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SINGLE-VALUED NEUTROSOPHIC SET WITH HYBRID NUMBER INFORMATION: AN INTRODUCTORY STUDY

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Abstract. In this paper, we introduce the concept of hybrid single-valued neutrosophic number, whose basic units are truth, falsity and indeterminacy memberships, and their properties are investigated. Then, we give the hybrid single-valued neutrosophic whose coefficients are consecutive Fibonacci and Lucas. Especially for consecutive coefficient Fibonacci and Lucas hybrid single-valued neutrosophic numbers, fundamental properties and identities such as Tagiuri, d'Ocagne, Catalan, and Cassini are given. We obtain the Binet formula and generating function formula for these numbers. Moreover, we give some sums of the consecutive coefficient Fibonacci and Lucas hybrid single-valued neutrosophic numbers.

1. Introduction

Number sequences arise in many different theoretical and applied areas, as well as in mathematical modeling of all the problems where there is a kind of invariance to shift in terms of space or of time. As in the computation of spline functions, signal and image processing, queueing theory, time series, analysis, polynomial and power series computations and many other areas, typical problems modelled by number sequences are the numerical solution of certain differential and integral equations. Ide and Renault [16] investigated integer sequences H satisfying the Fibonacci recurrence relation $H_n = H_{n-1} + H_{n-2}$ that also have the property that $H \equiv a^n$, $n \in \mathbb{N}$, for some modulus m. Sacco [25] investigated the complex relationship between fractal structures and cultural evolution using the Fibonacci time series model. Bhattacharya and Kumar [2] took over some of the popular technical analysis methodologies based

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on Fibonacci sequences and also advanced a theoretical rationale as to why security prices may be seen to follow such sequences. The Fibonacci sequence has delighted mathematics and scientists alike for centuries because of its beauty and tendency to appear in unexpected places such as the Pascal triangle, fractal types, graph theory, computer algorithms, geometry, stock market analysis, and graphic design. Music, finance, engineering, art, geostatistics, architecture, signaling, physics, and biology are some of the main fields of study. In Conti's [6] study, some mathematical and historical properties of Fibonacci numbers were shown, focused on their applications in art, music, and geometry. Falcon and Plaza [10] introduced a general Fibonacci sequence by studying the recursive application of two geometrical transformations used in the well-known four-triangle longest-edge partition. Keçilioğlu and Akkuş [19] obtained the Fibonacci and Lucas octonions and gave the generating function and Binet formula for these octonions. Vajda [28] brought together in his book some of the studies on the theory and applications of Fibonacci and Lucas numbers. They are unique in that they are emerging in other areas as well.

Fibonacci and Lucas numbers are defined by the following recurrence relations

$$\begin{aligned} F_0 &= 0, \quad F_1 = 1, \quad F_{n+2} = F_{n+1} + F_n, \quad n \geq 0, \\ L_0 &= 2, \quad L_1 = 1, \quad L_{n+2} = L_{n+1} + L_n, \quad n \geq 0, \end{aligned}$$

respectively. Besides, the nth Fibonacci and Lucas numbers are formulized as

$$F_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad L_n = \alpha^n + \beta^n, \quad n \geq 0,$$

where $\alpha = \frac{1+\sqrt{5}}{2}$ and $\beta = \frac{1-\sqrt{5}}{2}$ [20].

Complex numbers, Hyperbolic numbers, Dual numbers and Hybrid numbers arise in many areas such as velocity analysis, coordinate transformation, displacement analysis, matrix modeling, rigid body dynamics, static analysis, mechanics, dynamic analysis, transformation, kinematics, physics, biology, mathematics, and geometry [9, 13, 15, 27].

Horadam [14] introduced the concept, the complex Fibonacci numbers $C = F_n + iF_{n+1}$ where $F_n \in \mathbb{R}$, $i^2 = -1$ and F_n, nth Fibonacci numbers. Fjelstad and Gal [11] defined the hyperbolic numbers $H = a + jb$ where $a, b \in \mathbb{R}$, $j^2 = 1$ and $j \neq \mp 1$. Clifford [5] described the dual numbers $D = a + \varepsilon b$ where $a, b \in \mathbb{R}$, $\varepsilon^2 = 0$ and $\varepsilon \neq 0$. Özdemir [24] presented the hybrid numbers $J = a + ib + \varepsilon c + jd$, where $a, b, cd \in \mathbb{R}$, $i^2 = -1$, $\varepsilon^2 = 0$, $\varepsilon \neq 0$ and $j^2 = 1$, $j \neq \mp 1$. The hybrid numbers are a number system. The hybrid numbers of the form $H = a + ib + \varepsilon c + jd$, where a, b, c, d are real numbers and i, ε, j are basic units with the properties form Table 1.

The journey that started with inferring meaning and mathematical results from uncertainty has continued with intuitive uncertainty, including membership and non-membership states. It has reached the present day by adding indeterminacy to its current journey. Neutrosophic Logic is a nascent field of study in which each proposition is estimated to have a rate of truth in a subset of T, an uncertainty rate

TABLE 1. The multiplication properties of basic units.

$\cdot$	i	ε	j
i	-1	1-j	ε+i
ε	j+1	0	-ε
j	-ε-i	ε	1

in a subset of I, and a falsity rate in a subset of F. In [26], M being a space of points, a single-valued neutrosophic set A on a non-empty set M is characterized by a truth-membership function $T_A : M \to [0,1]$, an indeterminacy-membership function $I_A : M \to [0,1]$ and a falsity-membership function $F_A : M \to [0,1]$. Thus, $A = \Big\{\Big\langle x, T_A(x), I_A(x), F_A(x)\Big\rangle \mid x \in M\Big\}$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ for all $x \in M$. Obviously, every ordinary neutrosophic fuzzy set have the form

$$\begin{aligned} N =& \Big\{\Big\langle x, T_A(x), I_A(x), F_A(x)\Big\rangle \mid x \in M\Big\}, \\ 0 \leq & T_A(x) + I_A(x) + F_A(x) \leq 1 \text{ (or } \leq 2 \text{ or } \leq 3). \end{aligned}$$

The concept of fuzzy subsets was introduced by Zadeh [30] and later applied in various branches of mathematics. Zadeh's expansion principle allowed us to describe arithmetical operations between fuzzy numbers by expanding classical ones. Kandasamy and Smarandache [18] defined the standard form of the neutrosophic complex number. Dubois and Prade [8] drew attention to their arithmetic properties. Buckley [4] took the first steps from fuzzy real numbers to complex fuzzy numbers. Zhang [12] introduced a new definition for fuzzy complex numbers and obtained some results. Moura et al. [22] proposed to extend the real fuzzy numbers to quaternions fuzzy numbers and to investigate their properties. Yormaz et al. [29] defined the basic operations of fuzzy split quaternions and gave some geometric properties of this fuzzy quaternion.

Fuzzy, intuitionistic, and neutrosophic set to decision-making have been widely applied in many fields in recent years and receive increasing attention, like engineering, linguistics, medical treatment, statistics, multicriteria, experimental sciences, and so on. They [1] investigated the suitability of combining the intuitionistic hesitant fuzzy set and set pair analysis theories in multi-attribute decision making and obtained the hybrid model named intuitionistic hesitant fuzzy connection number set. In the work of Broumi et al. [3] a new concept of complex fermatean neutrosophic graph was established, and various basic graph ideas such as the order, size, degree, and total degree of a vertex were introduced. Derrac [7] evaluated the most relevant approaches, applications, and theoretical studies on fuzzy nearest neighbor classification and stated various defining features to create a complete taxonomy. In[17], Jian et al. focused on the global Mittag-Leffler boundedness for fractional-order fuzzy quaternion-valued neural networks with linear threshold neurons. Liu and Wen [21] gave a new distance measure based on the distance of interval numbers in interval-valued intuitionistic fuzzy sets. Ngan et al. [23] generalized and expanded the utility of complex intuitionistic fuzzy sets using the space of quaternion numbers. The purposed study received composite features and conveyed multi-dimensional fuzzy information via the functions of real membership, imaginary membership, real non-membership, and imaginary non-membership. In [31], Zulqarnain et al. aimed to develop cosine and set theoretic similarity measures for the generalized multipolar neutrosophic soft set. Hybrid single-valued neutrosophic numbers provide a way to extend the neutrosophic set theory based on number fields. The field of hybrid numbers is another fundamental number field that we cannot ignore, so we continue to extend the hybrid single-valued neutrosophic numbers theory based on number fields.

In the following sections, the hybrid single-valued neutrosophic number is defined; the definition of a number whose basic units contain neutrosophic set elements has not been studied so far. Hybrid single-valued neutrosophic numbers, whose coefficients consist of consecutive Fibonacci and Lucas numbers, are defined. In this work, a variety of algebraic properties of the consecutive coefficient Fibonacci and Lucas hybrid single-valued neutrosophic numbers are presented in a unified manner. Some identities will be given for the consecutive coefficient Fibonacci and Lucas hybrid single-valued neutrosophic numbers such as Binet's formula, generating function formula, Tagiuri's identity, d'Ocagne's identity, Catalan's identity, Cassini's identity, and some sum formulas.

2. The Hybrid Neutrosophic Numbers

In this part, analogies with complex numbers, hyperbolic numbers, and dual neutrosophic numbers will be used to construct complex fuzzy numbers, hyperbolic intuitionistic numbers, and dual neutrosophic numbers. Additionally, the definition of hybrid single-valued neutrosophic numbers, which are created when these three numbers are combined, will be provided. Neutrosophic numbers, whose constituents exhibit the characteristics of well-known number sequences, appear to have a variety of uses.

Definition 2.1. The complex fuzzy numbers are defined by

$$\mathbb{C} = a + ib,$$

where a, b are real numbers and $i^2 = T_A(x)$, $T_A(x) \in [0, 1]$.

Definition 2.2. The hyperbolic intuitionistic numbers are defined by

$$\mathbb{H} = a + jb,$$

where a, b are real numbers and $j^2 = F_A(x)$, $j \neq 1$ and $F_A(x) \in [0, 1]$.

Definition 2.3. The dual neutrosophic numbers are defined by

$$\mathbb{D} = a + \varepsilon b,$$

where a, b are real numbers and $\varepsilon^2 = I_A(x)$, $\varepsilon \neq 0$ and $I_A(x) \in [0, 1]$.

Table 2. Some properties of complex fuzzy numbers, hyperbolic intuitionistic numbers, and dual neutrosophic numbers.

	Complex fuzzy numbers	Hyperbolic intuitionistic numbers	Dual neutrosophic numbers
Property	$\mathbb{C}$=a+ib, $i^2 = T_A(x)$	$\mathbb{H}$=a+jb, $j^2 = F_A(x)$	$\mathbb{D}$=a+εb, $\varepsilon^2 = I_A(x)$
Conjugate	$\overline{\mathbb{C}}$=a-ib	$\overline{\mathbb{H}}$=a-jb	$\overline{\mathbb{D}}$=a-εb
Norm	$\|\mathbb{C}\|=\sqrt{\|a^2 - T_A(x)b^2\|}$	$\|\mathbb{H}\|=\sqrt{\|a^2 - F_A(x)b^2\|}$	$\|\mathbb{D}\|=\sqrt{\|a^2 - I_A(x)b^2\|}$
Geometry	Euclidean geometry	Lorentzian geometry	Galilean geometry
Rotation type	Elliptic rotation	Hyperbolic rotation	Parabolic rotation

Definition 2.4. The hybrid neutrosophic numbers are defined by

$$\mathfrak{H} = a + ib + \varepsilon c + jd,$$

where a, b, c, d are real numbers and i, ε, j are hybrid number units which satisfy the equalities

$$\begin{aligned} i^2 =& T_A(x), \quad \varepsilon^2 = I_A(x), \quad j^2 = F_A(x), \\ \varepsilon \neq& 0, \quad j \neq 1, \quad ij = -ji = \varepsilon + i, \end{aligned}$$

where $T_A(x)$, $I_A(x)$ and $F_A(x) \in [0, 1]$ are the truth-membership, the indeterminacy-membership and the falsity-membership values of the single-valued neutrosophic set and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 1$ (or ≤ 2 or ≤ 3).

The hybrid neutrosophic numbers of the form $\mathfrak{H} = a + ib + \varepsilon c + jd$, where a, b, c, d are real numbers and i, ε, j are basic units.

Table 3. The multiplication properties of basic units.

$\cdot$	i	ε	j
i	$T_A(x)$	1-j	ε+i
ε	j+1	$I_A(x)$	-ε
j	-ε-i	ε	$F_A(x)$

Definition 2.5. The consecutive coefficient Fibonacci hybrid single-valued neutrosophic numbers ($\mathfrak{H}^{\mathfrak{F}}$) are defined by

$$\mathfrak{H}^{\mathfrak{F}}{}_n = F_n + iF_{n+s} + \varepsilon F_{n+2s} + jF_{n+3s}, \tag{2.1}$$

where F_n is nth Fibonacci numbers and i, ε, j are hybrid units which satisfy the equalities

$$\begin{aligned} i^2 =& T_A(x), \quad \varepsilon^2 = I_A(x), \quad j^2 = F_A(x), \\ \varepsilon \neq& 0, \quad j \neq 1, \quad ij = -ji = \varepsilon + i, \end{aligned}$$

where $T_A(x)$, $I_A(x)$ and $F_A(x) \in [0,1]$ are the truth-membership, the indeterminacy-membership and the falsity-membership values of the single-valued neutrosophic set and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 1$ (or ≤ 2 or ≤ 3).

Remark 2.1. The consecutive coefficient Fibonacci hybrid single-valued neutrosophic numbers whose terms are in the form a decreasing sequence are called consecutive coefficient Gaussian Fibonacci hybrid single-valued neutrosophic numbers. The consecutive coefficient Gaussian Fibonacci hybrid single-valued neutrosophic numbers ${\mathfrak{G}^{\mathfrak{F}}}_n$ are defined by

$$ {\mathfrak{G}^{\mathfrak{F}}}_n = F_n + iF_{n-s} + \varepsilon F_{n-2s} + jF_{n-3s}. \tag{2.2} $$

Remark 2.2. Since the Lucas sequence is also obtained from the roots of the characteristic equation of the Fibonacci sequence, similar hybrid numbers can be defined in the Lucas sequence by

$$ \begin{aligned} {\mathfrak{H}^{\mathfrak{L}}}_n &= L_n + iL_{n+s} + \varepsilon L_{n+2s} + jL_{n+3s}, \\ {\mathfrak{G}^{\mathfrak{L}}}_n &= L_n + iL_{n-s} + \varepsilon L_{n-2s} + jL_{n-3s}. \end{aligned} $$

Remark 2.3. The recurrence relation between Fibonacci and Lucas numbers also applies to consecutive coefficient Fibonacci and Lucas hybrid single-valued neutrosophic numbers.

$$ \begin{aligned} {\mathfrak{H}^{\mathfrak{F}}}_{n+2} &= {\mathfrak{H}^{\mathfrak{F}}}_{n+1} + {\mathfrak{H}^{\mathfrak{F}}}_n, \\ {\mathfrak{H}^{\mathfrak{L}}}_n &= {\mathfrak{H}^{\mathfrak{F}}}_{n+1} + {\mathfrak{H}^{\mathfrak{F}}}_{n-1}, \\ {\mathfrak{H}^{\mathfrak{L}}}_n &= {\mathfrak{H}^{\mathfrak{F}}}_{n+2} - {\mathfrak{H}^{\mathfrak{F}}}_{n-2}, \\ {\mathfrak{H}^{\mathfrak{L}}}_{n+2} &= {\mathfrak{H}^{\mathfrak{L}}}_{n+1} + {\mathfrak{H}^{\mathfrak{L}}}_n, \\ 5{\mathfrak{H}^{\mathfrak{F}}}_n &= {\mathfrak{H}^{\mathfrak{L}}}_{n+1} + {\mathfrak{H}^{\mathfrak{L}}}_{n-1}, \\ 5{\mathfrak{H}^{\mathfrak{L}}}_n &= {\mathfrak{H}^{\mathfrak{L}}}_{n+2} - {\mathfrak{H}^{\mathfrak{L}}}_{n-2}. \end{aligned} $$

The same equations apply to consecutive coefficient Gaussian Fibonacci and Lucas hybrid single-valued neutrosophic numbers.

Theorem 2.1. *The Binet formula for the consecutive coefficient Fibonacci hybrid single-valued neutrosophic number is*

$$ {\mathfrak{H}^{\mathfrak{F}}}_n = \frac{\alpha^* \alpha^n - \beta^* \beta^n}{\alpha - \beta}, \tag{2.3} $$

where $\alpha = \frac{1+\sqrt{5}}{2}$, $\beta = \frac{1-\sqrt{5}}{2}$, $\alpha^* = 1 + i\alpha^s + \varepsilon\alpha^{2s} + j\alpha^{3s}$ *and* $\beta^* = 1 + i\beta^s + \varepsilon\beta^{2s} + j\beta^{3s}$.

Proof. We have

$$ {\mathfrak{H}^{\mathfrak{F}}}_n = \frac{\alpha^n - \beta^n}{\alpha - \beta} + i\frac{\alpha^{n+s} - \beta^{n+s}}{\alpha - \beta} + \varepsilon\frac{\alpha^{n+2s} - \beta^{n+2s}}{\alpha - \beta} + j\frac{\alpha^{n+3s} - \beta^{n+3s}}{\alpha - \beta}. $$

Then, some basic calculations are made and the desired result can be easily archieved. $\square$

The Binet formula consecutive coefficient Lucas hybrid single-valued neutrosophic number is verified similarly.

Theorem 2.2. *The generating function formula for the consecutive coefficient Fibonacci hybrid single-valued neutrosophic number is*

$$\sum_{i=0}^{+\infty} \mathfrak{H}^{\mathfrak{F}}{}_i t^i = \frac{\mathfrak{H}^{\mathfrak{F}}{}_0 + (\mathfrak{H}^{\mathfrak{F}}{}_1 - \mathfrak{H}^{\mathfrak{F}}{}_0)t}{1-t-t^2}. \tag{2.4}$$

Proof. Let $h(t)$ function be the generating function formula of the consecutive coefficient Fibonacci hybrid single-valued neutrosophic number. We have

$$h(t) = \sum_{i=0}^{+\infty} \mathfrak{H}^{\mathfrak{F}}{}_i t^i = \mathfrak{H}^{\mathfrak{F}}{}_0 + \mathfrak{H}^{\mathfrak{F}}{}_1 t + \mathfrak{H}^{\mathfrak{F}}{}_2 t^2 + \cdots + \mathfrak{H}^{\mathfrak{F}}{}_n t^n + \cdots .$$

Then, $th(t) = \sum_{i=0}^{+\infty} \mathfrak{H}^{\mathfrak{F}}{}_i t^{i+1}$ and $t^2h(t) = \sum_{i=0}^{+\infty} \mathfrak{H}^{\mathfrak{F}}{}_i t^{i+2}$. After the necessary calculations, the statement of the theorem follows

$$\sum_{i=0}^{+\infty} \mathfrak{H}^{\mathfrak{F}}{}_i t^i = \frac{\mathfrak{H}^{\mathfrak{F}}{}_0 + (\mathfrak{H}^{\mathfrak{F}}{}_1 - \mathfrak{H}^{\mathfrak{F}}{}_0)t}{1-t-t^2}. \qquad \square$$

The generating function formula consecutive coefficient Lucas hybrid single-valued neutrosophic number is verified similarly.

Remark 2.4. Some special sequences well-known for the Fibonacci sequence have also been calculated for the consecutive coefficient Fibonacci hybrid single-valued neutrosophic numbers. The proofs of these equations are omitted. $\mathfrak{H}^{\mathfrak{F}}{}_n$ be the nth consecutive coefficient Fibonacci hybrid single-valued neutrosophic number such that $n \geq 1$ integer. Then, the following equalities hold.

(a) Tagiuri's Identity:

$$\begin{aligned}
\mathfrak{H}^{\mathfrak{F}}{}_{m+r}\mathfrak{H}^{\mathfrak{F}}{}_{n-r} - \mathfrak{H}^{\mathfrak{F}}{}_m\mathfrak{H}^{\mathfrak{F}}{}_n =& (-1)^m F_r F_{n-m-r} + (-1)^{m+s} F_r F_{n-m-r} T_A(x) \\
&+ (-1)^{m+2s} F_r F_{n-m-r} I_A(x) + (-1)^{m+3s} F_r F_{n-m-r} F_A(x) \\
&+ i(-1)^m F_r \Big[F_{n-m+s-r} + (-1)^s F_{n-m-s-r}\Big] \\
&+ \varepsilon(-1)^m F_r \Big[F_{n-m+2s-r} + (-1)^{2s} F_{n-m-2s-r}\Big] \\
&+ j(-1)^m F_r \Big[F_{n-m+3s-r} + (-1)^{3s} F_{n-m-3s-r}\Big] \\
&+ i\varepsilon\Big[(-1)^{m+s} F_r F_{n-m+s-r}\Big] \\
&+ ij(-1)^{m+s} F_r \Big[F_{n-m+2s-r} - F_{n-m-2s-r}\Big] \\
&+ \varepsilon i\Big[(-1)^{m+2s} F_r F_{n-m-s-r}\Big] + \varepsilon j\Big[(-1)^{m+2s} F_r F_{n-m+s-r}\Big] \\
&+ j\varepsilon\Big[(-1)^{m+3s} F_r F_{n-m-s-r}\Big].
\end{aligned}$$

(b) d'Ocagne's Identity:

$$\mathfrak{H}^{\mathfrak{F}}{}_{m-1}\mathfrak{H}^{\mathfrak{F}}{}_{n+1} - \mathfrak{H}^{\mathfrak{F}}{}_m\mathfrak{H}^{\mathfrak{F}}{}_n = (-1)^m F_{n-m+1} + (-1)^{m+s} F_{n-m+1} T_A(x)$$

$$
\begin{aligned}
&+(-1)^{m+2s}F_{n-m+1}I_A(x)+(-1)^{m+3s}F_{n-m+1}F_A(x)\\
&+i(-1)^m\Big[F_{n-m+s+1}+(-1)^sF_{n-m-s+1}\Big]\\
&+\varepsilon(-1)^m\Big[F_{n-m+2s+1}+(-1)^{2s}F_{n-m-2s+1}\Big]\\
&+j(-1)^m\Big[F_{n-m+3s+1}+(-1)^{3s}F_{n-m-3s-r}\Big]\\
&+i\varepsilon\Big[(-1)^{m+s}F_{n-m+s+1}\Big]\\
&+ij(-1)^{m+s}\Big[F_{n-m+2s+1}-F_{n-m-2s+1}\Big]\\
&+\varepsilon i\Big[(-1)^{m+2s}F_{n-m-s+1}\Big]+\varepsilon j\Big[(-1)^{m+2s}F_{n-m+s+1}\Big]\\
&+j\varepsilon\Big[(-1)^{m+3s}F_{n-m-s+1}\Big].
\end{aligned}
$$

(c) Catalan's Identity:

$$
\begin{aligned}
\mathfrak{H}^{\mathfrak{F}}{}_{n+r}\mathfrak{H}^{\mathfrak{F}}{}_{n-r}-\mathfrak{H}^{\mathfrak{F}}{}_{n}\mathfrak{H}^{\mathfrak{F}}{}_{n}=&(-1)^nF_rF_{-r}+(-1)^{n+s}F_rF_{-r}T_A(x)\\
&+(-1)^{n+2s}F_rF_{-r}I_A(x)+(-1)^{n+3s}F_rF_{-r}F_A(x)\\
&+i(-1)^nF_r\Big[F_{s-r}+(-1)^sF_{-s-r}\Big]\\
&+\varepsilon(-1)^nF_r\Big[F_{2s-r}+(-1)^{2s}F_{-2s-r}\Big]\\
&+j(-1)^nF_r\Big[F_{3s-r}+(-1)^{3s}F_{-3s-r}\Big]+i\varepsilon\Big[(-1)^{n+s}F_rF_{s-r}\Big]\\
&+ij(-1)^{n+s}F_r\Big[F_{2s-r}-F_{-2s-r}\Big]+\varepsilon i\Big[(-1)^{n+2s}F_rF_{-s-r}\Big]\\
&+\varepsilon j\Big[(-1)^{n+2s}F_rF_{s-r}\Big]+j\varepsilon\Big[(-1)^{n+3s}F_rF_{-s-r}\Big].
\end{aligned}
$$

(d) Cassini's Identity:

$$
\begin{aligned}
\mathfrak{H}^{\mathfrak{F}}{}_{n+1}\mathfrak{H}^{\mathfrak{F}}{}_{n-1}-\mathfrak{H}^{\mathfrak{F}}{}_{n}\mathfrak{H}^{\mathfrak{F}}{}_{n}=&(-1)^n+(-1)^{n+s}T_A(x)\\
&+(-1)^{n+2s}I_A(x)+(-1)^{n+3s}F_A(x)\\
&+i(-1)^n\Big[F_{s-1}+(-1)^sF_{-s-1}\Big]\\
&+\varepsilon(-1)^n\Big[F_{2s-1}+(-1)^{2s}F_{-2s-1}\Big]\\
&+j(-1)^n\Big[F_{3s-1}+(-1)^{3s}F_{-3s-1}\Big]+i\varepsilon\Big[(-1)^{n+s}F_{s-1}\Big]\\
&+ij(-1)^{n+s}\Big[F_{2s-1}-F_{-2s-1}\Big]+\varepsilon i\Big[(-1)^{n+2s}F_{-s-1}\Big]\\
&+\varepsilon j\Big[(-1)^{n+2s}F_{s-1}\Big]+j\varepsilon\Big[(-1)^{n+3s}F_{-s-1}\Big].
\end{aligned}
$$

Remark 2.5. The identities Tagiuri, d'Ocagne, Catalan and Cassini can be calculated by the same method for the consecutive coefficient Lucas hybrid single-valued neutrosophic numbers.

3. Sums of the Consecutive Coefficient Fibonacci Hybrid Single-Valued Neutrosophic Numbers

In this section, we present some results concercing sums of terms of the consecutive coefficient Fibonacci hybrid single-valued neutrosophic number. Some known equations about Fibonacci numbers will be given in the following lemma.

Lemma 3.1. *Let F_n, $n \geq 0$, be the Fibonacci number. We have*

$$\begin{aligned}
\sum_{m=0}^{n} F_{m+s} =& F_{n+s+2} - F_{s+1},\\
\sum_{m=0}^{n} F_{m+2s} =& F_{n+2s+2} - F_{2s+1},\\
\sum_{m=0}^{n} F_{m+3s} =& F_{n+3s+2} - F_{3s+1},\\
\sum_{m=0}^{n} F_{2m+s} =& F_{2n+s+1} - F_{s-1},\\
\sum_{m=0}^{n} F_{2m+2s} =& F_{2n+2s+1} - F_{2s-1},\\
\sum_{m=0}^{n} F_{2m+3s} =& F_{2n+3s+1} - F_{3s-1},\\
\sum_{m=0}^{n} F_{2m+s+1} =& F_{2n+s+2} - F_{s},\\
\sum_{m=0}^{n} F_{2m+2s+1} =& F_{2n+2s+2} - F_{2s},\\
\sum_{m=0}^{n} F_{2m+3s+1} =& F_{2n+3s+2} - F_{3s}.
\end{aligned}$$

Proof. We give

$$\begin{aligned}
F_s &= F_{s-1} + F_{s-2},\\
F_{s+1} &= F_s + F_{s-1},\\
F_{s+2} &= F_{s+1} + F_s,\\
\cdots &= \cdots\\
F_{s+n} &= F_{s+n-1} + F_{s+n-2}.
\end{aligned}$$

Taking the sum of the equalities above, we obtained

$$\sum_{m=0}^{n} F_{m+s} = F_{n+s+2} - F_{s+1}.$$

Two other statements of the theorem are verified similarly. □

Some known equations for Lucas numbers can also be shown in a similar way.

Theorem 3.1. *Let $\mathfrak{H}^{\mathfrak{F}}{}_n$, $n \geq 0$, be the consecutive coefficient Fibonacci hybrid single-valued neutrosophic number. Then,*

$$\begin{aligned}\sum_{m=0}^{n} \mathfrak{H}^{\mathfrak{F}}{}_m =&(F_{n+2}-1)+i(F_{n+s+2}-F_{s+1})+\varepsilon(F_{n+2s+2}-F_{2s+1})\\&+j(F_{n+3s+2}-F_{3s+1}),\\ \sum_{m=0}^{n} \mathfrak{H}^{\mathfrak{F}}{}_{2m} =&(F_{2n+1}-1)+i(F_{2n+s+1}-F_{s-1})+\varepsilon(F_{2n+2s+1}-F_{2s-1})\\&+j(F_{2n+3s+1}-F_{3s-1}),\end{aligned}$$

$$\sum_{m=0}^{n} \mathfrak{H}^{\mathfrak{F}}{}_{2m+1} =(F_{2n+2}+1)+i(F_{2n+s+2}-F_{s})+\varepsilon(F_{2n+2s+2}-F_{2s})+j(F_{2n+3s+2}-F_{3s}).$$

Proof. We have

$$\begin{aligned}\mathfrak{H}^{\mathfrak{F}}{}_0 &= F_0+iF_s+\varepsilon F_{2s}+jF_{3s},\\ \mathfrak{H}^{\mathfrak{F}}{}_1 &= F_1+iF_{1+s}+\varepsilon F_{1+2s}+jF_{1+3s},\\ \mathfrak{H}^{\mathfrak{F}}{}_2 &= F_2+iF_{2+s}+\varepsilon F_{2+2s}+jF_{2+3s},\\ \cdots &= \cdots\\ \mathfrak{H}^{\mathfrak{F}}{}_n &= F_n+iF_{n+s}+\varepsilon F_{n+2s}+jF_{n+3s}.\end{aligned}$$

Taking the sum of the equalities above, we obtained

$$\sum_{m=0}^{n} \mathfrak{H}^{\mathfrak{F}}{}_m = (F_{n+2}-1)+i(F_{n+s+2}-F_{s+1})+\varepsilon(F_{n+2s+2}-F_{2s+1})+j(F_{n+3s+2}-F_{3s+1}).$$

Two other statements of the theorem are verified similarly. □

The sum values in the above theorem can be shown in a similar way for the consecutive coefficient Lucas hybrid single-valued neutrosophic numbers.

Theorem 3.2. *For $n \geq 0$, let $\mathfrak{H}^{\mathfrak{F}}{}_n$ and $\mathfrak{H}^{\mathfrak{L}}{}_n$ be the consecutive coefficient Fibonacci and Lucas hybrid single-valued neutrosophic numbers. Then,*

$$\begin{aligned}\sum_{m=0}^{n}\binom{n}{m}\mathfrak{H}^{\mathfrak{F}}{}_m &= F_{2n}+iF_{2n+s}+\varepsilon F_{2n+2s}+jF_{2n+3s},\\ \sum_{m=0}^{n}\binom{n}{m}\mathfrak{H}^{\mathfrak{L}}{}_m &= L_{2n}+iL_{2n+s}+\varepsilon L_{2n+2s}+jL_{2n+3s}.\end{aligned}$$

Proof. We have

$$\begin{aligned}\sum_{m=0}^{n}\binom{n}{m}\mathfrak{H}^{\mathfrak{F}}{}_m =& \sum_{m=0}^{n}\binom{n}{m}F_m+i\sum_{m=0}^{n}\binom{n}{m}F_{m+s}+\varepsilon\sum_{m=0}^{n}\binom{n}{m}F_{m+2s}\\&+j\sum_{m=0}^{n}\binom{n}{m}F_{m+3s},\end{aligned}$$

and we obtained

$$\sum_{m=0}^{n} \binom{n}{m} \mathfrak{H}^{\mathfrak{F}}{}_{m} = F_{2n} + iF_{2n+s} + \varepsilon F_{2n+2s} + jF_{2n+3s}.$$

The other statement of the theorem is verified similarly. □

4. Conclusion

This study presents the hybrid single-valued neutrosophic number, the consecutive coefficient Fibonacci and Lucas hybrid single-valued neutrosophic numbers. We obtained these new numbers not defined in the literature before. We have given a comprehensive introductory study of hybrid neutrosophic numbers as a guide. Since this study includes some new results, it contributes to literature by providing essential information concerning these new numbers. Hybrid numbers are an excellent mathematical tool in a number of different fields, and hybrid neutrosophic numbers may be a new mathematical tool for processing ambiguous information. Therefore, we hope that these new numbers and properties that we have found will offer a new perspective to the researches. As a future direction, we plan to study other identities and properties of number sequences by increasing the diversity of these number sequences. Researchers who are far from neutrosophic studies may think at first glance that the scope of this paper is limited to neutrosophics, but since Neutrosophy is a nice fuzzy generalization, we offer a working area for all fuzzy types, and we will continue to work with picture fuzzy, type fuzzy sets, extensions, and algebraic properties. We think that it would be useful to look at studies such as quaternion-valued fuzzy (intuitionistic fuzzy) and quaternionic fuzzy (intuitionistic fuzzy) from a hybridistic studies approach.

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