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FUZZY ALMOST HYPERIDEALS AND FUZZY ALMOST QUASI-HYPERIDEALS IN SEMIHYPERGROUPS

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Abstract. Studying fuzzy hyperideals is necessary for comprehending semihypergroups. The idea of fuzzy hyperideals is expanded upon by several concepts. The notion of almost fuzzy hyperideals is one of them. In this article, we first define the notions of fuzzy almost hyperideals and fuzzy almost quasi-hyperideals in semihypergroups. We investigate the fundamental characteristics of fuzzy almost hyperideals and fuzzy quasi-hyperideals. Additionally, we establish the connection between fuzzy (resp., quasi-) hyperideals and almost (resp., quasi-) hyperideals.

1. Introduction and Preliminaries

The idea of almost left (resp., right, two-sided) ideals plays a crucial role in characterizing semigroups that do not contain any proper left (resp., right, two-sided) ideals. Grošek and Satko [6,7] took on this issue for the first time. Bogdanović [1] considered a similar problem for almost bi-ideals in semigroups the following year. Researchers have studied a variety of almost ideals in semigroups and applied the concept of fuzzy sets, introduced by Zadeh [20], to several kinds of almost ideals (see [2,10,14,18]).

At the 8th International Congress of Scandinavian Mathematicians, Marty [11] introduced the concept of algebraic hyperstructures. Semihypergroups are a generalization of semigroups in that each product of two elements is a nonempty set rather than an element. This generalization of semihypergroups is applicable in many scientific disciplines, including biology (see [13]). Almost hyperideals, introduced by Suebsung et al. [17], were the ones that were first proposed the idea of almost ideals for

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semihypergroups. They looked into some of the essential properties of almost hyperideals. The concept of almost quasi-hyperideals in semihypergroups was defined, and their characteristics were given by Suebsung et al. [19] in 2021. Later, Muangdoo et al. [12] investigated a semihypergroup analog of the problem considered by Bogdanović. They introduced the idea of almost bi-hyperideals and fuzzy almost bi-hyperideals in semihypergroups. There were several significant studies and linkages made between these ideas.

We note that Suebsung et al. [17, 19] only considered the notion of almost (resp., quasi-) hyperideals into account in their studies. It is intriguing to consider whether we can use the concept of fuzzy sets in these kinds of analyses. In fact, in semihypergroups, we introduce the idea of fuzzy almost (resp., quasi-) hyperideals. There are given some essential properties of such introductory notions. Fuzzy almost (resp., quasi-) hyperideals and other kinds of fuzzy almost ideals have relationships. Additionally, the characteristic function is used to describe the relationship between fuzzy almost (resp., quasi-) hyperideals and almost (resp., quasi-) hyperideals.

2. Preliminaries

In this section we give some brief concepts and results, which will be helpful in next sections. Firstly, the concept of semihypergroups will be recalled as follows.

Let $\mathcal{H}$ be a non-empty set and $\mathcal{P}^*(\mathcal{H}) := \mathcal{P}(\mathcal{H}) \setminus \{\varnothing\}$ denotes the set of all non-empty subsets of $\mathcal{H}$. The map $\circ\colon \mathcal{H} \times \mathcal{H} \to \mathcal{P}^*(\mathcal{H})$ is called the *hyperoperation* or the *join operation* on the set $\mathcal{H}$. A couple $(\mathcal{H}, \circ)$ is called a *hypergroupoid* if $\circ$ is a hyperoperation on $\mathcal{H}$. For $\mathcal{A}$ and $\mathcal{B}$ be two non-empty subsets of a hypergroupoid $\mathcal{H}$, we will denote

$$\mathcal{A} \circ \mathcal{B} = \bigcup_{a \in \mathcal{A}, b \in \mathcal{B}} a \circ b, \quad a \circ \mathcal{A} = \{a\} \circ \mathcal{A} \quad \text{and} \quad a \circ \mathcal{B} = \{a\} \circ \mathcal{B}.$$

A hypergroupoid $(\mathcal{H}, \circ)$ is called a *semihypergroup* if for every $x, y, z \in \mathcal{H}$ we have $(x \circ y) \circ z = x \circ (y \circ z)$. Throughout this paper, we simply denote a semihypergroup $(\mathcal{H}, \circ)$ by $\mathcal{H}$, and $\mathcal{H}$ is understood to be a semihypergroup. A *subsemihypergroup* $\mathcal{Q}$ of $\mathcal{H}$ is a non-empty subset of $\mathcal{H}$ such that $\mathcal{Q} \circ \mathcal{Q} \subseteq \mathcal{Q}$. A *left* (*resp., right*) *hyperideal* $\mathcal{Q}$ of $\mathcal{H}$ if $\mathcal{H} \circ \mathcal{Q} \subseteq \mathcal{Q}$ (resp., $\mathcal{Q} \circ \mathcal{H} \subseteq \mathcal{Q}$). By a *hyperideal* $\mathcal{Q}$ of $\mathcal{H}$, we mean a non-empty set of $\mathcal{H}$ which is both a left and a right hyperideal of $\mathcal{H}$. A subsemihypergroup $\mathcal{Q}$ of $\mathcal{H}$ is called a *quasi-ideal* of $\mathcal{H}$ if $\mathcal{Q} \circ \mathcal{H} \cap \mathcal{H} \circ \mathcal{Q} \subseteq \mathcal{Q}$. In [5], the readers can find more information about the many types of hyperideals in semihypergroups From now on, we write $\mathcal{A}\mathcal{B}$ instead of $\mathcal{A} \circ \mathcal{B}$, for any nonempty subsets $\mathcal{A}$ and $\mathcal{B}$ of $\mathcal{H}$.

A non-empty subset $\mathcal{Q}$ of $\mathcal{H}$ is said to be:

(1) an *almost ideal* [17] of $\mathcal{H}$ if $h_1\mathcal{Q} \cap \mathcal{Q} \neq \emptyset$ and $\mathcal{Q}h_2 \cap \mathcal{Q} \neq \emptyset$ for all $h_1, h_2 \in \mathcal{H}$;
(2) an *almost quasi-hyperideal* [19] of $\mathcal{H}$ if $(h\mathcal{Q} \cap \mathcal{Q}h) \cap \mathcal{Q} \neq \emptyset$ for all $h \in \mathcal{H}$.

Example 2.1. Let $\mathcal{H} = \{a, b, c, d\}$. Define a hyperoperation $\circ$ on $\mathcal{H}$ by the following table:

$\circ$	a	b	c	d
a	a	$\{a,b\}$	$\{a,c\}$	H
b	b	b	$\{b,d\}$	$\{b,d\}$
c	c	$\{c,d\}$	c	$\{c,d\}$
d	d	d	d	d

.

Then $\mathcal{H}$ is a semihypergroup (see [8]). We can carefully calculate that $\{a,b,d\}$ is an almost hyperideal of $\mathcal{H}$ but it is not a hyperideal of $\mathcal{H}$. Furthermore, $\{a,d\}$ is an almost quasi-hyperideal of $\mathcal{H}$ but it is not a quasi-hyperideal of $\mathcal{H}$.

The above example illustrates the difference between (resp., quasi-) hyperideals and almost (resp., quasi-) hyperideals in semihypergroups. Now, we recall the concept of fuzzy sets.

For any $h_i \in [0,1]$, $i \in \mathcal{F}$, where $\mathcal{F}$ is a nonempty indexed set, we define

$$\bigvee_{i\in\mathcal{F}} h_i := \sup_{i\in\mathcal{F}}\{h_i\} \quad \text{and} \quad \bigwedge_{i\in\mathcal{F}} h_i := \inf_{i\in\mathcal{F}}\{h_i\}.$$

We observe that if $\mathcal{F}$ is finite, then

$$\bigvee_{i\in\mathcal{F}} h_i := \max_{i\in\mathcal{F}}\{h_i\} \quad \text{and} \quad \bigwedge_{i\in\mathcal{F}} h_i := \min_{i\in\mathcal{F}}\{h_i\}.$$

Let $\mathcal{T}$ be a non-empty set. We call a mapping $\eta\colon \mathcal{T} \to [0,1]$ a *fuzzy set* of $\mathcal{T}$ (see [20]). For any non-empty subset A of $\mathcal{T}$, the *characteristic function* λ_A of A in $\mathcal{T}$ is a fuzzy set of $\mathcal{T}$ defined by $\lambda_A(x) := 1$ if $x \in A$ and $\lambda_A(x) := 0$ if $x \notin A$ for all $x \in \mathcal{T}$. For any $\alpha \in [0,1]$ can be regarded as a fuzzy set of $\mathcal{T}$ by assigning $\alpha(x) := \alpha$ for all $x \in \mathcal{T}$.

For any two fuzzy sets η and ν of a non-empty set $\mathcal{T}$, define the symbol as follows:

(1) $\eta \subseteq \nu \Leftrightarrow \eta(h) \leq \nu(h)$ for all $h \in \mathcal{T}$;
(2) $\eta = \nu \Leftrightarrow \eta \subseteq \nu$ and $\nu \subseteq \eta$;
(3) $(\eta \cap \nu)(h) = \min\{\eta(h), \nu(h)\} = \eta(h) \wedge \nu(h)$ for all $h \in \mathcal{T}$;
(4) $(\eta \cup \nu)(h) = \max\{\eta(h), \nu(h)\} = \eta(h) \vee \nu(h)$ for all $h \in \mathcal{T}$;

We note here that the symbol $\eta \supseteq \nu$, we mean $\nu \subseteq \eta$.

The concept of semihypergroups can be studied in terms of fuzzy sets by the following setting. Let η and ν be fuzzy sets of $\mathcal{H}$. Define the product $\eta \circ \nu$ by

$$(\eta \circ \nu)(h) = \begin{cases} \bigvee\limits_{h=h_1h_2} \{\eta(h_1) \wedge \nu(h_2)\}, & \text{if } h = h_1h_2 \text{ for some } h_1, h_2 \in \mathcal{H}, \\ 0, & \text{otherwise}, \end{cases}$$

for all $h \in \mathcal{H}$.

By the above definition, one can prove the following important result.

Lemma 2.1 ([12]). *Let $\mathcal{K}$ and $\mathcal{L}$ be non-empty subsets of $\mathcal{H}$. Then the following holds:*

(1) $\mathcal{K} \subseteq \mathcal{L}$ *if and only if* $\lambda_{\mathcal{K}} \subseteq \lambda_{\mathcal{L}}$;
(2) $\lambda_{\mathcal{K}} \cap \lambda_{\mathcal{L}} = \lambda_{\mathcal{K}\cap\mathcal{L}}$;
(3) $\lambda_{\mathcal{K}} \circ \lambda_{\mathcal{L}} = \lambda_{\mathcal{K}\mathcal{L}}$.

Definition 2.1 ([15])**.** Let $u \in \mathcal{H}$ and $t \in (0,1]$. A fuzzy set u_t of $\mathcal{H}$ defined by

$$u_t(x) := \begin{cases} t, & \text{if } u = x, \\ 0, & \text{otherwise,} \end{cases}$$

for all $x \in \mathcal{H}$, is called a *fuzzy point* of $\mathcal{H}$.

We observe that for any characteristic function of a singleton set of $\mathcal{H}$ can be regarded as a fuzzy point of $\mathcal{H}$. That is, for any $a \in \mathcal{H}$, we have $\lambda_{\{a\}} = a_1$.

3. On Fuzzy Almost (resp., quasi-) Hyperideals

The concepts of fuzzy almost hyperideals and fuzzy quasi-hyperideals in semihypergroups are defined in this section. This section will demonstrate how these notions are distinct from fuzzy hyperideals and fuzzy quasi-hyperideals in semihypergroups. The properties of the notions we defined are investigated.

Definition 3.1. A fuzzy set η of $\mathcal{H}$ is said to be:

(1) a *fuzzy almost left* (*resp., right*) *hyperideal* of $\mathcal{H}$ if for any fuzzy point h_t of $\mathcal{H}$ there exists $x \in \mathcal{H}$ such that $(\eta \circ h_t)(x) \wedge \eta(x) \neq 0$ (resp., $(h_t \circ \eta)(x) \wedge \eta(x) \neq 0$);
(2) a *fuzzy almost* (*two-sided*) *hyperideal* of $\mathcal{H}$ if it is both a fuzzy left almost hyperideal and a fuzzy right almost hyperideal of $\mathcal{H}$.

Example 3.1. Let $\mathcal{H} = \{a, b, c, u, v\}$. Define a hyperoperation $\circ$ on $\mathcal{H}$ by the following table:

$\circ$	a	b	c	u	v
a	a	a	$\{a,b,c\}$	a	$\{a,b,c\}$
b	a	a	$\{a,b,c\}$	a	$\{a,b,c\}$
c	a	a	$\{a,b,c\}$	a	$\{a,b,c\}$
u	$\{a,b,u\}$	$\{a,b,u\}$	$\mathcal{H}$	$\{a,b,u\}$	$\mathcal{H}$
v	$\{a,b,u\}$	$\{a,b,u\}$	$\mathcal{H}$	$\{a,b,u\}$	$\mathcal{H}$

Then $\mathcal{H}$ is a semihypergroup (see [5]). We define a fuzzy set η of $\mathcal{H}$ by

$$\eta(a) = 0, \quad \eta(b) = 0, \quad \eta(c) = 0.6, \quad \eta(u) = 0.4 \quad \text{and} \quad \eta(v) = 0.$$

We can see that for any $t \in (0,1]$:

(1) $(a_t \circ \eta)(c) \wedge \eta(c) \neq 0$ and $(\eta \circ a_t)(u) \wedge \eta(u) \neq 0$;
(2) $(b_t \circ \eta)(c) \wedge \eta(c) \neq 0$ and $(\eta \circ b_t)(u) \wedge \eta(u) \neq 0$;
(3) $(c_t \circ \eta)(c) \wedge \eta(c) \neq 0$ and $(\eta \circ c_t)(c) \wedge \eta(c) \neq 0$;
(4) $(u_t \circ \eta)(u) \wedge \eta(u) \neq 0$ and $(\eta \circ u_t)(u) \wedge \eta(u) \neq 0$;
(5) $(v_t \circ \eta)(c) \wedge \eta(c) \neq 0$ and $(\eta \circ v_t)(c) \wedge \eta(c) \neq 0$.

Therefore, η is a fuzzy almost hyperideal of $\mathcal{H}$. Since $(1 \circ \eta)(a) = 0.6 > 0 = \eta(a)$, we have that η is not a fuzzy left hyperideal of $\mathcal{H}$. That is, η is not a fuzzy hyperideal of $\mathcal{H}$.

It is not difficult to verify that any fuzzy left (resp., right, two-sided) hyperideal is a fuzzy almost left (resp., right, two-sided) hyperideal. In addition, Example 3.1 illustrates that a fuzzy almost hyperideal may not be a fuzzy hyperideal. This example demonstrates how fuzzy hyperideals in semihypergroups are generalized by the concept of fuzzy almost hyperideals. We refer the readers to [3, 4] for more information about fuzzy left (resp., right, two-sided) hyperideals.

Definition 3.2. A fuzzy set η on $\mathcal{H}$ is said to be a *fuzzy almost quasi-hyperideal* of $\mathcal{H}$ if for any fuzzy point h_t of $\mathcal{H}$ there exists $x \in \mathcal{H}$ such that $(\eta \circ h_t)(x) \wedge (h_t \circ \eta)(x) \wedge \eta(x) \neq 0$.

Example 3.2. Let $\mathcal{H} = \{a, b, c, d\}$. Define a hyperoperation $\circ$ on $\mathcal{H}$ by the following table:

$\circ$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	$\{a, b\}$
d	a	a	$\{a, b\}$	$\{a, b, c\}$

Then $\mathcal{H}$ is a semihypergroup (see [5]). We define a fuzzy set η of $\mathcal{H}$ by

$$\eta(a) = 0.7, \quad \eta(b) = 0, \quad \eta(c) = 0.2 \quad \text{and} \quad \eta(d) = 0.4.$$

We can see that there exists $a \in \mathcal{H}$ such that $(h_t \circ \eta)(a) \wedge (\eta \circ h_t)(a) \wedge \eta(a) \neq 0$ for all fuzzy point h_t of $\mathcal{H}$. Therefore, η is a fuzzy almost quasi-hyperideal of $\mathcal{H}$. Since $(1 \circ \eta)(b) \wedge (\eta \circ 1)(b) = 0.4 > 0 = \eta(b)$, we have that η is not a fuzzy quasi-hyperideal of $\mathcal{H}$.

We can observe that any fuzzy quasi-hyperideal of semihypergroups is a fuzzy almost fuzzy quasi-hyperideal. We can see from the preceding example that the converse does not hold. For further detail on fuzzy quasi-hyperideals, we recommend readers to [16].

Remark 3.1. Examples 3.1 and 3.2 indicate how fuzzy almost (resp., quasi-) hyperideals extend on the idea of fuzzy (resp., quasi-) hyperideals. Verifying a relationship between fuzzy almost hyperideals and fuzzy almost quasi-hyperideals is not complicated. In semihypergroups, any fuzzy almost quasi-hyperideal is also a fuzzy almost hyperideal. Example 3.1 illustrates how these concepts differ from one another. Indeed, for any $t \in (0, 1]$, we have $(a_t \circ \eta)(x) \wedge (\eta \circ a_t)(x) \wedge \eta(x) = 0$ for all $x \in \mathcal{H}$.

In the following paper, we focus only on fuzzy almost hyperideals and fuzzy almost quasi-hyperideals in semihypergroups. However, the verification of our subsequent results is limited to fuzzy almost hyperideals since each fuzzy most quasi-hyperideal is a fuzzy almost hyperideal. The following result is required to examine the features of fuzzy almost (resp., quasi-) hyperideals in semihypergroups.

Lemma 3.1. *Let η, ν and θ be fuzzy sets of $\mathcal{H}$. We have that if $\eta \subseteq \nu$, then $\eta \circ \theta \subseteq \nu \circ \theta$ and $\theta \circ \eta \subseteq \theta \circ \nu$.*

Proof. We illustrate only that $\eta \circ \theta \subseteq \nu \circ \theta$. For verifying that $\theta \circ \eta \subseteq \theta \circ \nu$, it can be done similarly. Assume that $\eta \subseteq \nu$. Let $x \in \mathcal{H}$. If there is no $u, v \in \mathcal{H}$ such that $x \in uv$, then $(\eta \circ \theta)(x) \leq (\nu \circ \theta)(x)$. On the other hand, we have that

$$(\eta \circ \theta)(x) = \bigvee_{x \in uv} \{\eta(u) \wedge \theta(v)\} \leq \bigvee_{x \in uv} \{\nu(u) \wedge \theta(v)\} = (\nu \circ \theta)(x).$$

Therefore, we obtain our claim. □

Here is our initial significant conclusion. When determining if a fuzzy set is a fuzzy almost (resp., quasi-) hyperideal, we do not always need to check with the definition. The result examines whether there is a fuzzy almost (resp., quasi-) hyperideal less than it, in which case it is also a fuzzy almost (resp., quasi-) hyperideal.

Theorem 3.1. *Let η and ν be fuzzy sets of $\mathcal{H}$. We have that if η is a fuzzy almost (resp., quasi-) hyperideal of $\mathcal{H}$ such that $\eta \subseteq \nu$, then ν is a fuzzy almost (resp., quasi-) hyperideal of $\mathcal{H}$.*

Proof. Suppose that η is a fuzzy almost hyperideal of $\mathcal{H}$ such that $\eta \subseteq \nu$. By Lemma 3.1 and the definition of fuzzy almost hyperideal of $\mathcal{H}$, we obtain that there exist $x, y \in \mathcal{H}$ such that

$$0 \neq (h_t \circ \eta)(x) \wedge \eta(x) \leq (h_t \circ \nu)(x) \wedge \nu(x)$$

and

$$0 \neq (\eta \circ h'_{t'})(x) \wedge \eta(x) \leq (\eta \circ h'_{t'})(x) \wedge \nu(x),$$

for any fuzzy points h_t and $h'_{t'}$ of $\mathcal{H}$. This shows that ν is a fuzzy almost hyperideal of $\mathcal{H}$. For illustrating that ν is a fuzzy almost quasi-hyperideal of $\mathcal{H}$ can be done similarly. □

By Theorem 3.1, we obtain the following consequence immediately.

Corollary 3.1. *Let η be a fuzzy set of $\mathcal{H}$ and ν be a fuzzy almost (quasi-) hyperideal of $\mathcal{H}$. Then $\eta \cup \nu$ is a fuzzy almost (resp., quasi-) hyperideal of $\mathcal{H}$.*

Proof. By Theorem 3.1 and the fact that $\eta \subseteq \eta \cup \nu$, we obtain our claim. □

The following example shows the contrast of Corollary 3.1.

Example 3.3. Let $\mathcal{H} = \{a, b, c\}$. Define a hyperoperation $\circ$ on $\mathcal{H}$ by the following table:

$\circ$	a	b	c
a	$\{a\}$	$\{b, c\}$	$\{c\}$
b	$\{b, c\}$	$\{b, c\}$	$\{c\}$
c	$\{b, c\}$	$\{b, c\}$	$\{c\}$

Then $\mathcal{H}$ is a semihypergroup. Define fuzzy sets η and ν of $\mathcal{H}$ by

$$\eta(a) = 0, \quad \eta(b) = 0, \quad \eta(c) = 0.1, \quad \nu(a) = 0, \quad \nu(b) = 0.6 \quad \text{and} \quad \nu(c) = 0.$$

We can carefully calculate that η and ν are fuzzy almost hyperideals of $\mathcal{H}$, but $\eta \cap \nu$ is not a fuzzy almost hyperideal of $\mathcal{H}$. Similarly, we can show that η and ν are fuzzy almost quasi-hyperideals of $\mathcal{H}$, but $\eta \cap \nu$ is not a fuzzy almost quasi-hyperideal of $\mathcal{H}$.

In the next couple results, we study relationships between almost (resp., quasi-) hyperideals and fuzzy almost (resp., quasi-) hyperideals in semihypergroups. Firstly, we represent almost (resp., quasi-) hyperideals in terms of fuzzy almost (resp., quasi-) hyperideals.

Theorem 3.2. *Let $\mathcal{Q}$ be a non-empty subset of $\mathcal{H}$. Then the following statements are equivalent:*

(1) *$\mathcal{Q}$ is an almost (resp., quasi-) hyperideal of $\mathcal{H}$;*
(2) *$\lambda_{\mathcal{Q}}$ is a fuzzy almost (resp., quasi-) hyperideal of $\mathcal{H}$.*

Proof. (1) $\Rightarrow$ (2). Suppose that $\mathcal{Q}$ is an almost hyperideal of $\mathcal{H}$. Let h_t be a fuzzy point of $\mathcal{H}$. By our assumption, we have that $h\mathcal{Q} \cap \mathcal{Q} \neq \emptyset$. This means that there exists $x \in \mathcal{Q}$ such that $x \in hq_1$ for some $q_1 \in \mathcal{Q}$. Therefore,

$$(h_t \circ \lambda_{\mathcal{Q}})(x) = \bigvee_{x \in uv} \{h_t(u) \wedge \lambda_{\mathcal{Q}}(v)\} = 1.$$

Similarly, we have that $\mathcal{Q}h \cap \mathcal{Q} \neq \emptyset$. This means that there exists $y \in \mathcal{Q}$ such that $y \in q_2h$ for some $q_2 \in \mathcal{Q}$. Therefore,

$$(\lambda_{\mathcal{Q}} \circ h_t)(x) = \bigvee_{x \in uv} \{\lambda_{\mathcal{Q}}(u) \wedge h_t(v)\} = 1.$$

This shows that $\lambda_{\mathcal{Q}}$ is a fuzzy almost hyperideal of $\mathcal{H}$.

(2) $\Rightarrow$ (1). Assume that $\lambda_{\mathcal{Q}}$ is a fuzzy almost hyperideal of $\mathcal{H}$. Let $h, h' \in \mathcal{H}$. By our presumption, for any $t, t' \in (0, 1]$ there exist $x, y \in \mathcal{H}$ such that

$$(h_t \circ \lambda_{\mathcal{Q}})(x) \wedge \lambda_{\mathcal{Q}}(x) \neq 0 \tag{3.1}$$

and

$$(\lambda_{\mathcal{Q}} \circ h'_{t'})(y) \wedge \lambda_{\mathcal{Q}}(y) \neq 0. \tag{3.2}$$

By (3.1), we have that $x \in hu$ for some $u \in \mathcal{Q}$ and $x \in \mathcal{Q}$. That is, $x \in h\mathcal{Q} \cap \mathcal{Q}$, so $h\mathcal{Q} \cap \mathcal{Q} \neq \emptyset$. On the other hand, by (3.2), we also conclude that $\mathcal{Q}h' \cap \mathcal{Q} \neq \emptyset$. Therefore, $\mathcal{Q}$ is an almost hyperideal of $\mathcal{H}$.

In showing that $\mathcal{Q}$ is an almost quasi-hyperideal if and only if $\lambda_{\mathcal{Q}}$ is a fuzzy quasi-hyperideal can be completed in a similar way. ☐

In order to describe fuzzy almost (resp., quasi-) hyperideals using almost (resp., quasi-) hyperideals, we need the following notion. Let η be a fuzzy set of $\mathcal{H}$. The *support* of η, denoted by $\mathrm{supp}(\eta)$, is defined to be the set $\{h \in \mathcal{H} \mid \eta(h) \neq 0\}$.

Theorem 3.3. *Let η be a fuzzy set of $\mathcal{H}$. Then the following statements are equivalent:*

(1) *η is a fuzzy almost (resp., quasi-) hyperideal of $\mathcal{H}$;*
(2) $\mathrm{supp}(\eta)$ *is an almost (resp., quasi-) hyperideal of $\mathcal{H}$.*

Proof. (1) $\Rightarrow$ (2). Assume that η is a fuzzy almost hyperideal of $\mathcal{H}$. Let $h \in \mathcal{H}$. Then there exists $x \in \mathcal{H}$ such that $(\eta \circ h_t)(x) \wedge \eta(x) \neq 0$, where $t \in (0,1]$. Hence, $(\eta \circ h_t)(x) \neq 0$ and $\eta(x) \neq 0$. That is, $x = uh$ for some $u \in \mathcal{H}$ with $\eta(u) \neq 0$, and $\eta(x) \neq 0$. Thus, $x = uh \subseteq \operatorname{supp}(\eta)h$ and $x \in \operatorname{supp}(\eta)$. This means that $\operatorname{supp}(\eta)h \cap \operatorname{supp}(\eta) \neq \emptyset$. By similar arguments, we have that $h' \operatorname{supp}(\eta) \cap \operatorname{supp}(\eta) \neq \emptyset$ for any $h' \in \mathcal{H}$. This shows that $\operatorname{supp}(\eta)$ is an almost hyperideal of $\mathcal{H}$.

(2) $\Rightarrow$ (1). Assume that $\operatorname{supp}(\eta)$ is an almost hyperideal of $\mathcal{H}$. Let h_t be a fuzzy point of $\mathcal{H}$. By Theorem 3.2, $\lambda_{\operatorname{supp}(\eta)}$ is a fuzzy almost hyperideal of $\mathcal{H}$. Then, we have that there exists $x \in \mathcal{H}$ such that

$$(h_t \circ \lambda_{\operatorname{supp}(\eta)})(x) \wedge \lambda_{\operatorname{supp}(\eta)}(x) \neq 0.$$

This implies that $x = hu$ for some $u \in \operatorname{supp}(\eta)$ and $x \in \operatorname{supp}(\eta)$. Thus, we have that

$$(h_t \circ \eta)(x) \wedge \eta(x) \neq 0.$$

Similarly, for any fuzzy point $h'_{t'}$ of $\mathcal{H}$, we have that there exists $y \in \mathcal{H}$ such that $(\eta \circ h'_{t'})(y) \wedge \eta(y) \neq 0$. Altogether, η is a fuzzy almost hyperideal of $\mathcal{H}$.

Illustrating that η is a fuzzy almost quasi-hyperideal of $\mathcal{H}$ if and only if $\operatorname{supp}(\eta)$ is an almost quasi-hyperideal of $\mathcal{H}$ can be done similarly. □

The existence of proper almost (resp., quasi-) hyperideals in semihypergroups can be described using fuzzy almost (resp., quasi-) hyperideals by the following consequence.

Corollary 3.2. *The following statements are equivalent:*

(1) $\mathcal{H}$ *has no proper almost* (*resp., quasi-*) *hyperideal;*
(2) $\operatorname{supp}(\eta) = \mathcal{H}$ *for every fuzzy almost* (*resp., quasi-*) *hyperideal* η *of* $\mathcal{H}$.

4. Minimality and Maximality of Fuzzy Almost (resp., quasi-) Hyperideals

We define the minimalities of almost (resp., quasi-) hyperideals and fuzzy almost (resp., quasi-) hyperideals in semihypergroups. The relationship between minimal almost (resp., quasi-) hyperideals and minimal fuzzy almost (resp., quasi-) hyperideals is investigated.

Definition 4.1. An almost (resp., quasi-) hyperideal $\mathcal{Q}$ of $\mathcal{H}$ is said to be *minimal* if for any almost (resp., quasi-) hyperideal $\mathcal{M}$ of $\mathcal{H}$, we have $\mathcal{M} = \mathcal{Q}$ whenever $\mathcal{M} \subseteq \mathcal{Q}$.

Definition 4.2. A fuzzy almost (resp., quasi-) hyperideal η of $\mathcal{H}$ is said to be *minimal* if for any fuzzy almost (resp., quasi-) hyperideal ν of $\mathcal{H}$, we have $\operatorname{supp}(\nu) = \operatorname{supp}(\eta)$ whenever $\nu \subseteq \eta$.

Example 4.1. (a) By Example 3.1, we see that $\{a\}$ and $\{u\}$ are minimal almost hyperideals of $\mathcal{H}$. Moreover, for any $t \in (0,1]$, a fuzzy set η of $\mathcal{H}$ defined by $\eta(x) = 0$ if $x \in \{a, b, v\}$ and $\eta(x) = t$ if $x \in \{c, u\}$, is a minimal fuzzy almost hyperideal of $\mathcal{H}$.

(b) By Example 3.2, we see that $\{a\}$ is a minimal almost quasi-hyperideal of $\mathcal{H}$. Moreover, for any $t \in (0,1]$, a fuzzy set η of $\mathcal{H}$ defined by $\eta(x) = t$ if $x = a$ and $\eta(x) = 0$ if $x \in \{b, c, d\}$, is a minimal fuzzy almost quasi-hyperideal of $\mathcal{H}$.

Minimal almost (resp., quasi-) hyperideals are represented using fuzzy almost (resp., quasi-) hyperideals as follows.

Theorem 4.1. *Let $\mathcal{Q}$ be a non-empty subset of $\mathcal{H}$. Then the following statements are equivalent:*

(1) *$\mathcal{Q}$ is a minimal almost (resp., quasi-) hyperideal of $\mathcal{H}$;*
(2) *$\lambda_{\mathcal{Q}}$ is a minimal fuzzy almost (resp., quasi-) hyperideal of $\mathcal{H}$.*

Proof. (1) $\Rightarrow$ (2). Assume that $\mathcal{Q}$ is a minimal almost hyperideal of $\mathcal{H}$. By Theorem 3.2, $\lambda_{\mathcal{Q}}$ is a fuzzy almost hyperideal of $\mathcal{H}$. Let ν be a fuzzy almost hyperideal of $\mathcal{H}$ such that $\nu \subseteq \lambda_{\mathcal{Q}}$. Now, we know, by Theorem 3.3, that $\text{supp}(\nu)$ is an almost hyperideal of $\mathcal{H}$. Since $\text{supp}(\nu) \subseteq \text{supp}(\lambda_{\mathcal{Q}}) = \mathcal{Q}$, by the minimality of $\mathcal{Q}$, we have $\text{supp}(\nu) = \text{supp}(\lambda_{\mathcal{Q}})$. This shows that $\text{supp}(\lambda_{\mathcal{Q}})$ is a minimal fuzzy almost hyperideal of $\mathcal{H}$.

(2) $\Rightarrow$ (1). Assume that $\lambda_{\mathcal{Q}}$ is a minimal fuzzy almost hyperideal of $\mathcal{H}$. By Theorem 3.2, $\mathcal{Q}$ is an almost hyperideal of $\mathcal{H}$. Let $\mathcal{M}$ be an almost hyperideal of $\mathcal{H}$ such that $\mathcal{M} \subseteq \mathcal{Q}$. Then, by Lemma 2.1 and Theorem 3.2, $\lambda_{\mathcal{M}}$ is a fuzzy almost hyperideal of $\mathcal{H}$ such that $\lambda_{\mathcal{M}} \subseteq \lambda_{\mathcal{Q}}$. This implies that $\text{supp}(\lambda_{\mathcal{M}}) \subseteq \text{supp}(\lambda_{\mathcal{Q}})$. By the minimality of $\lambda_{\mathcal{Q}}$, we have $\text{supp}(\lambda_{\mathcal{M}}) = \text{supp}(\lambda_{\mathcal{Q}})$. That is, $\mathcal{M} = \mathcal{Q}$. Therefore, $\mathcal{Q}$ is minimal.

We can demonstrate that $\mathcal{Q}$ is a minimal almost quasi-hyperideal if and only if $\lambda_{\mathcal{Q}}$ is a minimal fuzzy almost quasi-hyperideal by the same technique. □

Next, we define the maximalists of almost (resp., quasi-) hyperideals and fuzzy almost (resp., quasi-) hyperideals in semihypergroups. The relationship between maximal almost (resp., quasi-) hyperideals and maximal fuzzy almost (resp., quasi-) hyperideals is investigated.

Definition 4.3. An almost (resp., quasi-) hyperideal $\mathcal{M}$ of $\mathcal{H}$ is said to be *maximal* if for all almost (resp., quasi-) hyperideal $\mathcal{L}$ of $\mathcal{H}$ such that $\mathcal{M} \subseteq \mathcal{L}$ implies $\mathcal{M} = \mathcal{L}$.

Definition 4.4. A fuzzy almost (resp., quasi-) hyperideal η of $\mathcal{H}$ is said to be *maximal* if for all fuzzy almost (resp., quasi-) hyperideal ν of $\mathcal{H}$ such that $\eta \subseteq \nu$ implies $\text{supp}(\eta) = \text{supp}(\nu)$.

Maximal almost (resp., quasi-) hyperideals are represented using fuzzy almost (resp., quasi-) hyperideals as follows.

Theorem 4.2. *Let $\mathcal{M}$ be a non-empty subset of $\mathcal{H}$. Then the following statements are equivalent:*

(1) *$\mathcal{M}$ is a maximal almost (resp., quasi-) hyperideal of $\mathcal{H}$;*
(2) *$\lambda_{\mathcal{M}}$ is a maximal fuzzy almost (resp., quasi-) hyperideal of $\mathcal{H}$.*

Proof. (1) $\Rightarrow$ (2). Assume that $\mathcal{M}$ is a maximal almost hyperideal of $\mathcal{H}$. By Theorem 3.2, $\lambda_{\mathcal{M}}$ is a fuzzy almost hyperideal of $\mathcal{H}$. Let ν be a fuzzy almost hyperideal of $\mathcal{H}$ such that $\lambda_{\mathcal{M}} \subseteq \nu$. Now, we know, by Theorem 3.3, that $\text{supp}(\nu)$ is an almost hyperideal of $\mathcal{H}$. Since $\text{supp}(\lambda_{\mathcal{M}}) \subseteq \text{supp}(\nu) = \mathcal{M}$, by the maximality of $\mathcal{M}$, we have $\text{supp}(\nu) = \text{supp}(\lambda_{\mathcal{M}})$. This shows that $\text{supp}(\lambda_{\mathcal{M}})$ is a maximal fuzzy almost hyperideal of $\mathcal{H}$.

(2) $\Rightarrow$ (1). Assume that $\lambda_{\mathcal{M}}$ is a maximal fuzzy almost hyperideal of $\mathcal{H}$. By Theorem 3.2, $\mathcal{M}$ is an almost hyperideal of $\mathcal{H}$. Let $\mathcal{L}$ be an almost hyperideal of $\mathcal{H}$ such that $\mathcal{M} \subseteq \mathcal{L}$. Then, by Lemma 2.1 and Theorem 3.2, $\lambda_{\mathcal{L}}$ is a fuzzy almost hyperideal of $\mathcal{H}$ such that $\lambda_{\mathcal{M}} \subseteq \lambda_{\mathcal{L}}$. This implies that $\text{supp}(\lambda_{\mathcal{M}}) \subseteq \text{supp}(\lambda_{\mathcal{L}})$. By the of $\lambda_{\mathcal{M}}$, we have $\text{supp}(\lambda_{\mathcal{M}}) = \text{supp}(\lambda_{\mathcal{L}})$. That is, $\mathcal{M} = \mathcal{L}$. Therefore, $\mathcal{M}$ is maximal.

We can demonstrate that $\mathcal{M}$ is a maximal almost quasi-hyperideal if and only if $\lambda_{\mathcal{M}}$ is a maximal fuzzy almost quasi-hyperideal by the same technique. □

5. Prime of (fuzzy) almost (resp., quasi-) hyperideals

We introduce various notions of prime almost (resp., quasi-) hyperideals and prime fuzzy almost (resp., quasi-) hyperideals in semihypergroups. Their fundamental related property is provided.

First of all the primes of almost (reps., quasi-) hyperideals are defined.

Definition 5.1. Let $\mathcal{Q}$ be an almost (resp., quasi-) hyperideal of $\mathcal{H}$. Then $\mathcal{Q}$ is said to be:

(1) *prime* if for any almost (resp., quasi-) hyperideals $\mathcal{M}$ and $\mathcal{L}$ of $\mathcal{H}$, we have $\mathcal{M} \subseteq \mathcal{Q}$ or $\mathcal{L} \subseteq \mathcal{Q}$ whenever $\mathcal{M}\mathcal{L} \subseteq \mathcal{Q}$;
(2) *semiprime* if for any almost (resp., quasi-) hyperideal $\mathcal{M}$ of $\mathcal{H}$, we have $\mathcal{M} \subseteq \mathcal{Q}$ whenever $\mathcal{M}^2 \subseteq \mathcal{Q}$;
(3) *strongly prime* if for any almost (resp., quasi-) hyperideals $\mathcal{M}$ and $\mathcal{L}$ of $\mathcal{H}$, we have $\mathcal{M} \subseteq \mathcal{Q}$ or $\mathcal{L} \subseteq \mathcal{Q}$ whenever $\mathcal{M}\mathcal{L} \cap \mathcal{L}\mathcal{M} \subseteq \mathcal{Q}$.

The following definition, we provide the primes of fuzzy almost (resp., quasi-) hyperideals.

Definition 5.2. Let η be a fuzzy almost (resp., quasi-) hyperideal of $\mathcal{H}$. Then η is said to be:

(1) *prime* if for any two fuzzy almost hyperideals ν and ϑ of $\mathcal{H}$, we have $\nu \subseteq \eta$ or $\vartheta \subseteq \eta$ whenever $\nu \circ \vartheta \subseteq \eta$;
(2) *semiprime* if for any fuzzy almost (resp., quasi-) hyperideal ν of $\mathcal{H}$, we have $\nu \subseteq \eta$ whenever $\nu \circ \nu \subseteq \eta$;
(3) *strongly prime* if for any two fuzzy almost (resp., quasi-) hyperideals ν and ϑ of $\mathcal{H}$, we have $\nu \subseteq \eta$ or $\vartheta \subseteq \eta$ whenever $(\nu \circ \vartheta) \cap (\vartheta \circ \nu) \subseteq \eta$.

It is clear that every fuzzy strongly prime almost (resp., quasi-) hyperideal is a fuzzy prime almost (resp., quasi-) hyperideal, and every fuzzy prime almost (resp., quasi-) hyperideal is a fuzzy semiprime almost (resp., quasi-) hyperideal.

A necessary auxiliary result should be presented without proof before we can start our theorem.

Lemma 5.1. *Let η and ν be fuzzy sets of $\mathcal{H}$. Then the following statements hold:*

(a) $\operatorname{supp}(\eta) \cap \operatorname{supp}(\nu) \subseteq \operatorname{supp}(\eta \cap \nu)$;
(b) $\operatorname{supp}(\eta)\operatorname{supp}(\nu) \subseteq \operatorname{supp}(\eta \circ \nu)$.

Theorem 5.1. *Let $\mathcal{Q}$ be a non-empty subset of $\mathcal{H}$. Then the following statements are equivalent:*

(1) *$\mathcal{Q}$ is a strongly prime (resp., prime, semiprime) almost (resp., quasi-) hyperideal of $\mathcal{H}$;*
(2) *$\lambda_{\mathcal{Q}}$ is a strongly prime (resp., prime, semiprime) fuzzy almost (resp., quasi-) hyperideal of $\mathcal{H}$.*

Proof. (1) $\Rightarrow$ (2). Assume that $\mathcal{Q}$ is an almost hyperideal of $\mathcal{H}$. Then, by Theorem 3.2, $\lambda_{\mathcal{Q}}$ is a fuzzy almost hyperideals of $\mathcal{H}$. Let η and ν be fuzzy almost hyperideals of $\mathcal{H}$ such that $(\eta \circ \nu) \cap (\nu \circ \eta) \subseteq \lambda_{\mathcal{Q}}$. By Lemma 5.1, we have that

$$\begin{aligned}\operatorname{supp}(\eta)\operatorname{supp}(\nu) \cap \operatorname{supp}(\nu)\operatorname{supp}(\eta) &\subseteq \operatorname{supp}(\eta \circ \nu) \cap \operatorname{supp}(\nu \circ \eta)\\ &\subseteq \operatorname{supp}((\eta \circ \nu) \cap (\nu \circ \eta)) \subseteq \operatorname{supp}(\lambda_{\mathcal{Q}}).\end{aligned}$$

By Theorem 3.3, we have $\operatorname{supp}(\eta)$ and $\operatorname{supp}(\nu)$ are almost hyperideals of $\mathcal{H}$. Thus, by our presumption, we have $\operatorname{supp}(\eta) \subseteq \operatorname{supp}(\lambda_{\mathcal{Q}})$ or $\operatorname{supp}(\nu) \subseteq \operatorname{supp}(\lambda_{\mathcal{Q}})$. This implies that $\eta \subseteq \lambda_{\mathcal{Q}}$ or $\nu \subseteq \lambda_{\mathcal{Q}}$. Therefore, $\lambda_{\mathcal{Q}}$ is strongly prime.

(2) $\Rightarrow$ (1). Assume that $\lambda_{\mathcal{Q}}$ is a strongly prime fuzzy almost hyperideal of $\mathcal{H}$. Then, by Theorem 3.2, $\mathcal{Q}$ is an almost hyperideal of $\mathcal{H}$. Let $\mathcal{L}$ and $\mathcal{M}$ be almost hyperideals of $\mathcal{H}$ such that $\mathcal{ML} \cap \mathcal{LM} \subseteq \mathcal{Q}$. By Lemma 2.1 and 5.1, we have that

$$(\lambda_{\mathcal{M}} \circ \lambda_{\mathcal{L}}) \cap (\lambda_{\mathcal{L}} \circ \lambda_{\mathcal{M}}) = \lambda_{\mathcal{ML}} \cap \lambda_{\mathcal{LM}} = \lambda_{\mathcal{ML} \cap \mathcal{LM}} \subseteq \lambda_{\mathcal{Q}}.$$

By Theorem 3.2, we have $\lambda_{\mathcal{M}}$ and $\lambda_{\mathcal{L}}$ are fuzzy almost hyperideals of $\mathcal{H}$. Thus, by our assumption, we have $\lambda_{\mathcal{M}} \subseteq \lambda_{\mathcal{Q}}$ or $\lambda_{\mathcal{L}} \subseteq \lambda_{\mathcal{Q}}$. According to Lemma 2.1, it implies that $\mathcal{M} \subseteq \mathcal{Q}$ or $\mathcal{L} \subseteq \mathcal{Q}$. This shows that $\mathcal{Q}$ is strongly prime.

Using a similar methodology, we can show the connection between prime almost hyperideals and prime fuzzy almost hyperideals. We may demonstrate this for the semiprime property by applying $\mathcal{M} = \mathcal{L}$ in the proof. Since the hyperideality and fuzzy hyperideality do not act in the proof, we do not present the evidence of almost quasi-hyperideals and fuzzy almost quasi-hyperideals. □

6. Conclusion

We introduce concepts that we introduce in this study, fuzzy almost hyperideals and fuzzy almost quasi-hyperideals in semihypergroups. We investigate the properties of fuzzy almost (resp., quasi-) hyperideals. Additionally, we establish the connection between almost (resp., quasi-) hyperideals and fuzzy almost (resp., quasi-) hyperideals. Investigated are the minimality, maximality and primes properties of the concepts we defined. Future research will expand this study to include some fuzzy set generalizations.

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