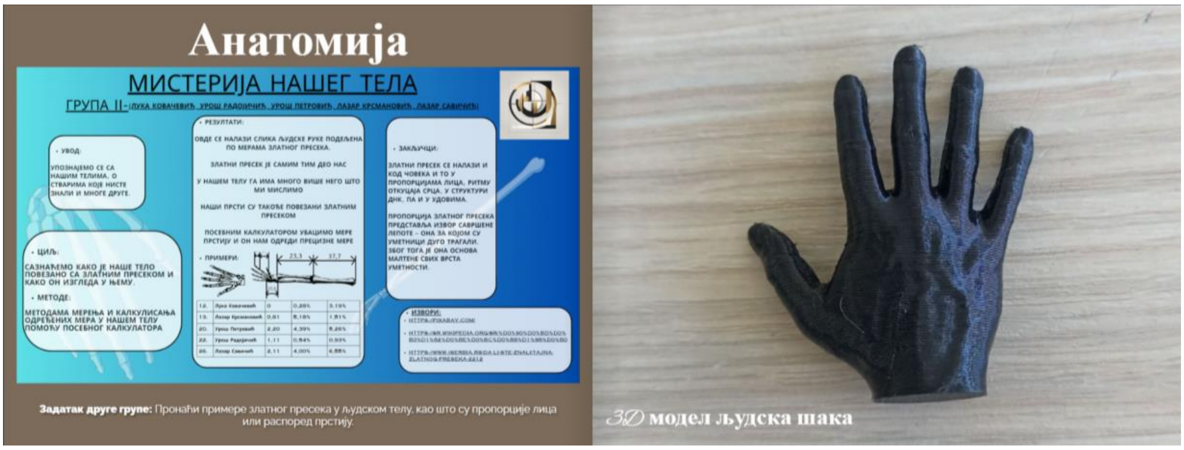

Fig. 4. Poster and 3D object made by group II

Fig. 5. Poster and 3D object made by group III

Fig. 6. Poster and 3D object made by group IV

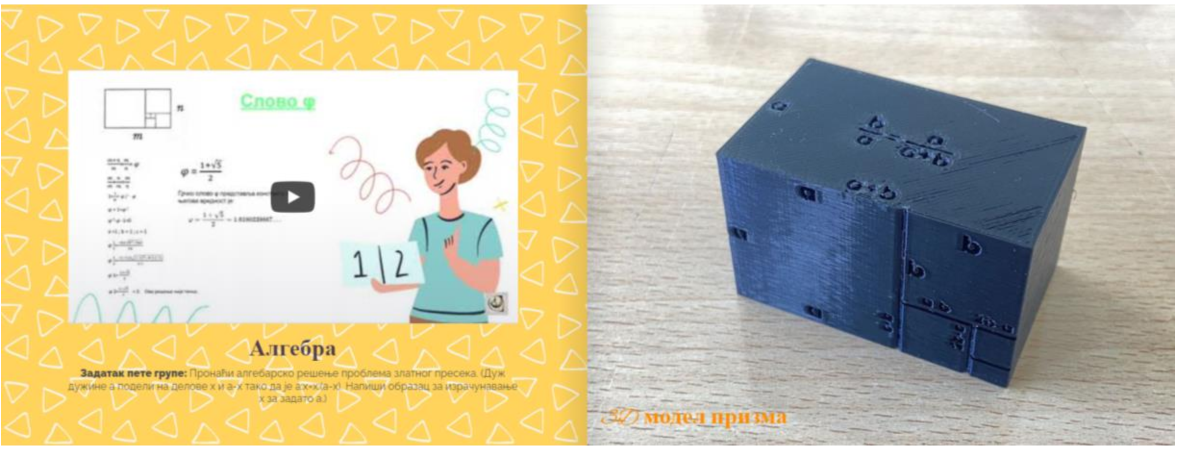

Fig. 7. Video presentation and 3D object made by group V

Fig. 8. Video presentation and 3D object made by group VI

3. Evaluation of Project-Based Learning

After completing all the research and project tasks, the students filled out a Google questionnaire to share their experiences with project-based learning, the knowledge they gained, the problems they encountered, and their impressions of the tasks.

When asked what they liked about the project-based learning experience, some of the students' answers were:

- "I liked the project and the people I worked with, and the project topic was also interesting."
- "The teamwork that allowed us to complete the tasks."
- "I liked everything, but my favorite task was watching the movie."
- "How the Fibonacci sequence is everywhere around us."
- "I liked the way we worked and the organization of the whole project."
- "I enjoyed working with my friends and the various activities."
- "I mostly liked everything. The tasks were clear and interesting, and we had enough time for each one. My favorite part was 3D printing the models."
- "We get good grades."

When asked what they didn't like about project-based learning, some of the students' answers were:

- "The organization of the group."
- "I didn't like the tasks 'Project Name and Logo' and 'The Human Body Through Measurements.' It was boring to measure myself and input the values into a table."
- "I didn't like that not everyone participated."
- "Some tasks were boring."
- "The deadlines were sometimes too short."
- "I didn't like the number of tasks."
- "The grading system."
- "Some people in our group didn't do anything and weren't interested, and in the end, only the coordinator would be blamed if the task wasn't completed."

When asked what they didn't know before, some of the students' answers were:

- "I didn't know that math and biology could be interesting and that they could be combined."
- "What the golden ratio and Fibonacci sequence are."
- "How a 3D printer works."
- "The connection between math and anatomy."

- “That the golden ratio is present in so many works of art and in nature.”
- “Well, I knew everything.”

When asked what they know now and what they’ve learned, some of the students’ answers were:

- “That I can learn a lot of interesting things through math and biology.”
- “I know that the golden ratio is present in many places in the human body.”
- “How to work as a team.”
- “I know more about project-based learning now.”
- “How a 3D printer works.”
- “I learned what the golden ratio, golden rectangle, and golden spiral are.”

When asked what they personally found important among the things they learned, some of the students’ answers were:

- “It was important to me that I spent time with my friends.”
- “Learning how to work as a team.”
- “How we can apply math in nature and learn something about math from nature.”
- “Getting a good grade.”
- “That I expanded my knowledge of math, especially about the similarity of triangles.”
- “The effort I put in.”
- “I know how to use a 3D printer.”

At the end, students gave suggestions that project-based learning should be implemented more often and across more subjects. They also suggested that they should participate in deciding on the project topics and that students should be allowed to work on individual projects, which would also be graded.

Finally, the evaluation of project-based learning outcomes was conducted through an Escape Classroom activity: Escape Classroom for Mathematical Fascination.

Fig. 9. Escape Classroom activity

In groups, the students worked on solving tasks from mathematics, biology, and geography for two class periods. Only when they found all the tasks in the room, successfully solved them, and recorded their answers could they proceed to arrange the answers in the given order. This was because the groups did not solve the tasks in the same order but followed clues around the room. The answers to the tasks were the numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, and 55. Finally, the students had to use logical reasoning to find the key to unlock the “escape room”. The key was the next number in the Fibonacci sequence, which was 89. All groups found the exit from the room within the allotted time.

The entire project was documented in an e-book, which can be viewed at the following link: Mathematical Fascination E-Book.

4. Conclusion

The project focuses on identifying and analyzing examples of the golden ratio and the golden spiral in nature, mathematics, and art. By researching literature and information sources, students illustrated the golden ratio and the golden spiral through examples, explained the Fibonacci sequence, and created 3D models of selected objects. The primary goal of the project was to motivate students to apply STEM learning and highlight its significance in connecting with everyday life.

During the project implementation, students had the opportunity to learn how to use various web tools and programs, gaining new knowledge and skills in the fields of modern technologies and other technological resources. The applied teaching methods enriched the educational process, making it more interactive and engaging for students. This contributes to the quality of education and better prepares them for future professional and academic challenges.

The project was presented on November 7, 2024, during a webinar titled “Two Approaches to Science Through Experiments: Science and Knowledge = Playing!” organized by the “STEM Staffroom” teacher network and the “STEM Education Center”. The webinar was attended by teachers, educators, preschool teachers, and science enthusiasts from Serbia, Montenegro, North Macedonia, Bosnia and Herzegovina, and Croatia. A total of 6,031 participants attended the webinar, providing an excellent opportunity to showcase an innovative teaching approach and how, through play and practical examples, we can motivate students to actively engage in the learning process.

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Through Mathematical Calculations and Computational Visualizations into the Microscale World of Molecules

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Abstract

This demonstrative lesson investigates the reaction of salts with ethanol and aluminum foil, emphasizing the role of mathematics and computer science in data analysis and visualization. The aim is to provide students with an opportunity to develop and apply mathematical and computational skills through practical work and direct observation of chemical reactions. Students use mathematical tools to calculate concentrations, yield percentages, and stoichiometric ratios, enabling precise quantitative data analysis. Computational tools, such as Microsoft Excel and Python (with libraries Matplotlib and Pandas), were utilized for tabular analysis and data visualization. The experiment's results demonstrate the formation of new substances in the presence of aluminum foil, and students create graphical representations and models of the reactions. This experiment not only enhances the understanding of chemical reactions but also develops key mathematical and computational skills, encouraging interdisciplinary thinking and preparing students to solve complex scientific problems.

Keywords: Chemical reactions, Ethanol, Salts, Microsoft Excel, Python.

1. Introduction

The main motives for writing this paper are, on the one hand, the misalignment of physics and mathematics curricula at almost all levels of education, both in Serbia and in most other countries, and on the other hand, the strong insistence on linking the content of different subjects in a formal and bureaucratic manner within our educational system. The most significant consequences of this misalignment are difficulties in mastering physics and related disciplines during the first year of studies at faculties in the fields of technical and natural sciences, as well as poor integration and application of differential and integral calculus in the description of immediate natural phenomena. The mentioned bureaucratization of the teaching process leads to a lack of motivation among teachers and inadequate organization of the educational process in schools in terms of introducing interdisciplinary approaches.

In a world where the boundaries of science are increasingly blurred, an interdisciplinary approach becomes key to understanding complex phenomena. Imagine a classroom where chemistry, mathematics, and computer science merge in an engaging experiment, where students observe chemical reactions and deeply analyze and visualize them using modern technologies.

This demonstrative lesson explores the reaction of salts in an ethanol environment, emphasizing the role of mathematics and computer science in data analysis and visualization. The research aims to provide students with an opportunity to develop and apply mathematical and computational skills through practical work and direct observation of chemical reactions. In this context, the methods applied include quantitative analysis and data visualization, where students use mathematical tools to calculate concentrations, yield percentages, and stoichiometric ratios.

The project involves the use of programming languages such as Python and Microsoft Excel. Python, a powerful language for data processing, will be used to analyze the results obtained and create graphical representations through libraries such as Matplotlib and Pandas. Matplotlib serves for data visualization, enabling students to create interactive graphics representing experimental results, including charts illustrating concentration changes during reactions.

Moreover, students will apply image processing to analyze visual aspects of the experiment. By using image processing libraries in Python, such as OpenCV, they will be able to analyze pixels in solution images, identifying changes in color and intensity occurring during reactions. This pixel analysis will provide insights into the molecular microscale, enabling students to visualize and quantitatively analyze concentration changes at the molecular level.

Through these methods, students will be able to model molecular interactions and reactions, creating a link between observable laboratory results and theoretical chemistry concepts. Excel will be used for systematically recording data and calculating basic statistical parameters, such as arithmetic means and standard deviations, giving students a better understanding of the experiment's quantitative aspects.

Data sources include experimental results obtained from practical work and data analysis through computational tools. This innovative approach lies at the heart of the demonstrative lesson, challenging students to go beyond traditional learning frameworks and engage in research integrating science and technology in a new and exciting way. *How can we delve into the microscale world of two salts' solutions without auxiliary instruments and transfer their reaction into the realm of computational records and mathematical numbers?*

Fig. 1. The solution of two salts and its visual appearance extracted from the video recording*

2. Research Methodology

The section headings are numbered in boldface capital and lowercase letters. Second-level headings are typed as subsection headings of this paragraph.

The microscale world of solutions represents one of the most fascinating aspects of chemistry, where complex physico-chemical processes occur within molecular structures. To convey these abstract concepts to students and enable them to participate in scientific research actively, we designed an innovative and visually adapted methodology.

This experiment begins with recording an innovative demonstration using a mobile phone in a laboratory, capturing each second of the experiment as a new image. From these extracted images,

* https://youtu.be/mKTGpc5MgOU https://youtu.be/LKYuZyMG4U0

we analyze colors, frequencies, and flame intensities during the combustion of two salts dissolved in ethanol. Our key data include blue and green flames representing ion concentrations in the solution. This visualization allows us to explore the dynamics of molecular interactions in real time.

When salts react in ethanol, they generate various ions and new products while releasing heat. Without visualization, it would be extremely challenging to notice the dynamics and changes in reactions. This visualization technique reveals how ionic relationships transform and new substances form before our eyes.

Using computational tools, we translate complex chemical processes into clear and comprehensible visual forms. Excel serves as a foundation for systematically recording data, where students input information on reagent concentrations, reaction times, and other key variables. This creates a logical system for tracking and comparing data based on scientific methodology principles.

As our advanced tool, Python enters the picture with advanced algorithms and libraries such as Pandas and NumPy. These libraries enable students to process large datasets, perform complex calculations, and analyze trends that might otherwise remain unnoticed.

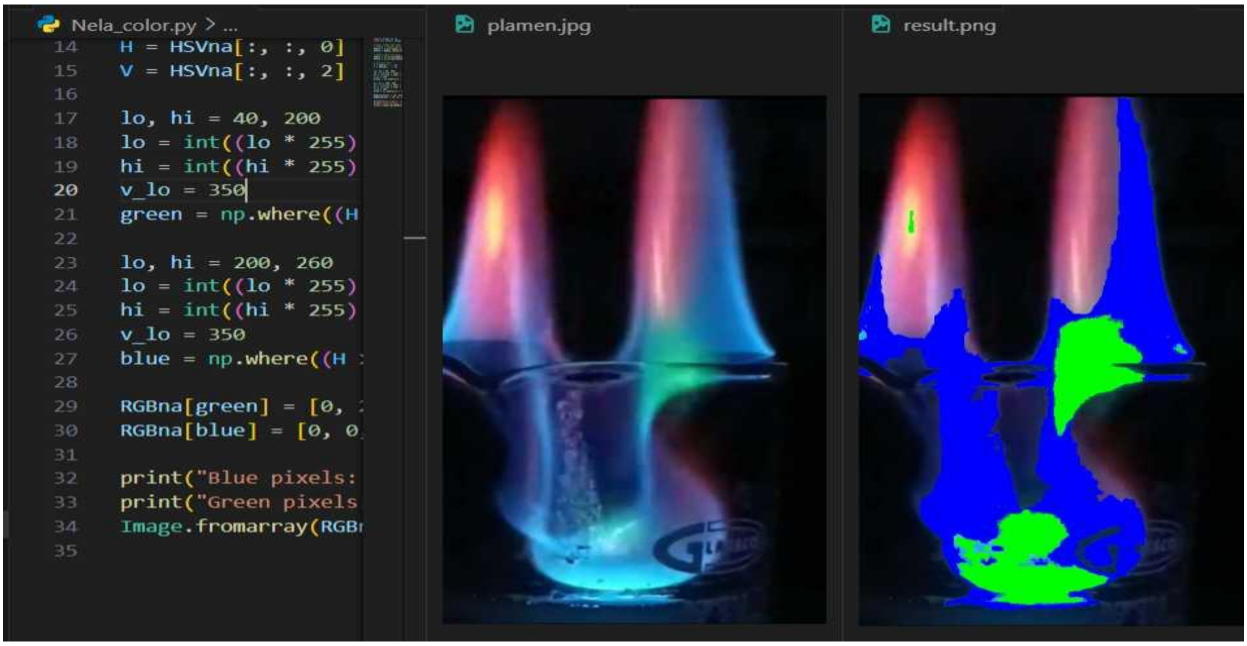

Fig. 2. Illustration of translating flame intensity images into computational data

The Matplotlib library plays a key role in data visualization in this research context. Students can create elegant charts – from line graphs to three-dimensional plots – enabling them to represent reaction dynamics and visible changes in ion concentrations over time. This visualization contributes to a deeper understanding of the processes occurring at the molecular level.

Additionally, Python is utilized to simulate chemical reactions. Students program models that predict reaction outcomes based on input parameters, helping them learn to understand and forecast the behavior of substances in the microscale world of solutions. This scientific-research approach not only empowers students with scientific skills but also fosters critical thinking and creativity.

If the ratio of blue and green flames is directly proportional, it may indicate the presence of stoichiometric relationships in the reaction, aligning with Proust's Law. By analyzing data, you can examine how consistently these relationships hold and whether there are conditions under which deviations from this theory occur.

Through this project, students gain not only a deeper understanding of chemical reactions but also practical experience in applying mathematical and computational tools, opening new horizons in scientific research.

3. Research Results

The results reveal unexpected and intriguing phenomena in the reaction of salts in ethanol in the presence of aluminum foil, indicating complex interactions within the microscale world of solutions. Students successfully analyzed and visualized these interactions using mathematical and computational methods, shedding light on aspects of the reaction that were otherwise challenging to observe with traditional methods.

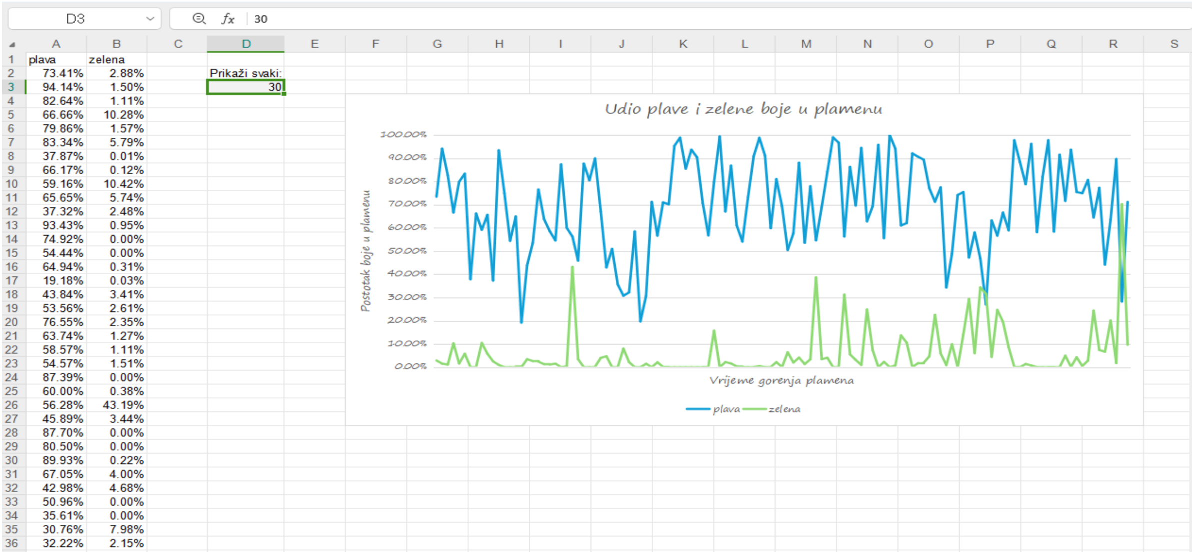

Fig. 3. Results of blue and green flames expressed as percentages over time, presented and integrated in the form of a graph using Excel software

Based on the available data on the percentages of blue and green flames, an analysis was conducted that included the results of mathematical calculations in table below.

Table 1. the results of mathematical calculations.

Indicator	Blue Flame (%)	Green Flame (%)
Arithmetic Mean	62,63	5,06
Standard Deviation	18,92	5,97
Minimum Value	19,18	0,00
Maximum Value	99,99	70,23

In the graph, you can see the changes in the proportions of blue and green flames throughout the experiment. This graph was used to visualize trends and highlight periods in which significant changes in concentrations occurred.

By applying Proust's Law, it was determined that the mass ratios of the components in the reaction closely align with theoretical predictions. The data analysis confirms that the blue and green flames show a proportional change depending on the reactants present.

In this experiment, the flame is produced as a result of chemical reactions that occur when salts (nickel chloride and copper sulfate) dissolved in alcohol are exposed to heat. The flame color formed depends on the presence of different metal ions in the solution, as well as the temperature applied during the experiment. When dissolved salts such as nickel chloride and copper sulfate are exposed to high temperatures in the flame, electron excitation occurs in the metal atoms. This means that electrons within the metal atoms gain energy from the high temperatures and move to higher energy levels. When the electrons return to lower energy levels, they release energy in the form of light, resulting in the flame color. The different flame colors depend on the type of metal because different metals have different energy levels.

The experiment was conducted with second-year high school students and later discussed in subsequent lessons, as it required mathematical calculations—arithmetic mean, maximum and minimum values, as well as standard deviations. Supersaturated solutions were prepared arbitrarily to create conditions in which the reactions would clearly show the flame color. Supersaturated alcoholic solutions serve as a medium that allows easy evaporation and combustion of the salts, resulting in the detection of characteristic ion colors.

Software tools used included a camera for recording flame color and computer programs such as Python. Students arrived at the following results:

- If the values obtained for the percentage of blue and green in the flame were much higher or lower than expected, it could indicate unexpected factors in the reaction. For example, if the flame was overly emphasized with one color (blue or green) relative to the other, this might suggest variations in salt concentrations, flame temperature, or the influence of other components in the solution. Additionally, if some colors were weakly expressed or absent altogether, it could be considered whether the salt concentrations were sufficiently high to produce a clear flame color.

Based on the calculated Pearson correlation coefficient, which was: In the full table: -0.27; In the dynamic table: -0.35, this indicates that the relationship is not strong. The correlation is not close to -1, (which would mean that if one variable increases, the other decreases) as shown by the flame colors in the graph, and this also serves as evidence for Proust's Law of constant mass ratio.
Did the aluminum foil play a role as a variable factor (if it was contaminated) in this experiment and not just as a conductor?

The fact that the aluminum foil darkened and emitted sparks may indicate a strong reaction with the salt solution, which could be surprising in the context of this experiment. The assumption that aluminum would only transfer heat might be incorrect if the solution and temperature are such that they cause a more aggressive reaction with aluminum. This discovery was further confirmed by reaction models created in Python, which enabled the simulation and prediction of the reaction flow depending on the initial conditions. These results suggest complex interactions between the salts, ethanol, and aluminum. By using mathematical and computational methods, the students gained a deeper understanding of the dynamics of the reactions. Interestingly, the amount of aluminum ions in the solution changed significantly over time, indicating that aluminum influences the reaction dynamics in an unforeseen way. It is recommended that these findings be explored further in future research, including the study of other components and their impact on reaction dynamics. By analyzing the obtained data, we can see how modern techniques confirm the fundamental principles of chemistry.

4. Conclusion

By using advanced methods and tools such as Microsoft Excel and Python, the students were able to analyze and visualize the data from the chemical reactions in detail. Excel allowed for systematic recording and tabular analysis, while Python, with the Matplotlib and Pandas libraries, provided the ability for advanced graphical visualization and simulation of reactions.

The experiment's results confirmed that the mass ratios of elements in chemical compounds remain constant regardless of the source or conditions of the reaction by Proust's Law. The graphical representations showed that the masses of the components remained stable throughout the reaction, although the dynamics of the reaction varied depending on the concentration of the reagents. The integration of chemistry, mathematics, and computer science in this experiment demonstrated the significance of a multidisciplinary approach to scientific research. By using mathematical and computational methods, the students gained a deeper understanding of chemical processes and confirmed fundamental scientific principles. They were particularly interested in practically applying theoretical knowledge and were fascinated by the ability to observe color changes in the flame as evidence of the presence of specific ions. Many were surprised by the amount of color formed, especially the blue, which helped them better understand how different salts can influence the strength of the reaction. The students were also intrigued by the changes in color intensity, which they associated with the concentration of salts in the solution, helping them understand the importance of Proust’s Law and the relationship between colors and ions. Several students pointed out the importance of precision in preparing the solutions (although the experiment was conducted with arbitrarily prepared supersaturated solutions) and the potential application of these principles and factors in chemical and industrial processes. This study not only confirms historical scientific laws, such as Proust’s Law, but also provides new insights into complex chemical processes, which is of great importance for future research and education in science.

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STEM approach as a tool for motivation in teaching mathematics

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Abstract

One of the biggest problems facing today's math teachers is motivating students and giving meaning to their learning. This problem is all the greater if the student perceives mathematics a priori as a difficult science to master.

As an adequate response to existing problems, the STEM (science, technology, engineering, mathematics) approach to teaching is imposed by its general characteristics. By placing the student in the center, this approach significantly contributes to the improvement of the quality of the teaching process itself, and thus to raising student competencies and achievements to a higher level. The student researches, experiments, solves problems, makes conclusions and decisions, and sometimes makes mistakes and learns from them. With its interdisciplinarity, STEM provides an answer to the famous question: "Why am I learning this?" and develops in him the appropriate STEM and math skills necessary to solve real-life as well as abstract problems (Anonymous, 2023).

On the other hand, the STEM approach requires appropriate competencies of the teacher as well as his permanent professional development through various training and exchange of experiences with other colleagues (Amir et al.,2015).

This paper presents a good practice example of the STEM approach in the teaching of mathematics in elementary school and analyzes how and to what extent its application contributes to students' acquisition of permanent and functional knowledge. It also discusses the possibility of implementing the STEM approach into the regular curriculum and its long-term positive impact on the development of skills necessary for the 21st century.

Keywords: STEM, mathematics, teaching, didactics, methodology.

1. Introduction

Modern teaching in general, and therefore the teaching of mathematics, faces numerous challenges, such as lack of motivation among students, overload of numerous information, and lack of functional knowledge in the sense of not recognizing the possibility of applying certain contents in other fields of science, but also in everyday life. Mathematics is a subject that students often perceive as an abstract and hard-to-master scientific discipline, which further complicates their engagement and understanding of mathematical concepts and content. This fact is supported by the results of the final exam for the 2022/2023 school year (Nedeljković, 2023) and the results of the PISA test for 2022 (Čaprić & Videnović, 2024). In the mathematics test, more than 75% of districts in Serbia achieved a below-average result, while in the PISA test only 4% of our students reach the highest level where the application of what has been learned is required.

Geometry is the area that poses the most problems for students. When it comes to understanding basic geometric concepts, the concept of angle and the concept of parallelism, as well as measuring and calculating the area and perimeter of geometric figures, the achievements of our students are at a fairly low level, and the results of the TIMSS test even show that they have a decreasing trend

(Đerić et al.,2021). Given that this problem is already growing into a multi-decade problem (Pejić & Todorović, 2007), it is necessary to take steps that would lead to positive changes in this matter.

The traditional approach to teaching mainly relies on memorizing and reproducing data and practicing algorithms to solve certain problems. As such, it less and less meets the needs of today's student who lives in a world of accelerated technological development and digitization. Also, due to a lack of understanding of basic concepts, students try to memorize content, which does not lead to functional knowledge, especially in mathematics. In this way, the problems that arise at the lowest level of studying mathematical content only multiply and grow, and the student acquires aversion to the subject.

On the other hand, STEM (Science, Technology, Engineering, Mathematics) represents an innovative approach to education that, by integrating these disciplines, encourages the spirit of inquiry in students and develops critical thinking, creativity, and practical skills. As a result, the student becomes more motivated and goes from being a passive listener to an active participant in the teaching process (Tomić, 2023; Гурчиновски, 2023).

2. An example of good practice

One example of good practice of implementing STEM in mathematics education is the study of axial symmetry in the fifth grade. Within this area, the very concept of axial symmetry, axial symmetry of two points, axial symmetry of two figures, axial symmetry of one figure, length bisector, angle bisector, and some basic features of axial symmetry are studied.

The traditional approach to teaching through the implementation of this topic is focused on the definitions and construction of axially symmetrical figures, which at a certain point becomes boring for students and abstract for many of them.

Integrating STEM makes this topic significantly more interesting for students and, therefore, easier to master. Students are now able to study axial symmetry through various activities: research, science experiments, use of technology, engineering thinking, and mathematical methods.

We begin the introduction to the topic with a simple experiment, Fig. 1. We fold a rectangular sheet of paper in half so that all its edges overlap. Then we open the paper, draw a dot on one side of the line along which the paper was previously folded (with tempera, watercolor or nail polish), and fold the paper again so that the dot is painted on the other side of the paper and open the paper again. The students are given the task to assemble the obtained points into one length, to measure the distance of each of those points to the line along which the paper is folded, and to measure the angle at which the line is drawn along, and the line along which the paper is folded intersect. After completing the task, a discussion follows, on the basis of which we arrive at the definition of axisymmetric points and the axis of symmetry.

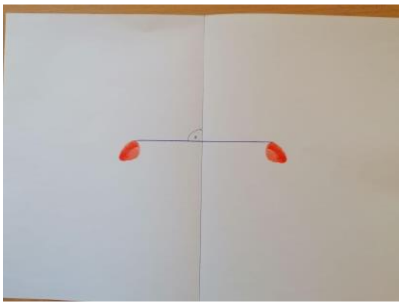

Fig. 1. Axial symmetry of two points

Then, the students are asked to give some examples of axisymmetric points from life. The most frequently cited examples are eyes, ears, dimples on the cheeks when someone laughs, holes in the socket, headlights on vehicles, and threads in jeans' pockets.

In order to understand the axial symmetry of the two figures, we repeat the experiment with folded paper, only this time, instead of a point, we first draw a length in an oblique position and then an arbitrary polygon, Fig. 2. Using the points that represent the ends of the lines, i.e. the vertices of the polygon, the students are given the task of making all the measurements from the previous example. After completing the task, through discussion, we come to the definition of the axial symmetry of the two figures.

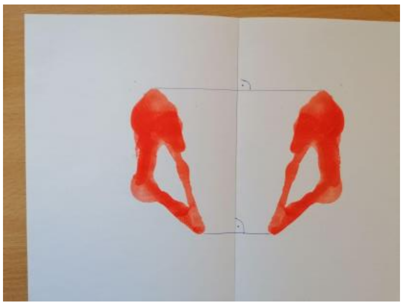

Fig. 2. Axial symmetry of two figures

Then the students get assignments.

Each bench receives two cards. Each card has two figures, one of which is axially symmetrical and the other not (Fig. 3 and Fig. 4). Students from the group should show the card on which the axisymmetric figures are presented and explain why the figures on the other card are not axisymmetric.

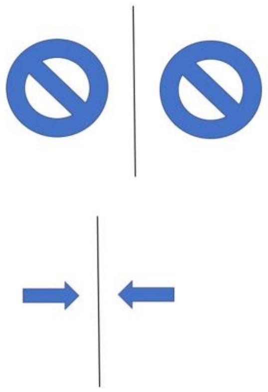

Fig. 3. Pair work cards 1

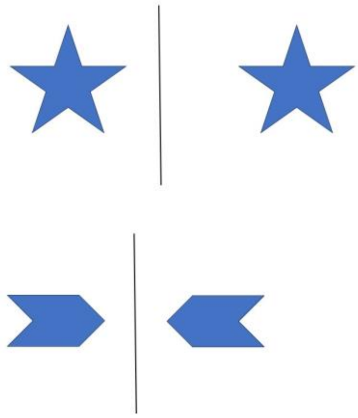

Fig. 4. Cards for pair work 2

In the following, each student has the task of mapping the initial letter of his name with axial symmetry in relation to the axis of symmetry that he assigns (Fig. 5).

Fig. 5. Construction of axisymmetric letters

As a final task, students in a square grid map the given figure with axial symmetry in relation to the given axis of symmetry (collection of tasks), Fig. 6.

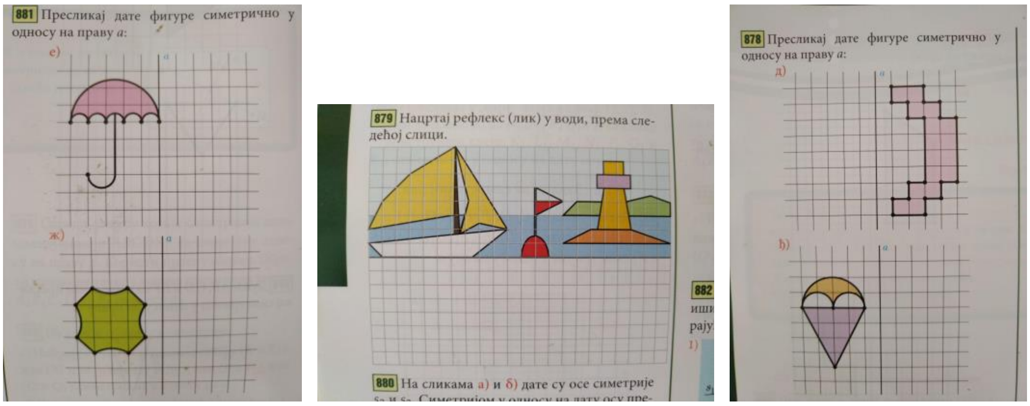

Fig. 6. Axial symmetry in a square grid (Popović et al., 2018., page 100)

For homework, students are asked to bring photographs showing the axial symmetry of the two figures (Fig. 7).

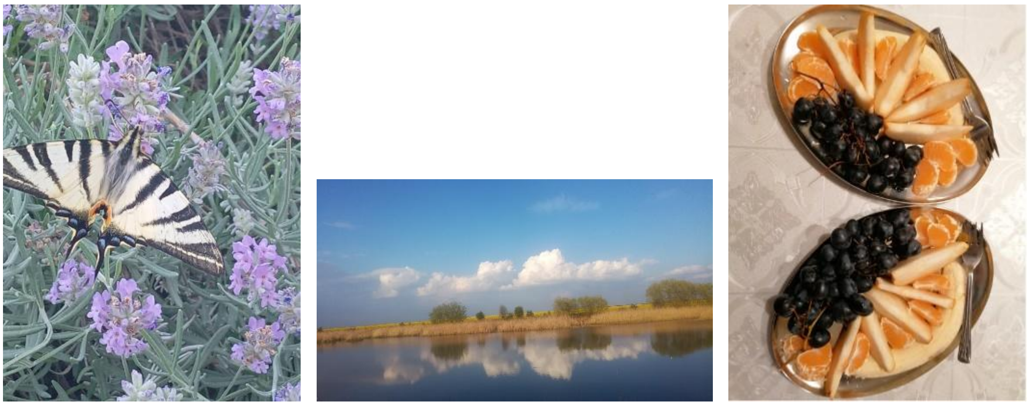

Fig. 7. Photographs of two axisymmetric figures

We further investigate when a figure is axisymmetric. Students are divided into groups of four members each. Each group receives three figures - one that is not axisymmetric and two that is, where one of the two has only one axis of symmetry, while the other has more, Fig. 8. Based on the matching of the edges of the figure when it is folded, the students should determine whether the given figure is axisymmetric and, if so, how many axes of symmetry it has. When the students solve the given task, through discussion, we come to the conclusion when a figure is axially symmetrical.

Fig. 8. Materials for determining the axial symmetry of a figure

Students are then asked to name some examples of axisymmetric objects from their environment.

Most often, students mention the school blackboard, the clock we have in the classroom, a bench, a leaf, a four-leaf clover, a flower, a heart, a butterfly, a wheel, numbers, letters. As axisymmetric numbers, students often mention 101, 202, 303 and the like, so it is important to explain the difference between an axisymmetric number and a number that represents a palindrome. The same problem occurs when students begin to list axisymmetric words.

In the following, students in groups are given tasks to draw at least four axisymmetric ones:

- flags containing only red and yellow, Fig. 9;
- flags containing only blue, red and white, Fig. 10;
- flags containing red and white colors, Fig. 11;
- flags containing red or blue, Fig. 12;
- round traffic signs, Fig 13;
- triangular traffic sign.

Fig. 9. Flags containing only red and yellow colors

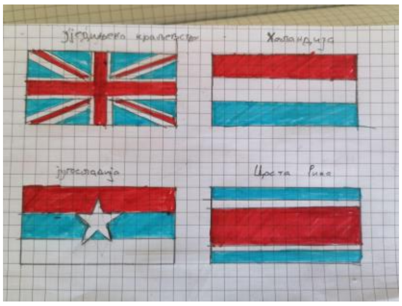

Fig. 10. Flags containing only blue, red and white colors

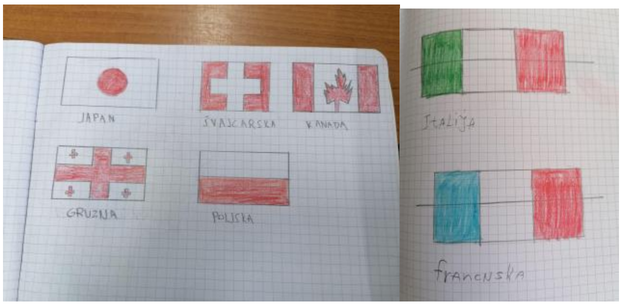

Fig. 11 Flags containing red and white colors

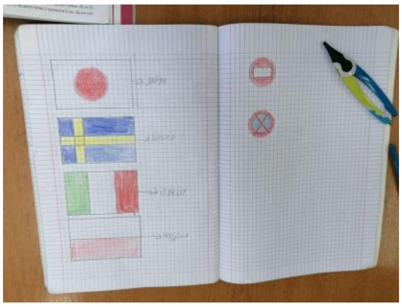

Fig. 12. Flags containing red or blue

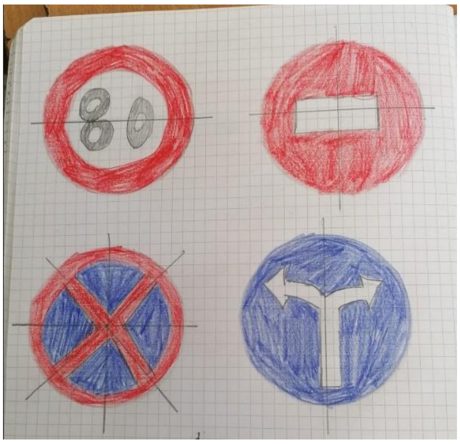

Fig. 13. Round traffic signs

Through this activity, we also repeat the use of the words “and”, “or” and “only”.

For homework, students should draw an axisymmetric figure of their choice, Fig. 14. and find a photo of an axisymmetric building, Fig. 15, where students are required to know which building is in the photo and where it is located.

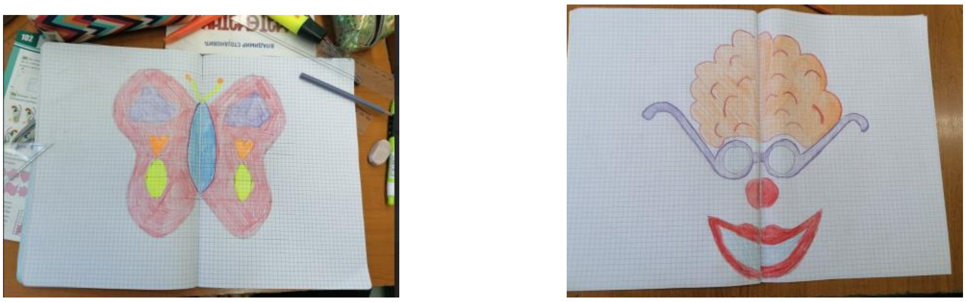

Fig. 14. Axisymmetric drawings

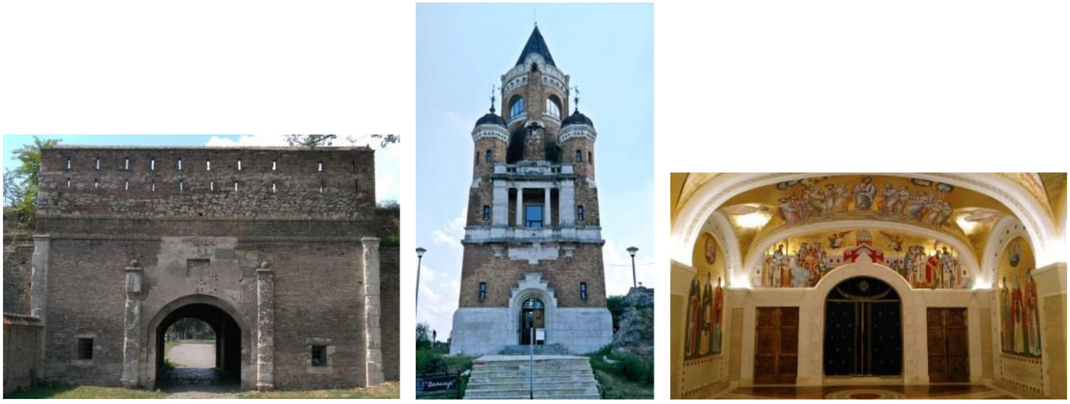

Fig. 15. Photos of axisymmetric buildings

When analyzing the photos brought by the students, we referred to the notion of copyright. We have explained what copyright is and why they should not download photos that we are not sure are not protected by copyright, and they were directed to some of the sites where some of the photos can be freely downloaded.

We practice the construction of the length bisector and angle bisector with standard geometric equipment and then use Mathigon, Fig. 16, and GeoGebra tools. In order to get better practice, students are given the task of dividing a given length and a given angle into four equal parts using a geometric tool and one of the ICT tools.

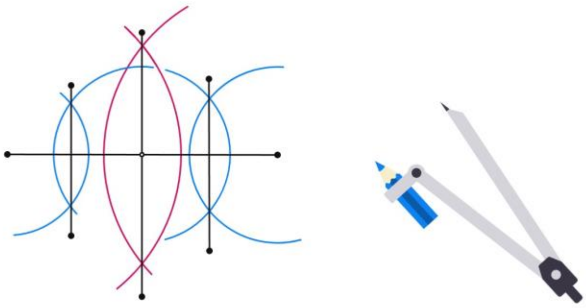

Fig. 16. Split length into 4 equal parts in the Mathigon tool

In order to determine and systematize the knowledge at the end of the field, students solve tasks in groups that require the application of knowledge from the field of axial symmetry, linking materials from other fields and the use of ICT tools.

The tasks are as follows:

- in the Mathigon tool, create two axisymmetric figures and construct the axis of symmetry;
- using the GeoGebra tool, determine a point equidistant from three given points (lines);
- using the Geometry Toolbox tool, draw an arbitrary quadrilateral, set the axis of symmetry, then map the drawn quadrilateral with axial symmetry in relation to the given axis, Fig. 18;

- Serbia, Hungary, and Bosnia and Herzegovina submitted a joint application to organize the European basketball championship, but they have to build a modern basketball hall that will be equidistant from Belgrade, Budapest, and Sarajevo. Determine the place where the hall will be built and calculate how far it will be from each of the mentioned cities, Fig. 19;
- The three main roads are located as shown in the picture, Fig. 17.

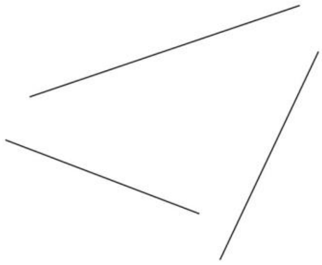

Fig. 17. Location of three main roads

It is necessary to build a hotel that is equidistant from each of these roads. Determine the exact location of the hotel's construction.

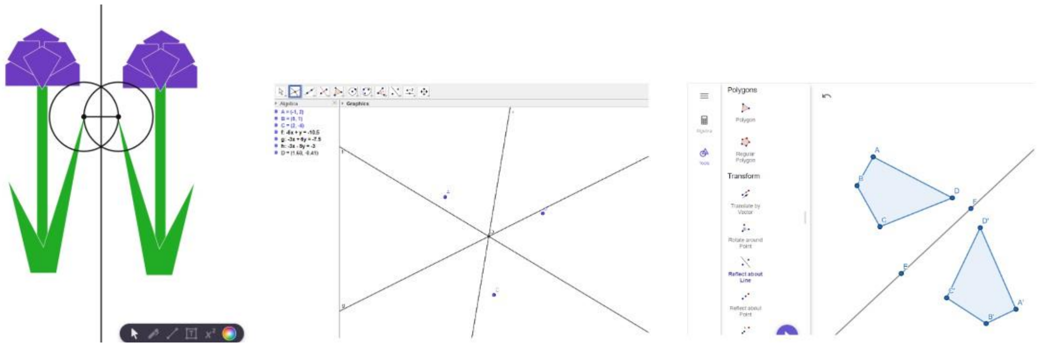

Fig. 18. Application of ICT tools (Mathigon, GeoGebra, Geometry Toolbox)

Fig. 19. Developing problem-solving competencies

For homework, students should make a model of a building that contains elements of axial symmetry, Fig. 20. They can use cardboard, styrofoam, toothpicks,... In this way, students apply the acquired knowledge through an engineering challenge.

Fig. 20. A bridge with axial symmetry elements

3. Discussion

Several important differences were observed in comparing the results of students who studied axial symmetry through the STEM approach with those who studied the same topic in a traditional way.

The students who studied the topic through the STEM approach achieved a slightly better result on the control task, but on the initial testing of the following school year, all (except for two students who attend classes according to the IEP) correctly answered the question in the area of axial symmetry, Table 1.

Table 1. Comparative presentation of student achievements through STEM and traditional teaching approaches.

	The class that studied the area of Axisymmetrical Geometry through a STEM approach (academic year 2017/2018)	**The class that studied the area of Axisymmetrical symmetry using the traditional approach (academic year 2019/2020)**
The average grade on the test	3,41	4,03
Percentage of students who correctly solved the axial symmetry task on the initial test of the following school year (without IEP)	70,83	100

What is even more important, their knowledge was not limited only to the template construction of axisymmetric figures, but they were able to apply the acquired knowledge in some real situations, but also to the further study of mathematical content such as, for example, coordinate system and construction of the described and inscribed circle of the triangle, Table 2.

Table 2. Comparative overview of the application of acquired knowledge.

	The class that studied the area of Axisymmetrical Symmetry through a STEM approach (academic year 2018/2019)	**The class that studied the area of Axisymmetric in the traditional way (academic year 2020/2021.)**
Percentage of students who recognize axisymmetric figures in the coordinate system	83,33	88,46
Percentage of students who, based on the given coordinates of a point, determine the coordinates of a point symmetrical to it in relation to a given coordinate axis	66,67	73,08
Percentage of students who independently conclude where the center of the inscribed/circumscribed circle of a triangle is located, based on the properties of axial symmetry	29,17	42,31

On the other hand, students who studied the topic in the traditional way, in addition to showing a slightly lower result on the tests, also showed a weaker connection of the acquired knowledge with concrete real problems and, therefore, a lower level of motivation for further research of mathematical content related to this topic.

4. Conclusion

The analysis of the obtained results leads to the conclusion that the implementation of STEM in the regular teaching of mathematics could significantly improve the teaching process and thus increase the level of motivation and interest of students in studying mathematical content. The STEM approach helps students to understand better and apply mathematical concepts but also develops in students the skills necessary for functioning in the modern world - creativity, critical thinking, and the ability to solve problems. The STEM approach is widely represented in education in Asian countries, which can be considered one of the main reasons for the extremely good results achieved by their students in various mathematics tests and competitions (Chang et al., 2020). Therefore, STEM should be implemented in regular classes so that students become ready for the challenges of the future.

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