

Semiclassical orthogonal polynomials, Painlevé equations and applications to quadrature formulae

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In this lecture we overview the theory of semiclassical orthogonal polynomials following [5]. Some illustrative examples (see [1], [2]) are presented with a special emphasis on the Laguerre-Freud equations associated with the coefficients of the three term recurrence relation they satisfy. The connection with discrete and continuous Painlevé equations has been deeply analyzed in [6],

The truncated Hermite case (see [2]) is related to the so called Rys polynomials, whose zeros and quadrature formulas have been studied in [4], among others. In the framework of random matrices, Gaussian unitary ensembles with two jump discontinuities have been studied in [3], among others.

Some open problems will be stated.

References

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