Generalization of the Levin-Stečkin inequality

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V. I. Levin and S. B. Stečkin proved the following theorem

Theorem 1. Let f be defined on $[0,1]$ satisying the conditions:

$$f(x) \text{ is nondecreasing for } 0 \leq x \leq \frac{1}{2}, \tag{1}$$

and

$$f(x) = f(1-x), x \in [0,1]. \tag{2}$$

Then for any convex function ϕ we have

$$\int_0^1 f(x)\phi(x)dx \leq \int_0^1 f(x)dx \int_0^1 \phi(x)dx. \tag{3}$$

Using the Green function we obtain some interesting results concerning the difference of the integral arithmetic means

$$\frac{\int_a^b f(x)d\lambda_2(x)}{\int_a^b d\lambda_2(x)} - \frac{\int_a^b f(x)d\lambda_1(x)}{\int_a^b d\lambda_1(x)}, \tag{4}$$

where $\lambda_1, \lambda_2 \colon [a,b] \to \mathbf{R}$ are continuous functions or the functions of bounded variation, such that $\lambda_i(a) \neq \lambda_i(b)$, $i = 1, 2$.
First we derive results for integral means with general measures, using them we obtain the results for integral weighted means with functions as weights and with unique measure, and finally we show that we have really obtained generalization of the Levin-Stečkin inequality.