The essentially left and right generalized Drazin invertible operators and generalized Saphar decomposition

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In this paper we define and study the classes of the essentially left and right generalized Drazin invertible operators and of the left and right Weyl-g-Drazin invertible operators by means of the analytical core and the quasinilpotent part of an operator. We show that the essentially left (right) generalized Drazin invertible operator can be represented as a sum of a left (right) Fredholm and a quasinilpotent operator. Analogously, the left (right) Weyl-g-Drazin invertible operator can be represented as a sum of a left (right) Weyl and a quasinilpotent operator. We also characterize these operators in terms of their generalized Saphar decompositions, accumulation and interior points of various spectra of operator pencils. Furthermore, we expand the results from [7], on generalized left and right Drazin invertible operators. Special attention is devoted to the investigation of the corresponding spectra of operator pencils.

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