Existence of Approximating Solution of Second Order Nonlinear Integro-differential Equations via. Iteration Method

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In this paper the existence theorem for the second order functional integro-differential equations in Banach algebras is proved under the mixed generalized Lipschitz and Caratheodory conditions. The existence of extremal solutions is also proved under certain monotonicity conditions usings Dhage iteration Method. As a generalization of ordinary integro-differential equations, there is a series of papers dealing with the abstract measure integro-differential equations in which ordinary derivative is replaced by the derivative of set functions, namely, the Radon-Nokodym derivative of a measure with respect to another measure. See Bellale [10], Dhage [4], Dhage and Bellale [8] and the references therein. The above mentioned papers also include some already known abstract measure differential equations those considered in P. C. Das and Sharma [2] , Shendge and Joshi studied as special cases. The origin of the quadratic integral equations appears in the works of Chandrasekhar's H-equation in radioactive heat transfer, but the study of nonlinear integral equations via operator theoretic techniques seems to have been started by Dhage in the year 1988. Similarly, the study of nonlinear quadratic differential equations is relatively new and initiated by Dhage and O'Regan in the year 2000. In this paper we discussed the following problem for external solution For a given closed and bounded interval J = [0; a] in $\mathbb{R}$, the set of real numbers, consider the following integro-differential equation.

$$
\left\{ \begin{array}{l} \left(\dfrac{x(t)}{f(t, x(t))} \right)' = g\left(t, x(t), \int_0^t k(s, x(s)) \mathrm{d}s \right) \quad \text{a.e. } t \in J \\ x(0) = x_0 \in \mathbb{R} \end{array} \right. \tag{1}
$$

where $f : J \times \mathbb{R} \to \mathbb{R} \setminus \{0\}$ is continuous, $g : J \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and $k : J \times \mathbb{R} \to \mathbb{R}$.

The existence of the solutions to (1) is proved in Dhage and Ballale by using a new nonlinear alternative of Leray-Schauder type developed in same paper. In this paper we apply a nonlinear alternative of Leray-Schauder type due to Dhage and Bellale [7] involving the product of two operators in a Banach algebra under some weaker conditions than that given in Dhage and Regan [9] to a quadratic abstract measure differential equation related to (1) for proving the existence results. The existence of extremal solutions is also proved using a fixed point theorem of Banas [1] in ordered Banach algebras.

References

[1] J. Banas , K. Goebel, Measures of non compactness in Banach space, in: Lecture Notes in pure and Applied Mathematics , **Vol 60** , Dekker, New York,(1980)

[2] P. C. Das and R. R. Sharma, Existence and stability of measure differential equations, Zech. Math. J., **22** (1972), 145 –158.

[3] B. C. Dhage, On abstract measure integro-differental equations, J. Math. Phy. Sci. 20(1986), 367–380.

[4] B.C. Dhage, On system of abstract measure integro-differential inequalities and applications, Bull. Inst. Math. Acad. Sinica 18 (1989), 65–75.

[5] B. C. Dhage, Bellale and S. K. Ntouyas, Abstract measure differential equations, Dynamic Systems & Appl. 13 (2010), 105–108.

[6] S. S Bellale and B. C. Dhage Abstract measure integro-differential equations, Global Jour. Math. Anal. 1 (1-2) (2007), 91–108.

[7] B. C. Dhage and S. S. Bellale, Exitence theorem for perturbed Abstract measure differential equations Nonlinear Analysis, Theory; methods and Applications (2008) doi:1016j.na.2008.11.057.

[8] B. C. Dhage and S. S. Bellale, Local asymptotic stability for Nonlinear quadratic functional equations, Electronic Journal of Qualitative Theory of differential Equations, **No. 1-13** (2008)

[9] B. C. Dhage and O Regan, A fixed point theorem in Banach Algebras with applicationto the nonlinear integral equations , Functional differential equations 7(3-4) (2000), 259–267.

[10] S. S. Bellale , Hybrid fixed point theorem for Abstract Measure Integro Differential Equations, World Academy of Science and Technology, **73** (2013), 782–785.