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# ON UNIFORMLY STRONGLY PRIME Γ-SEMIHYPERRING

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Abstract. The Γ-semihyperring is a generalization of the concepts of a semiring, a semihyperring and a Γ-semiring. The concepts of uniformly strongly (weak) prime Γ-semihyperring and essential extension for the Γ-semihyperring are introduced and studied some important properties in this respect. It is proved that any essential extension of a uniformly strongly prime Γ-semihyperring is a uniformly weak prime Γ-semihyperring. Also strongly prime radical of a Γ-semihyperring is introduced and its characterization is made with the help of a super sp-system. A necessary and sufficient condition for a ideal of Γ-semihyperring to be a right strongly prime ideal is provided with the help of sp-system and super sp-system.

## 1. Introduction and Preliminaries

In 1975, Hadelman and Lawrence [4] introduced the notion of strongly prime ring motivated by the notion of primitive group ring and proved some properties of strongly prime rings. In 2006, Dutta and Das [2] introduced the notion of strongly prime ideal in a semiring and strongly prime semiring. Again in 2006, Dutta and Dhara [3] introduced the concept of uniformly strongly prime Γ-semirings and studied uniformly strongly prime k-radical of a Γ-semiring as special class via its operator semiring. The notion of essential ideal and essential extension for semirings was introduced and studied some important properties in this respect by Pawar and Deore [7].

The notion of hypergroup was introduced by Marty [5] in 1934. After that, many authors studied algebraic hyperstructure which are generalization of classical algebraic structure. In classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure composition of two elements is a

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set. Let $H$ be a non-empty set. Then, the map $\circ : H \times H \to \wp^*(H)$ is called a hyperopertion, where $\wp^*(H)$ is the family of all non-empty subsets of $H$ and the couple $(H, \circ)$ is called a hypergroupoid. Moreover, the couple $(H, \circ)$ is called a semihypergroup if for every $a, b, c \in H$ we have $(a \circ b) \circ c = a \circ (b \circ c)$. The notion of $\Gamma$-semihyperrings as a generalization of semiring, semihyperring and $\Gamma$-semiring was introduced by Dehkordi and Davvaz [8]. Also, Pawar et al. [6] introduced regular (strongly regular) $\Gamma$-semihyperrings and made it's characterization with the help of ideals in $\Gamma$-semihyperrings.

In now days hyperstructure theory was studied widely as it has vast applications in various streams of sciences. In this paper, we extended various concepts of classical algebraic structure to a $\Gamma$-semihyperring. In Section 2, we introduced the notion of uniformly strongly prime $\Gamma$-semihyperring, essential ideal and essential extension for $\Gamma$-semihyperring and proved some important properties. In Section 3, we introduced the notion of right strongly prime ideal and super sp-system. These concepts are studied analogously with the concepts of classical algebraic structures which are studied in [2, 3].

Here are some useful definitions and the readers are requested to refer [8].

**Definition 1.1.** Let $R$ be a commutative semihypergroup and $\Gamma$ be a commutative group. Then, $R$ is called a $\Gamma$-semihyperring if there is a map $R \times \Gamma \times R \to \wp^*(R)$ (images to be denoted by $a\alpha b$ for all $a, b \in R$ and $\alpha \in \Gamma$) and $\wp^*(R)$ is the set of all non-empty subsets of $R$ satisfying the following conditions:

(1) $a\alpha(b + c) = a\alpha b + a\alpha c$;
(2) $(a + b)\alpha c = a\alpha c + b\alpha c$;
(3) $a(\alpha + \beta)c = a\alpha c + a\beta c$;
(4) $a\alpha(b\beta c) = (a\alpha b)\beta c$,

for all $a, b, c \in R$ and for all $\alpha, \beta \in \Gamma$.

**Definition 1.2.** A $\Gamma$-semihyperring $R$ is said to be commutative if $a\alpha b = b\alpha a$ for all $a, b \in R$ and $\alpha \in \Gamma$.

**Definition 1.3.** A $\Gamma$-semihyperring $R$ is said to be with zero, if there exists $0 \in R$ such that $a \in a + 0$ and $0 \in 0\alpha a$, $0 \in a\alpha 0$ for all $a \in R$ and $\alpha \in \Gamma$.

Let $A$ and $B$ be two non-empty subsets of a $\Gamma$-semihyperring $R$ and $x \in R$. Then

$$A + B = \{x \mid x \in a + b, a \in A, b \in B\},$$
$$A\Gamma B = \{x \mid x \in a\alpha b, a \in A, b \in B, \alpha \in \Gamma\}.$$

**Definition 1.4.** A non-empty subset $R_1$ of $\Gamma$-semihyperring $R$ is said to be a $\Gamma$-subsemihyperring if it is closed with respect to the addition and multiplication, that is, $R_1 + R_1 \subseteq R_1$ and $R_1\Gamma R_1 \subseteq R_1$.

**Definition 1.5.** A right (left) ideal $I$ of a $\Gamma$-semihyperring $R$ is an additive sub semihypergroup of $(R, +)$ such that $I\Gamma R \subseteq I(R\Gamma I \subseteq I)$. If $I$ is both right and left ideal of $R$, then we say that $I$ is a two sided ideal or simply an ideal of $R$.

## 2. Uniformly Strongly (Weak) Prime Γ-Semihyperrings

**Definition 2.1.** A $\Gamma$-semihyperring $R$ is said to be a finitely multiplicative if $F$ and $G$ are finite subsets of $R$ and $\Delta$ is finite subset of $\Gamma$, then $F\Delta G$ is finite subset of $R$.

**Definition 2.2.** A $\Gamma$-semihyperring $R$ is said to be a finitely additive if $F$ and $G$ are finite subsets of $R$, then $F+G$ is finite subset of $R$.

*Example* 2.1 ([6]). Consider the following:

$$R = \left\{ \begin{pmatrix} x & y \\ z & w \end{pmatrix} \mid x, y, z, w \in \mathbb{R} \right\},$$
$$\Gamma = \{z \mid z \in \mathbb{Z}\},$$
$$A_\alpha = \left\{ \begin{pmatrix} \alpha a & 0 \\ 0 & \alpha b \end{pmatrix} \mid a, b \in \mathbb{R}, \alpha \in \Gamma \right\}.$$

Then, $R$ is a $\Gamma$-semihyperring under the matrix addition with hyperoperation $M\alpha N \mapsto MA_\alpha N$ for all $M, N \in R$ and $\alpha \in \Gamma$. Here $R$ is a finitely additive but not finitely multiplicative.

*Example* 2.2. Let $X$ be a non-empty set and $\tau$ is a topology on $X$. We define the hyperoperation of the addition and the multiplication on $\tau$ as $A, B \in \tau, A + B = A \cup B, A \cdot B = A \cap B$. Then $\tau$ is a $\Gamma$-semihyperring, where $\Gamma$ is a commutative group, if we define $x\alpha y \mapsto x \cdot y$ for every $x, y \in \tau, \alpha \in \Gamma$. Here $\tau$ is a finitely additive as well as finitely multiplicative.

Throughout this paper we consider that a $\Gamma$-semihyperring $R$ is always finitely multiplicative, finitely additive and contains a zero element.

**Definition 2.3.** A $\Gamma$-semihyperring $R$ is called uniformly right strongly prime if there exist a finite subset $F$ of $R$ and a finite subset $\Delta$ of $\Gamma$ if $0 \notin A \subseteq R$ and $0 \in A\delta_1 f\delta_2 B$ for all $\delta_1, \delta_2 \in \Delta$ and $f \in F$ implies that $0 \in B$. The pair $(F, \Delta)$ is called a uniform right insulator for $R$.

**Definition 2.4.** A $\Gamma$-semihyperring $R$ is called uniformly right weak prime if there exist a finite subset $F$ of $R$ and a finite subset $\Delta$ of $\Gamma$ if $x(\neq 0) \in R$ and $0 \in x\delta_1 f\delta_2 y$ for all $\delta_1, \delta_2 \in \Delta$ and $f \in F$ implies that $y = 0$. The pair $(F, \Delta)$ is called a uniform right insulator for $R$.

Analogously we can define uniformly left strongly (weak) prime $\Gamma$-semihyperring. It is obvious that a uniformly right (left) strongly prime $\Gamma$-semihyperring $R$ is uniformly right (left) weak prime.

**Definition 2.5** ([1]). A $\Gamma$-semihyperring $R$ with zero is called prime if $0 \in x\alpha r\beta y$ for all $r \in R$ and $\alpha, \beta \in \Gamma$ implies that either $x = 0$ or $y = 0$.

**Theorem 2.1.** *A $\Gamma$-semihyperring $R$ is uniformly right weak prime if and only if there exist finite subsets $F$ of $R$ and $\Delta$ of $\Gamma$ such that for any two nonzero elements $x$ and $y$ of $R$, there exists $f \in F$ and $\delta_1, \delta_2 \in \Gamma$ such that $0 \notin x\delta_1 f\delta_2 y$.*

*Proof.* Let $R$ be a uniformly right weak prime $\Gamma$-semihyperring and $(F, \Delta)$ be a uniform right insulator for $R$. Suppose $x$ and $y$ be a two nonzero elements of $R$ and $0 \in x\delta_1 f\delta_2 y$, for all $\delta_1, \delta_2 \in \Gamma$ and $f \in F$. Then we get $y = 0$, a contradiction. So there exist $f \in F$ and $\delta_1, \delta_2 \in \Gamma$ such that $0 \notin x\delta_1 f\delta_2 y$.

Conversely, let for any two nonzero elements $x$ and $y$ of $R$ there exist $f \in F$ and $\delta_1, \delta_2 \in \Gamma$ such that $0 \notin x\delta_1 f\delta_2 y$. Consider $a(\neq 0) \in R$ and $0 \in a\delta_1 f\delta_2 b$, for all $\delta_1, \delta_2 \in \Gamma$ and $f \in F$ so by our hypothesis $b$ must be 0. Therefore, by definition $\Gamma$-semihyperring $R$ is uniformly right weak prime. □

**Theorem 2.2.** *A $\Gamma$-semihyperring $R$ is uniformly right strongly prime if and only if there exist finite subsets $F$ of $R$ and $\Delta$ of $\Gamma$ such that for any two non-empty subsets $A$ and $B$ of $R$ and $0 \notin A, 0 \notin B$, there exist $f \in F$ and $\delta_1, \delta_2 \in \Delta$ such that $0 \notin A\delta_1 f\delta_2 B$.*

**Corollary 2.1.** *A $\Gamma$-semihyperring $R$ is uniformly right weak (strongly) prime if and only if $R$ is uniformly left weak (strongly) prime.*

So, we can use uniformly strongly (weak) prime instead of uniformly right (left) strongly (weak) prime and uniform insulator instead of uniform right (left) insulator.

**Proposition 2.1.** *A uniformly weak prime $\Gamma$-semihyperring is prime.*

*Proof.* Let $R$ be a uniformly weak prime $\Gamma$-semihyperring and $(F, \Delta)$ is a uniform insulator for $R$. Let $x(\neq 0) \in R$ and $0 \in x\alpha r\beta y$ for all $\alpha, \beta \in \Gamma$ and $r \in R$. Now, $F \subseteq R$ and $\Delta \subseteq \Gamma$, so $0 \in x\delta_1 f\delta_2 y$, for all $\delta_1, \delta_2 \in \Delta$ and $f \in F$. Since $R$ is a uniformly weak prime $\Gamma$-semihyperring and $(F, \Delta)$ is a uniform insulator for $R$, then $y = 0$. Therefore, by definition, $R$ is a prime $\Gamma$-semihyperring. □

**Proposition 2.2.** *If $R$ is uniformly weak prime $\Gamma$-semihyperring, then for nonzero ideal $I$ of $R$, there exist finite subsets $F$ of $I$ and $\Delta$ of $\Gamma$ such that $0 \in f\delta y$ for all $f \in F$ and $\delta \in \Delta$, then $y = 0$.*

*Proof.* Let $I$ be a nonzero ideal of a uniformly weak prime $\Gamma$-semihyperring $R$ and $(F, \Delta)$ is a uniform insulator for $R$. Let $x(\neq 0) \in I$. Then $F' = x\Delta F$ is finite subset of $I$. Also if $0 \in x\delta_1 f\delta_2 y$ for all $\delta_1, \delta_2 \in \Delta$ and $f \in F$, then $y = 0$. Then $0 \in f^{'}\delta y$, for all $f^{'} \in F^{'}, \delta \in \Delta$ implies that $0 \in x\delta_1 f\delta_2 y$, for all $\delta_1, \delta_2 \in \Delta$ and $f \in F$ gives that $y = 0$. This complete the proof. □

**Definition 2.6.** A nonzero ideal $I$ of a $\Gamma$-semihyperring $R$ is called an essential ideal of $R$ if for any nonzero ideal $J$ of $R$, $x(\neq 0) \in I \cap J$.

**Definition 2.7.** A $\Gamma$-semihyperring $T$ is said to be an essential extension of a $\Gamma$-semihyperring $R$ if $R$ is an essential ideal of $T$.

**Definition 2.8.** Let $A$ be a non-empty subset of a $\Gamma$-semihyperring $R$. Right annihilator of $A$ in $R$, denoted by $ann_r(A)$, is defined as $ann_r(A) = \{x \in R \mid 0 \in a\alpha x$ for all $a \in A, \alpha \in \Gamma\}$.

Similarly, we can define left annihilator of $A$ in $R$, i.e., $ann_l(A)$.

**Lemma 2.1.** *Let $R$ be a $\Gamma$-semihyperring and $T$ be its essential extension. If $R$ is a uniformly strongly prime $\Gamma$-semihyperring, then for each nonzero $x$ of $T$, $0 \in x\alpha f$ for all $\alpha \in \Gamma, f \in F$ implies that $x \in ann_r(R)$ and $0 \in f\alpha x$ for all $\alpha \in \Gamma, f \in F$ implies that $x \in ann_l(R)$, where $(F, \Delta)$ is a uniform insulator for $R$.*

*Proof.* Let $T$ be an essential extension of a uniformly strongly prime $\Gamma$-semihyperring $R$ and $(F, \Delta)$ is uniform insulator for $R$. Let $x(\neq 0) \in T$ and $0 \in x\alpha f$ for all $\alpha \in \Gamma, f \in F$. Then $0 \in (k\gamma x)\delta_1 f\delta_2(k\gamma x)$ for all $\delta_1, \delta_2 \in \Delta, \gamma \in \Gamma, f \in F$ and $k \in R$. Since $R$ is a uniformly strongly prime $\Gamma$-semihyperring and $(F, \Delta)$ is a uniform insulator for $R, 0 \in k\gamma x$ for all $k \in R, \gamma \in \Gamma$, i.e., $x \in ann_r(R)$.

On similar lines, we can prove $0 \in f\alpha x$ for all $\alpha \in \Gamma$, $f \in F$ implies that $x \in ann_l(R)$. □

**Lemma 2.2.** *If $R$ is a uniformly strongly prime $\Gamma$-semihyperring and $I$ is an ideal of $R$, then $I$ is a uniformly weak prime $\Gamma$-subsemihyperring.*

*Proof.* Let $R$ be a uniformly strongly prime $\Gamma$-semihyperring and $(F, \Delta)$ be a uniform insulator for $R$. If $I$ is zero ideal, then obviously $I$ is a uniformly weak prime $\Gamma$-subsemihyperring. Suppose $I \neq 0$ and $r$ be a fixed nonzero element of $I$. Let $F' = \{x \in f_1\alpha r\beta f_2 \mid f_1, f_2 \in F, \alpha, \beta \in \Delta\}$. Since $I$ is an ideal of $R$ and $F$, $\Delta$ are finite subsets, $F'$ is finite subset of $I$. Let $x(\neq 0) \in I$ and $y \in I$. If $0 \in x\delta_1 f'\delta_2 y$ for all $\delta_1, \delta_2 \in \Delta$ and $f' \in F'$, then $0 \in x\delta_1 f_1\alpha r\beta f_2\delta_2 y$ for all $f_1, f_2 \in F$ and for all $\delta_1, \delta_2, \alpha, \beta \in \Delta$, i.e., $0 \in x\delta_1 f_1\alpha(r\beta f_2\delta_2 y)$ for all $f_1, f_2 \in F$ and for all $\delta_1, \delta_2, \alpha, \beta \in \Delta$. Since $r\beta f_2\delta_2 y \subseteq R$, for all $f_2 \in F$ and for all $\beta, \delta_2 \in \Delta$ and $R$ is a uniformly strongly prime $\Gamma$-semihyperring with $x \neq 0$, then $0 \in r\beta f_2\delta_2 y$ for all $f_2 \in F$ and for all $\beta, \delta_2 \in \Delta$. But as $r \neq 0$ it gives $y = 0$. Hence, $I$ is a uniformly weak prime $\Gamma$-semihyperring and $(F', \Delta)$ is uniform insulator for $I$. □

**Definition 2.9.** An element $k$ of a $\Gamma$-semihyperring $R$ is additively aggressive with respect to subset $A$ of $R$ if $k$ belongs to $a\alpha b(b\alpha a)$ and $a\alpha c(c\alpha a)$ for all $a \in A$ and $\alpha \in \Gamma$, then for any $p \in b + c$, $k \in a\alpha p$ $(k \in p\alpha a)$ for all $a \in A$ and $\alpha \in \Gamma$.

**Definition 2.10.** An element $k$ of a $\Gamma$-semihyperring $R$ is multiplicatively aggressive with respect to subset $A$ of $R$ if $k$ belongs to $a\alpha b(b\alpha a)$ for all $a \in A$ and $\alpha \in \Gamma$, then for any $p \in b\alpha t(p \in t\alpha b)$, where $\alpha \in \Gamma$, $t \in R$, we have $k \in a\alpha p(k \in p\alpha a)$ for all $a \in A$ and $\alpha \in \Gamma$.

Example 2.1 zero element (zero matrix) is a multiplicatively aggressive.

**Definition 2.11.** An element $k$ of a $\Gamma$-semihyperring $R$ is additively and multiplicatively aggressive with respect to all subset $A$ of $R$, then $k$ is aggressive element of a $\Gamma$-semihyperring $R$.

*Example* 2.3 ([6]). Let $R = \{a, b, c, d\}$. Then $R$ is commutative semihypergroup with following hyperoperations

| $+$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $a$ | $\{a\}$ | $\{a, b\}$ | $\{a, c\}$ | $\{a, d\}$ |
| $b$ | $\{a, b\}$ | $\{b\}$ | $\{b, c\}$ | $\{b, d\}$ |
| $c$ | $\{a, c\}$ | $\{b, c\}$ | $\{c\}$ | $\{c, d\}$ |
| $d$ | $\{a, d\}$ | $\{b, d\}$ | $\{c, d\}$ | $\{d\}$ |

,

| $\cdot$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $a$ | $\{a\}$ | $\{a, b\}$ | $\{a, b, c\}$ | $\{a, b, c, d\}$ |
| $b$ | $\{a, b\}$ | $\{b\}$ | $\{b, c\}$ | $\{b, c, d\}$ |
| $c$ | $\{a, b, c\}$ | $\{b, c\}$ | $\{c\}$ | $\{c, d\}$ |
| $d$ | $\{a, b, c, d\}$ | $\{b, c, d\}$ | $\{c, d\}$ | $\{d\}$ |

.

Then $R$ be a $\Gamma$-semihyperring, where $\Gamma$-is any commutative group with operation $x\alpha y \mapsto x \cdot y$ for $x, y \in R$ and $\alpha \in \Gamma$. Here $a$ is a aggressive element of $R$.

**Theorem 2.3.** *If zero is an aggressive element of a $\Gamma$-semihyperring $R$, then $ann_r(A)$ is a right ideal of $R$ and $ann_l(A)$ is a left ideal of $R$. If $A$ is an ideal of $\Gamma$-semihyperring $R$, then both annihilators are ideals of $R$.*

Now in the rest part of the given section we consider zero as aggressive element of $\Gamma$-semihyperring $R$.

**Lemma 2.3.** *Let $R$ be a uniformly weak prime $\Gamma$-semihyperring and $T$ be its essential extension. Then both annihilators of $R$ in $T$ are zero.*

*Proof.* Let $(F, \Delta)$ is a uniform insulator for $R$. If possible let $ann_r(R) \neq 0$. Then $ann_r(R)$ is nonzero ideal of $T$. Since $R$ is an essential ideal of $T$, $ann_r(R) \cap R \neq 0$. Let $x(\neq 0) \in ann_r(R) \cap R$. Then $0 \in k\alpha x$, for all $k \in R, \alpha \in \Gamma$. As $\Delta \subseteq \Gamma$ and $F \subseteq R$, it gives $0 \in x\delta_1 r\delta_2 x$ for all $\delta_1, \delta_2 \in \Delta$ and $r \in F$. Since $R$ is a uniformly weak prime $\Gamma$-semihyperring, $x = 0$, a contradiction. Therefore $ann_r(R) = 0$.

Similarly, we can prove that $ann_l(R) = 0$. □

**Lemma 2.4.** *Let $R$ be a uniformly strongly prime $\Gamma$-semihyperring with pair $(F, \Delta)$ be a uniform insulator for $R$ and $T$ be its essential extension. Then for any nonzero element $x$ of $T$ there exist some $f \in F$, $\delta \in \Delta$ such that $0 \notin x\delta f$.*

*Proof.* Let $(F, \Delta)$ be a uniform insulator for $R$ and $T$ be an essential extension of $R$. Let $x$ be a nonzero element of $T$. Suppose that $0 \in x\delta f$ for all $\delta \in \Delta$, $f \in F$, then by Lemma 2.1, $x \in ann_r(R)$. Also, by Lemma 2.3, $ann_r(R) = 0$, which implies that $x = 0$, a contradiction. Therefore, $0 \notin x\delta f$ for some $f \in F, \delta \in \Delta$. □

**Lemma 2.5.** *Let $R$ be a uniformly strongly prime $\Gamma$-semihyperring with pair $(F, \Delta)$ be a uniform insulator for $R$ and $T$ be its essential extension. Then for any nonzero element $x$ of $T$ there exist some $f \in F$, $\delta \in \Delta$ such that $0 \notin f\delta x$.*

**Theorem 2.4.** *Any essential extension of a uniformly strongly prime* $\Gamma$*-semihyperring* $R$ *is a uniformly weak prime* $\Gamma$*-semihyperring.*

*Proof.* Let $(F, \Delta)$ be a uniform insulator for $R$ and $T$ be an essential extension of $R$. Let $y, z$ be two nonzero elements of $T$. Then by Lemmas 2.4 and 2.5, there exist $f_1, f_2 \in F$ and $\delta_1, \delta_2 \in \Delta$ such that $0 \notin y\delta_1 f_1$ and $0 \notin f_2\delta_2 z$. Since $R$ is an ideal of $T$, so $y\delta_1 f_1$ and $f_2\delta_2 z$ are subsets of $R$. Again since $R$ is uniformly strongly prime and $(F, \Delta)$ be a uniform insulator for $R$, then by Theorem 2.2, there exist $\alpha, \beta \in \Delta$ and $f \in F$ such that $0 \notin y\delta_1 f_1 \alpha f \beta f_2 \delta_2 z$. Let $F' = \{k \in f_1\alpha f\beta f_2 \mid 0 \notin y\delta_1 f_1\alpha f\beta f_2\delta_2 z; f_1, f, f_2 \in F, \alpha, \beta, \delta_1, \delta_2 \in \Delta, y, z \in T\}$. Then $F' \subseteq T$ is finite subset, since $F$ and $\Delta$ are finite subset. Hence, by Theorem 2.1, $T$ is uniformly weak prime $\Gamma$-semihyperring with insulator $(F', \Delta)$. □

## 3. Right Uniformly Strongly Prime Radical

**Definition 3.1.** An ideal $I$ of a $\Gamma$-semihyperring $R$ is said to be right strongly prime if $a \notin I$, then there are two finite sets $F \subseteq < a >$ and $\Delta \subseteq \Gamma$ such that $F\Delta b \subseteq I$ implies that $b \in I$.

**Definition 3.2.** A subset $G$ of a $\Gamma$-semihyperring $R$ is called an sp-system if for any $g \in G$ there are two finite sets $F \subseteq < g >$ and $\Delta \subseteq \Gamma$ such that $(f\delta z) \cap G \neq \emptyset$ for all $f \in F$, $\delta \in \Delta$ and $z \in G$.

**Proposition 3.1.** *An ideal* $I$ *of a* $\Gamma$*-semihyperring* $R$ *is a right strongly prime if and only if* $R \setminus I$ *is an sp-system.*

*Proof.* Let $I$ be a right strongly prime ideal of $R$ and let $g \in R \setminus I$. Then $g \notin I$. So there exists a finite subsets $F$ of $< g >$ and $\Delta$ of $\Gamma$ such that $F\Delta b \subseteq I$ implies that $b \in I$, i.e., $(f\delta z) \cap (R \setminus I) \neq \emptyset$ for all $f \in F$, $\delta \in \Delta$ and $z \in R \setminus I$. Therefore, $R \setminus I$ is an sp-system.

Conversely, suppose $R \setminus I$ is an sp-system. Let $a \notin I$. Then $a \in R \setminus I$. So there exists a finite subsets $F$ of $< a >$ and $\Delta$ of $\Gamma$ such that $(f\delta z) \cap (R \setminus I) \neq \emptyset$ for all $f \in F, \delta \in \Delta$ and $z \in R \setminus I$. Let $F\Delta b \subseteq I$. Then $F\Delta b \cap (R \setminus I) = \emptyset$. If possible let $b \notin I$. Then $b \in R \setminus I$ which implies that $(f\delta b) \cap (R \setminus I) \neq \emptyset$ for all $f \in F$, $\delta \in \Delta$, a contradiction. Hence, $b \in I$. Therefore, $I$ is a right strongly prime ideal of $R$. □

**Definition 3.3.** Right strongly prime radical of a $\Gamma$-semihyperring $R$ is a defined by $SP(R) = \cap\{I \mid I \text{ is a right strongly prime ideal of } R\}$.

**Definition 3.4.** A pair of subsets $(G, P)$ where $P$ is an ideal of a $\Gamma$-semihyperring $R$ and $G$ is a non-empty subset of $R$ is called a super sp-system of $R$ if $G \cap P$ contains no nonzero element of $R$ and for any $g \in G$ there are finite subsets $F$ of $< g >$ and $\Delta$ of $\Gamma$ such that $(f\delta z) \cap G \neq \emptyset$ for all $f \in F$, $\delta \in \Delta$ and $z \notin P$.

*Remark* 3.1. An ideal $I$ of a $\Gamma$-semihyperring $R$ is a right strongly prime ideal if and only if $(R \setminus I, I)$ is super sp-system.

**Theorem 3.1.** *Let any $\Gamma$-semihyperring $R$. Then $x \in SP(R)$ if and only if whenever $x \in G$ and $(G, P)$ is super sp-system for some ideal $P$ of $R$, then $0 \in G$.*

*Proof.* Let $x \in SP(R)$. If possible let $x \in G$ where $(G, P)$ is a super sp-system and $0 \notin G$. Then $G \cap P = \emptyset$. By Zorn's Lemma choose an ideal $Q$ with $P \subseteq Q$ and $Q$ is maximal with respect to $G \cap Q = \emptyset$. We now prove that $Q$ is a right strongly prime ideal of $R$. Let $a \notin Q$. Then there is a $g \in G$ such that $< g > \subseteq Q + < a >$. Since $(G, P)$ is a super sp-system there exists a finite subsets $F = \{f_1, f_2, \ldots, f_m\} \subseteq < g >$ and $\Delta \subseteq \Gamma$ such that $f_i \delta z \cap G \neq \emptyset$ for all $f_i \in F$, $\delta \in \Delta$ and $z \notin P$. Since $F \subseteq Q + < a >$ each $f_i \in q_i + a_i$ for some $q_i \in Q$ and $a_i \in < a >$. Let $F^* = \{a_1, a_2, \ldots, a_m\}$. Then $F^* \subseteq < a >$. Let $z \in R$ such that $f_i^* \delta z \subseteq Q$ for all $f_i^* \in F^*$, $\delta \in \Delta$. Then $f_i \delta z \subseteq (q_i + ai)\delta z \subseteq Q$ for all $f_i \in F$, $\delta \in \Delta$, i.e., $F \Delta z \subseteq Q$. If $z \notin Q$, then $f_i \delta z \cap G \neq \emptyset$, because $P \subseteq Q$. But this contradict $G \cap Q = \emptyset$. Hence, $z \in Q$ must hold. So, $Q$ is a right strongly prime ideal. But $x \notin Q$, since $x \in G$, which is a contradiction. Hence, $0 \in G$.

Conversely, let whenever $x \in G$ and $(G, P)$ is super sp-system for some ideal $P$ of $R$, then $0 \in G$. Then there exists a right strongly prime ideal $I$ of $R$ such that $x \notin I$. Then $(R \setminus I, I)$ is a super sp-system where $x \in R \setminus I$ but $0 \notin R \setminus I$, a contradiction. Hence, converse follows. $\square$

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