Partitions and automorphisms of finite Kurepa trees

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Đuro Kurepa introduced in [4] a finite partially ordered set $\mathbf{T}_n = (T_n, \leq, 0)$, a rooted tree of height n with root 0 in which each node of height $k < n$ splits into $k+1$ successors. This tree is related to the Kurepa left factorial hypothesis KHp which states that for no prime $p > 2$, $p \,|\, !p$, where $!n \equiv |T_{n-1}| = 0! + 1! + \ldots (n-1)!$, $n \geq 1$. Kurepa defined $!n$ in [3] and posed there KHp. The status of KHp is not yet resolved, but it is numerically checked by many authors, with the last achievement for $p \leq 2^{40}$, see [1]. History of the problem and related results are presented in [2] and [5]. We consider here KHp combinatorially studying partitions of $\mathbf{T}_{n-1}$ into n components having the same size. We proved that there are no such partitions $\mathcal{P}$ if all components in $\mathcal{P}$ are connected subsets of T_{n-1}, or there is a component invariant under all automorphisms of $\mathbf{T}_{n-1}$. Along the way, we found that $\mathrm{Aut}(\mathbf{T}_n)$ is the wreath product of factorial powers of permutation groups S_k, $k \leq n$. We also found an approximation of Kurepa function $K(z)$, an extension of $!n$ to complex domain.

References

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