

Landau and Grüss type inequalities for inner product type integral transformers in norm ideals

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For a probability measure μ and for square integrable fields (\mathcal{A}_t) and (\mathcal{B}_t) ($t \in \Omega$) of commuting normal operators we prove Landau type inequality

$$\begin{aligned} & \left\| \int_{\Omega} \mathcal{A}_t X \mathcal{B}_t d\mu(t) - \int_{\Omega} \mathcal{A}_t d\mu(t) X \int_{\Omega} \mathcal{B}_t d\mu(t) \right\| \\ & \leq \left\| \sqrt{\int_{\Omega} |\mathcal{A}_t|^2 d\mu(t) - \left| \int_{\Omega} \mathcal{A}_t d\mu(t) \right|^2} X \sqrt{\int_{\Omega} |\mathcal{B}_t|^2 d\mu(t) - \left| \int_{\Omega} \mathcal{B}_t d\mu(t) \right|^2} \right\| \end{aligned}$$

for all $X \in B(\mathcal{H})$ and for all unitarily invariant norms $\| \cdot \|$.

For Schatten p -norms similar inequalities are given for arbitrary double square integrable fields, i.e. in that case normality and commutativity conditions can be dropped. Also, for all bounded self-adjoint fields satisfying $C \leq \mathcal{A}_t \leq D$ and $E \leq \mathcal{B}_t \leq F$ for all $t \in \Omega$ and some bounded self-adjoint operators C, D, E and F , and for all $X \in C_{\| \cdot \|}(\mathcal{H})$ we prove Grüss type inequality

$$\left\| \int_{\Omega} \mathcal{A}_t X \mathcal{B}_t d\mu(t) - \int_{\Omega} \mathcal{A}_t d\mu(t) X \int_{\Omega} \mathcal{B}_t d\mu(t) \right\| \leq \frac{\|D - C\| \cdot \|F - E\|}{4} \cdot \|X\|.$$

References

- [1] Jocić Danko, Krtinić Đorđe, Moslehian Mohammad Sal, Landau and Grüss type inequalities for inner product type integral transformers in norm ideals, *Mathematical Inequalities and Applications* **16**, no. 1 (2013), 109–125.