Computing ε-Weak Bisimulations for Fuzzy Automata over Truncated Product Structures: Algorithms and Complexity Analysis

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Weak bisimulations have been widely studied as a generalization of bisimulations for fuzzy automata (FAs). These bisimulations maintain language equivalence and perform better in the state reduction of FAs. However, a significant limitation of weak bisimulations is their inability to be computed for all $(\vee, \cdot)$-FAs, where $\vee$ represents the maximum operation and $\cdot$ denotes the product t-norm on the real-unit interval $[0, 1]$. The reason is that weak bisimulations are solutions to specific linear systems of fuzzy relation inequalities, and such systems can consist of infinitely many inequalities when observed under such FAs.

We propose a solution to this problem by introducing a new type of weak bisimulation, named ε-weak bisimulation. By choosing a small value $\varepsilon > 0$, we define ε-weak bisimulations, which yield finite systems of fuzzy relation inequalities. These ε-weak bisimulations preserve a new form of approximation for language equivalence. Specifically, we demonstrate that two $(\vee, \cdot)$-FAs that are ε-weak bisimilar recognize each word with degrees that are either equal or both less than or equal to ε.

Furthermore, we present two algorithms developed for computing the ε-weak bisimulation. The first algorithm computes the ε-weak bisimulation for a single automaton, while the second algorithm extends the computation to two automata. We discuss the time complexity of these algorithms and analyze the complexity of storing the resulting bisimulations.

As ε-weak bisimulations possess the property of approximating equivalence for arbitrarily small values of $\varepsilon > 0$, they effectively model an "almost-equivalence" between two FAs. This means that words accepted with degrees smaller than or equal to ε can be considered irrelevant. Our proposed ε-weak bisimulations offer a practical approach to address the challenge of computing bisimulations for $(\vee, \cdot)$-FAs,

expanding the scope of bisimulation techniques in the field of fuzzy automata. The presented algorithms provide efficient methods for computing the ε-weak bisimulation, and the analysis of time complexity and storage requirements further contribute to the understanding and applicability of these techniques in practice.

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