Error estimates for Gaussian quadrature formulae

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We studied the error bound of Gaussian quadrature for analytic functions. The basic idea is to express the remainder of Gaussian quadrature as a contour integral, then the error bound is reduced to find the maximum of the kernel function:

$$K_n(z;\omega) = \frac{\varrho_n(z;\omega)}{\pi_n(z)}, \quad \varrho_n(z;\omega) = \int_{-1}^{1} \frac{\pi_n(t)}{z-t}\, dt, \quad z \in \mathbb{C} \setminus [-1,1]. \tag{1}$$

The integral representation of the error term leads directly to the error bound

$$|R_n(f)| \leq \frac{l(\Gamma)}{2\pi} \left(\max_{z\in\Gamma} |K_n(z)| \right) \left(\max_{z\in\Gamma} |f(z)| \right), \tag{2}$$

where $l(\Gamma)$ is the length of the choosen contour Γ.

A common choice for the contour Γ is one of the confocal ellipses with foci at the points ∓ 1, also known as the Bernstein ellipses, and the sum of semi-axes $\rho > 1$,

$$\mathcal{E}_\rho = \left\{ z \in \mathbb{C} : \; z = \frac{1}{2}\left(u + u^{-1}\right), u = \rho e^{i\theta}, \; 0 \leq \theta < 2\pi \right\}. \tag{3}$$

For such Γ we studied the estimates (2) when w is one of the four generalized Chebyshev weight functions.