MATRICE

1. Izračunati: (a) $\begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}^{50}$, (b) $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}^{n}, n \in N$, (c) $\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}^{n}, n \in N$.

2. Ako je $A = \begin{bmatrix} 2 & 4 & 6 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$ naći A^n, $n \in N$.

3. Dokazati da je proizvod dve gornje (donje) trougaone matrice gornja (donja) trougaona matrica.

4. Data je matrica $A = \begin{bmatrix} 5 & -1 \\ 6 & 2 \end{bmatrix}$. Dokazati da je skup $S = \{B \in M_2(R) | AB = BA\}$ potprostor vektorskog prostora $M_2(R)$. Odrediti jednu bazu i dimenziju ovog potprostora.

5. (a) Ako je $M_n^T(R) = \{A \in M_n(R) | A^T = A\}$, $-M_n^T(R) = \{A \in M_n(R) | A^T = -A\}$ pokazati da su $M_n^T(R)$ i $-M_n^T(R)$ potprostori vektorskog prostora $M_n(R)$ i da je $M_n(R) = M_n^T(R) \oplus -M_n^T(R)$.

(b) Odrediti po jednu bazu i dimenziju prostora $M_2^T(R)$ i prostora $-M_2^T(R)$.

6. Ako je A matrica tipa $m \times n$, a B tipa $n \times m$ pokazati da je $tr(AB) = tr(BA)$.

DETERMINANTE

1. Izračunati

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{nn} \end{vmatrix}.$$

2. Izračunati: (a) $\begin{vmatrix} 4 & 5 & 6 & 7 \\ 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 \end{vmatrix}$, (b) $\begin{vmatrix} 1 & 5 & 3 & 5 & -4 \\ 3 & 1 & 2 & 9 & 8 \\ -1 & 7 & -3 & 8 & -9 \\ 3 & 4 & 2 & 4 & 7 \\ 1 & 8 & 3 & 3 & 5 \end{vmatrix}$.

3. (a) $\begin{vmatrix} 1 & 2 & 3 & 4 & \dots & n-1 & n \\ -1 & 0 & 3 & 4 & \dots & n-1 & n \\ -1 & -2 & 0 & 4 & \dots & n-1 & n \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & -2 & -3 & -4 & \dots & 0 & n \\ -1 & -2 & -3 & -4 & \dots & -(n-1) & n \end{vmatrix}$,

(b) $\begin{vmatrix} a_1 & x & x & \dots & x \\ x & a_2 & x & \dots & x \\ x & x & a_3 & \dots & x \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x & x & x & \dots & a_n \end{vmatrix}, \quad x \neq a_i,\ i = 1, \dots, n,$

(c) $\begin{vmatrix} a_0 & a_1 & a_2 & \dots & a_{n-1} & a_n \\ -x & x & 0 & \dots & 0 & 0 \\ 0 & -x & x & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -x & x \end{vmatrix},$

(d) $\begin{vmatrix} x_1 & a_2 & a_3 & \dots & a_n \\ a_1 & x_2 & a_3 & \dots & a_n \\ a_1 & a_2 & x_3 & \dots & a_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_1 & a_2 & a_3 & \dots & x_n \end{vmatrix},$

(e) $\begin{vmatrix} a_1 & 1 & 1 & \dots & 1 \\ 1 & a_2 & 0 & \dots & 0 \\ 1 & 0 & a_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & a_n \end{vmatrix}, \quad a_i \neq 0,\ i = 1, \dots, n.$

4. Izračunati Vandermondovu determinantu

$$\begin{vmatrix} 1 & a_1 & a_1^2 & \dots & a_1^{n-1} \\ 1 & a_2 & a_2^2 & \dots & a_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & a_n & a_n^2 & \dots & a_n^{n-1} \end{vmatrix}.$$

5. Izračunati: (a) $D_n = \begin{vmatrix} 2 & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 2 & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 2 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 2 & 1 \\ 0 & 0 & 0 & 0 & \dots & 1 & 2 \end{vmatrix},$

(b) $D_{2n} = \begin{vmatrix} a & 0 & 0 & 0 & \dots & 0 & b \\ 0 & a & 0 & 0 & \dots & b & 0 \\ 0 & 0 & a & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & b & 0 & 0 & \dots & a & 0 \\ b & 0 & 0 & 0 & \dots & 0 & a \end{vmatrix}.$

6. Izračunati: (a) $D_n = \begin{vmatrix} \alpha+\beta & \alpha\beta & 0 & 0 & \dots & 0 & 0 \\ 1 & \alpha+\beta & \alpha\beta & 0 & \dots & 0 & 0 \\ 0 & 1 & \alpha+\beta & \alpha\beta & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \alpha+\beta & \alpha\beta \\ 0 & 0 & 0 & 0 & \dots & 1 & \alpha+\beta \end{vmatrix}$,

(b) $D_n = \begin{vmatrix} 3 & -2 & 0 & 0 & \dots & 0 & 0 \\ 5 & 3 & -2 & 0 & \dots & 0 & 0 \\ 0 & 5 & 3 & -2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 3 & -2 \\ 0 & 0 & 0 & 0 & \dots & 5 & 3 \end{vmatrix}$.