Multiple orthogonal polynomials for special function weights

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Multiple orthogonal polynomials are polynomials in one variable that satisfy orthogonality conditions with respect to r measures. They appear naturally in Hermite-Padé approximation to r functions. The case $r = 1$ corresponds to the usual orthogonal polynomials. Several systems of multiple orthogonal polynomials have been constructed using classical weight functions (multiple Hermite, multiple Laguerre, multiple Jacobi polynomials). In this talk I will use weight functions given by special functions satisfying a differential equation. The r weights then appear by writing the differential equation as a system of first order equations, which then generalizes the Pearson equation for classical orthogonal polynomials. The weights are in terms of modified Bessel functions $K_\nu(2\sqrt{x})$ [3], modified Bessel functions $I_\nu(2\sqrt{x})$ [4], hypergeometric functions [1] and confluent hypergeometric functions [2], and the exponential integral [5]. We give some applications where these multiple orthogonal polynomials appear, such as the eigenvalues of products of random matrices, non-intersecting Brownian motions, and rational approximations to real numbers.

References

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