Idempotent-aided factorizations of matrices over a field

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A factorization of a given matrix is its representation in the form of a product of two or more matrices and, as such, entails data analysis. Although matrix factorization typically gives a more compact representation than learning the full matrix, it is a simple embedding model. In the language of computer science, the expression of a matrix D as a product amounts to pre-processing of the data in D and organizing that data into two or more parts structured to be of better use in whichever way necessary, and more accessible for computation. There are many different matrix (factorisations) decompositions over fields such as rank factorization, LU factorization, QR factorisation, Cholesky factorization, singular value decomposition, spectral factorisation etc. and each finds use among a particular class of problems, such as SVD-like machine learning model, mathematical problems in social network analysis and real-world recommendation systems, airflow problems and so on... The concept of an idempotent-aided factorization, presented here, can be viewed as a semigroup theoretical generalization of the full rank factorization of matrices (cf. [1, 13]). Namely, Green's $\mathcal{D}$-classes of the semigroup of matrices, consist of all matrices of the same rank. If D is considered to be a matrix over a field, a factorization of the matrix D with respect to the idempotent matrix E (with the same rank as D) represents the decomposition of the matrix D into the product $D = UV$ of the matrices U and V such that U has the same null space (kernel) with E and the same range (image, column space) as D, while V has the same range as E and the same null space as D. This factorization is a full-rank factorization of D and, moreover, U is a left invertible and V is a right invertible matrix. Several effective algorithms related to idempotent-aided factorizations of matrices have been provided and the correctness of those algorithms has been proven.

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