On perfectness of systems of weights satisfying Pearson's equation with nonstandard parameters

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Measures generating classical orthogonal polynomials are determined by Pearson's equation, whose parameters usually provide the positivity of the measures. The case of general complex parameters (nonstandard) is also of interest; the non-Hermitian orthogonality with respect to (now complex-valued) measures is considered on curves in $\mathbb{C}$.

Some applications lead to multiple orthogonality with respect to a number of such measures. In this talk, we will introduce a unified approach allowing to prove the perfectness of the systems of complex measures satisfying Pearson's equation with nonstandard parameters. For a system of r orthogonality measures, the perfectness is an important property implying, in particular, the uniqueness for the whole family of corresponding multiple orthogonal polynomials, and for their $(r+2)$-term recurrence relations.

We will also consider the polynomials satisfying multiple orthogonality relations with respect to a system of classical discrete (Charlier, Krawtchouk, Meixner or Hahn) measures. The corresponding measures solve the difference counterpart of Pearson's equation. Using the same approach, we verify the perfectness of such systems for general parameters. For some values of the parameters, discrete measures should be replaced with the related continuous measures with non-real supports.

The work is supported by Moscow Center of Fundamental and Applied Mathematics, agreement with Ministry of Science and Higher Education of Russia No. 075-15-2022-283.