

# The Ramanujan integral and its derivatives: Computation and analysis

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The principal tool of computation used in this paper is classical Gaussian quadrature on the interval  $[0, 1]$ , which happens to be particularly effective here. Explicit expressions are found for the derivatives of the Ramanujan integral and it is proved that the latter is completely monotone on  $(0, \infty)$ . A new series expansion for the incomplete gamma function is found and conjectured to converge alternately from above and below. The paper also pays attention to another famous integral, the Euler integral — better known as the gamma function — revitalizing a largely neglected part of the function, the part corresponding to negative values of the argument, which plays a prominent role in our work.