Gauss-type quadrature rules for variable-sign weight functions

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When the Gauss quadrature formula G_n is applied, it is often assumed that the weight function (or the measure) is non-negative on the integration interval $[a, b]$. In the present paper, we introduce a Gauss-type quadrature formula Q_n for weight functions that change the sign in the interior of $[a, b]$. Construction of Q_n is based on the idea to transform the given integral into a sum of one integral which doesn't cause a quadrature error and the other integral with a property that the points from the interior of $[a, b]$ at which the weight function changes sign are the zeros of its integrand. It proves that all nodes of Q_n are pairwise distinct and contained in the interior of $[a, b]$. Moreover, G_n (with a non-negative weight function) turns out to be a special case of Q_n. Obtained results on the remainder term of Q_n suggest that the application of Q_n makes sense both when the points from the interior of $[a, b]$ at which the weight function changes sign are known exactly, as well as when those points are known approximately. The accuracy of Q_n is confirmed by numerical examples.

References

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