Fuzzy and rough approximations

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The world we live in is pervaded with uncertainty and imprecision, which are incorporated into every information system that attempts to provide a complete and accurate model of the real world. Until recently, almost all aspects of imperfect data were modeled by probability theory but in the last decades, many new models have been developed to represent uncertain, imprecise or incomplete data, and provide reasoning that is approximate rather than precise. Fuzzy sets and rough sets take a prominent place among those models. The rapid development of these two approaches provided a basis for soft computing, an area of computing that, along with fuzzy sets and rough sets, also includes neural networks, probabilistic reasoning, belief networks, machine learning, evolutionary computing, and chaos theory.

Fuzzy sets and rough sets are complementary generalizations of classical sets. Capability of fuzzy sets to cope with uncertainty is based on the concepts of graded truth and graded set membership, while rough sets are a formal approximation of conventional sets in terms of a pair of sets which give the lower and the upper approximation of the original set. In the standard version of rough set theory, the lower- and upper-approximation sets are conventional sets, but nowadays a specific combination of rough and fuzzy approaches is increasingly used, where the approximating sets are fuzzy sets. Our research is based on just such an approach.

The basic characteristic of our approach to fuzzy rough approximations is dealing with fuzzy approximation spaces defined by a family of fuzzy relations, rather than by a single fuzzy relation, which is common with other authors. This is very important due to applications in various fuzzy (multi)relational systems such as fuzzy transition systems, fuzzy networks and others [8, 9, 10, 12]. We consider fuzzy rough approximation operators both on one and two universes of discourse, and we relate them with particular systems of fuzzy relation equations and inequations with one and two unknown fuzzy sets. We describe various algebraic properties of the solution sets of these systems, and expanding the methodology developed in our previous papers dealing with fuzzy relation equations and inequations [2, 3, 4, 5, 11] we provide efficient algorithms for computing the greatest and the least solutions.

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