Dynamical sampling for shift-preserving operators acting on finitely generated shift-invariant subspaces of Sobolev spaces

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We analyze shift-invariant spaces V_s, subspaces of Sobolev spaces $H^s(\mathbb{R}^n)$, $s \in \mathbb{R}$, generated by the set of generators φ_i, $i \in I$, I is countable at most, by the use of range functions and characterize Bessel sequences, frames and Riesz basis of such spaces. We show that an $f \in \mathcal{D}'_{L^2}(\mathbb{R}^n)$ belongs to V_s if and only if its Fourier transform has the form $\widehat{f} = \sum_{i\in I} f_i g_i$, $f_i = \widehat{\varphi}_i \in L^2_s(\mathbb{R}^n)$, $\{\varphi_i(\cdot + k) : k \in \mathbb{Z}^n, i \in I\}$ is a frame and $g_i = \sum_{k\in\mathbb{Z}^n} a^i_k \mathrm{e}^{-2\pi\sqrt{-1}\langle\cdot,k\rangle}$, with $(a^i_k)_{k\in\mathbb{Z}^n} \in \ell^2$. Moreover, connecting two different approaches to shift-invariant spaces V_s and $\mathcal{V}^2_s$, $s > 0$, under the assumption that the finite number of generators belongs to $H^s \cap L^2_s$, we give the characterization of elements in V_s through the expansions with coefficients in ℓ^2_s. We also give the representation for shift-preserving operators $L : V_s \to V_s$ in terms of range operators. Using a range operator approach, we derive a result about dual frames and solve the dynamical sampling problem for a class of shift-preserving operators acting on a finitely generated shift-invariant space V_s.

References

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