Fixed Point Algorithms: Convergence, stability and data dependence results

Javid Ali[1]

[1]Aligarh Muslim University, India, Department of Mathematics, javid.mm@amu.ac.in

In this talk, we discuss a newly introduced two step fixed point iterative algorithm. We prove a strong convergence result for weak contractions. We also prove stability and data dependency of a proposed iterative algorithm. Furthermore, we utilize our main result to approximate the solution of a nonlinear functional Volterra integral equation. If time permits, then we will discuss Image recovery problem as well.

Theorem 1. Let $T : C \to C$ be a weak contraction satisfying (1.9), where C is a nonempty, closed and convex subset of a Banach space X. Then proposed iterative algorithm (1.21) is almost T-stable.

Theorem 2. Let S be an approximate operator of a weak contraction mapping T satisfying (1.9), $\{x_n\}$ be a sequence generated by proposed iterative algorithm (1.21) for T and define a sequence $\{u_n\}$ for S as follows:

$$\begin{cases} u_0 = u \in C, \\ u_{n+1} = Sv_n, \\ v_n = S((1-a_n)u_n + a_n Su_n), \ n \in \mathbb{Z}_+, \end{cases} \tag{1}$$

where $\{a_n\}$ is a sequence in $(0,1)$ satisfying $\frac{1}{2} \leq a_n$ for all $n \in \mathbb{Z}_+$ and $\sum_{n=0}^{\infty} a_n = \infty$. If $Tp = p$ and $Sq = q$ such that $u_n \to q$ as $n \to \infty$, then we have

$$\|p - q\| \leq \frac{5\epsilon}{1-\delta},$$

where $\epsilon > 0$ is a fixed number.

References

[1] J. Ali, M. Jubair, F. Ali, Stability and convergence of F iterative scheme with an application to the fractional differential equation, Engg. with Comp. **38** (2022), S693–S702.

[2] V. Berinde, Summable almost stability of fixed point iteration procedures, Carpathian J. Math. **19(2)** (2003), 81–88.

[3] A. M. Harder, Fixed point theory and stability results for fixed point iteration procedures, PhD thesis, University of Missouri-Rolla, Missouri, 1987.

[4] I. A. Rus, S. Muresan, Data dependence of the fixed points set of weakly Picard operators, Stud. Univ. Babeş-Bolyai Math. **43**, (1998), 79–83.