Gauss-type quadrature rules for variable-sign weight functions

Jelena Tomanović

1University of Belgrade, Faculty of Mechanical Engineering, Department of Mathematics, jtommanovic@mas.bg.ac.rs

When the Gauss quadrature formula $G_n$ is applied, it is often assumed that the weight function (or the measure) is non-negative on the integration interval $[a, b]$. In the present paper, we introduce a Gauss-type quadrature formula $Q_n$ for weight functions that change the sign in the interior of $[a, b]$. Construction of $Q_n$ is based on the idea to transform the given integral into a sum of one integral which doesn’t cause a quadrature error and the other integral with a property that the points from the interior of $[a, b]$ at which the weight function changes sign are the zeros of its integrand. It proves that all nodes of $Q_n$ are pairwise distinct and contained in the interior of $[a, b]$. Moreover, $G_n$ (with a non-negative weight function) turns out to be a special case of $Q_n$. Obtained results on the remainder term of $Q_n$ suggest that the application of $Q_n$ makes sense both when the points from the interior of $[a, b]$ at which the weight function changes sign are known exactly, as well as when those points are known approximately. The accuracy of $Q_n$ is confirmed by numerical examples.

References


