On optimal quadrature formulas for approximation of Fourier integrals and their application to CT image reconstruction

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This paper is devoted to construction of optimal quadrature formulas in the Hilbert space $\widetilde{W}_2^{(m,m-1)}$ of complex-valued, periodic functions for the numerical calculation of the integral $\int_0^1 e^{2\pi i\omega x}\varphi(x)dx$ for $\omega \in \mathbb{Z}$. In the cases $m = 1$ and $m = 2$, the exponentially weighted integrals of some functions at the values of some N and ω are approximated using the constructed optimal quadrature formulas, and it is shown in numerical results that the orders of convergence of this formulas are $O(h+|\omega|^{-1})$ and $O((h+|\omega|^{-1})^2)$, respectively. Also, in the space $\widetilde{W}_2^{(m,m-1)}$, the sharp upper bound of the error for the optimal quadrature formulas is obtained, and it is shown analytically that the order of convergence of the optimal quadrature formula is $O((h+|\omega|^{-1})^m)$. Furthermore, in the case $\omega \in \mathbb{R}$, effective quadrature formulas for the approximate calculation of Fourier integrals are obtained and they are used in the reconstruction of CT images.