On reverse triangle inequality and some its applications

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In Euclidean space the triangle inequality asserts that the sum of any two sides of a triangle is strictly bigger than the remaining third side. The reverse triangle inequality is its equivalent which say that any side of a triangle is greater than to the difference between the other two sides.

In first part of this talk we shall proved equivalence of triangle inequality and reverse triangle inequality on class of pseudo - metric spaces. In particular we shall presented the following pseudo - metric spaces characterization of Lindenbaum's type [1].

Proposition 1. Let X be non empty set and $d : X \times X \to [0, \infty)$, such that for any $x, y \in X$ $x = y$ implies $d(x, y) = 0$. Then the following statements are equivalent:

1) (X, d) is pseudo - metric space;

2) for any $x, y, z \in X$ holds $d(x, y) \leq d(x, z) + d(y, z)$;

3) for any $x, y, z \in X$ holds $|d(x, z) - d(y, z)| \leq d(x, y)$.

Further, as examles of applications of reverse triangle inequality, we shell proved one inequality of Serbian mathematician D. D. Adamović (see [2] page 281.) and some its generalizations. The main result of this section is:

Proposition 2. If sequences $(a_n), (b_n) \in L^p$ and $\|(b_n)\|_{L^p} = B$, then

$$|(\sum_{i=1}^{\infty} |a_n|^p)^{\frac{1}{p}} - (\sum_{i=1}^{\infty} |b_n|^p)^{\frac{1}{p}}| \leq \inf_{\|(c_n)\|_{L^p}=B} \sum_{i=1}^{\infty} |a_i - c_i|.$$

References

[1] A. Lindenbaum, Contributions à l'étude de l'espace métrique. I., Fundam. Math. **8** (1926), 209–222.

[2] D. S. Mitrinović, Analytic Inequalities, Springer-Verlag, Berlin Heidelberg 1970.