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A NEW CHARACTERIZATION OF PROJECTIVE SPECIAL LINEAR GROUPS $\mathbf{L_2(q)}$

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Abstract. In this paper, we prove that projective special linear groups $L_2(q)$, where $q \equiv \pm 2 \pmod 5$ (q is an odd prime) can be uniquely determined by $|L_2(q)|$ and $nse(L_2(q))$.

1. Introduction

Let G be a finite group, $\pi(G)$ be the set of prime divisors of the order of G and $\pi_e(G)$ be the set of the order of elements in G. If $k \in \pi_e(G)$, then we denote the number of elements of order k in G by $m_k(G)$ and the set of the numbers of elements with the same order in G by $nse(G)$. In other words, $nse(G) = \{m_k(G) \mid k \in \pi_e(G)\}$. Also we denote a sylow p-subgroup of G by G_p and the number of sylow p-subgroups of G by $n_p(G)$. The prime graph $\Gamma(G)$ of group G is a graph whose vertex set is $\pi(G)$, and two vertices u and v are adjacent if and only if $uv \in \pi_e(G)$. Moreover, assume that $\Gamma(G)$ has $t(G)$ connected components π_i, for $i = 1, 2, \ldots, t(G)$. In the case where G is of even order, we always assume that $2 \in \pi_1$.

One of the important problems in finite groups theory is, group characterization by specific property. Properties such as, elements order, set of elements with the same order, the largest elements order, etc. One of this methods is group characterization by using the order of group and $nse(G)$. In other words, we say the group G is characterizable by using the order of G and $nse(G)$, if there exists the group H, so that $nse(G) = nse(H)$ and $|G| = |H|$, then $G \cong H$. Next, in [1, 2, 4–8, 11, 12, 16, 17, 20–22] was proved the groups such as, $PGL_2(q)$, suzuki groups, sporadic groups, $PSL(3, q)$,

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Symmetric group, $U_4(2)$, $PSU(3,3)$ and projective special linear group $PSL(3,3)$, $L_2(p)$ with $p \in \{19, 23\}$, $L_2(q)$ where $q \in \{17, 27, 29\}$, the projective special linear group $L_2(2^a)$, where either $2^a - 1$ or $2^a + 1$ is a prime number, symplectic groups $C_2(3^n)$, projective special linear group $l_3(q)$, projective special unitary groups $U_3(3^n)$ and ${}^2G_2(q)$, where $q \pm \sqrt{3q} + 1$ are prime numbers are characterizable by using $nse(G)$ and the order of G. In this paper, we prove that projective special linear groups $L_2(q)$, where q is an odd prime can be uniquely determined by $|L_2(q)|$ and $nse(L_2(q))$. In fact, we prove the following main theorem.

Theorem 1.1 (Main Theorem)**.** *Let $L_2(q)$ be projective special linear groups, where $q \equiv \pm 2 \pmod 5$ (q is an odd prime) and G be a group with $nse(G) = nse(L_2(q))$, $|G| = |L_2(q)|$. Then $G \cong L_2(q)$.*

2. Notation and Preliminaries

Lemma 2.1 ([13])**.** *Let H be a finite soluble group all of whose elements are of a power prime order. Then $|\pi(H)| \leq 2$.*

Lemma 2.2 ([10])**.** *Let G be a Frobenius group of even order with kernel K and complement H. Then*

(a) *$t(G) = 2$, $\pi(H)$ and $\pi(K)$ are vertex sets of the connected components of $\Gamma(G)$;*
(b) *$|H|$ divides $|K| - 1$;*
(c) *K is nilpotent.*

Definition 2.1. A group G is called a 2-Frobenius group if there is a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that $\frac{G}{H}$ and K are Frobenius groups with kernels $\frac{K}{H}$ and H, respectively.

Lemma 2.3 ([14])**.** *Let G be a 2-Frobenius group of even order. Then*

(a) *$t(G) = 2$, $\pi(H) \cup \pi(\frac{G}{K}) = \pi_1$ and $\pi(\frac{K}{H}) = \pi_2$;*
(b) *$\frac{G}{K}$ and $\frac{K}{H}$ are cyclic groups satisfying $|\frac{G}{K}|$ divides $|Aut(\frac{K}{H})|$. In particular, every 2-Frobenius group is soluble group.*

Lemma 2.4 ([25])**.** *Let G be a finite group with $t(G) \geq 2$. Then one of the following statements holds:*

(a) *G is a Frobenius group;*
(b) *G is a 2-Frobenius group;*
(c) *G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H and $\frac{G}{K}$ are π_1-groups, K/H is a non-abelian simple group, H is a nilpotent group and $|\frac{G}{K}|$ divides $|Out(\frac{K}{H})|$.*

Lemma 2.5 ([9])**.** *Let G be a finite group and m be a positive integer dividing $|G|$. If $L_m(G) = \{g \in G \mid g^m = 1\}$, then $m \mid |L_m(G)|$.*

Lemma 2.6. *Let G be a finite group. Then, for every $i \in \pi_e(G)$, $\varphi(i)$ divides $m_i(G)$, and i divides $\sum_{j|i} m_j(G)$. Moreover, if $i > 2$, then $m_i(G)$ is even.*

Proof. By Lemma 2.5, the proof is straightforward. □

Lemma 2.7 ([24]). *Let G be a non-abelian simple group such that $(5, |G|) = 1$. Then G is isomorphic to one of the following groups:*

(a) $L_n(q)$, $n = 2, 3$, $q \equiv \pm 2 \pmod 5$;
(b) $G_2(q)$, $q \equiv \pm 2 \pmod 5$;
(c) ${}^2A_2(q)$, $q \equiv \pm 2 \pmod 5$;
(d) ${}^3D_4(q)$, $q \equiv \pm 2 \pmod 5$;
(e) ${}^2G_2(q)$, $q = 3^{2m+1}$, $m \geq 1$.

Remark 2.1. In previous lemma, we consider the projective special linear groups by $A_n(q) \cong L_{n+1}(q)$ and the projective special unitary groups by ${}^2A_{n-1}(q) \cong U_n(q)$.

Lemma 2.8 ([26]). *Let q, k, l be natural numbers. Then*

(a) $(q^k - 1, q^l - 1) = q^{(k,l)} - 1$;

(b) $(q^k + 1, q^l + 1) = \begin{cases} q^{(k,l)} + 1, & \text{if both } \frac{k}{(k,l)} \text{ and } \frac{l}{(k,l)} \text{ are odd,} \\ (2, q+1,) & \text{otherwise}; \end{cases}$

(c) $(q^k - 1, q^l + 1) = \begin{cases} q^{(k,l)} + 1, & \text{if } \frac{k}{(k,l)} \text{ is even and } \frac{l}{(k,l)} \text{ is odd,} \\ (2, q+1), & \text{otherwise.} \end{cases}$

In particular, for every $q \geq 2$ and $k \geq 1$, the inequality $(q^k - 1, q^k + 1) \leq 2$ holds.

Lemma 2.9. *Let L be projective special linear group $L_2(q)$, where q is an odd prime number and p is an isolated vertex in $\Gamma(G)$. Then $m_p(G) = \frac{(p-1)|G|}{(4p)}$ and for every $i \in \pi_e(G) - \{1, p\}$, p divides $m_i(G)$.*

Proof. Since that $|G_p| = p$, it follows that G_p is a cyclic group of order p. Thus $m_p(G) = \phi(G)n_p(G)$. Now, it is enough to show $n_p(G) = \frac{|G|}{(4p).}$ By [25], p is an isolated vertex of $\Gamma(G)$. Hence, $|C_G(G_p)| = p$ and $|N_G(G_p)| = xp$ for a natural number x. On the other hand, we know that $\frac{N_G(G_p)}{C_G(G_p)}$ embed in $Aut(G_p)$, which implies $x \mid p - 1$. Furthermore, by Sylow's theorem, $n_p(G) = |G : N_G(G_p)|$ and $n_p(G) \equiv 1 \pmod p$. Thus, p divides $\frac{|G|}{xp} - 1$ thus $\frac{q\pm1}{2}$ divides $\frac{q(q^2-1)}{xp} - 1$. It follows that $\frac{q\pm1}{2}$ divides $q^2 \pm q - x$ so we have $p \mid 4 - x$ and since $x \mid p - 1$ we deduce that $x = 4$, and the proof is finished. We prove that p is an isolated vertex of $\Gamma(G)$. Opposite there is $t \in \pi(G) - \{p\}$ such that $tp \in \pi_e(G)$. So $m_{tp}(G) = \phi(tp)n_p(G)k$, where k is the number of cyclic subgroups of order t in $C_G(G_p)$ and since $n_p(G) = n_p(L)$, it follows that $m_{tp}(G) = \frac{(t-1)(p-1)|L|k}{(4p)}$. If $m_{tp}(G) = m_p(L)$, then $t = 2$ and $k = 1$. Furthermore lemma 2.6 yields $p \mid m_2(G) + m_{2p}(G)$ and since $m_2(G) = m_2(L)$ and $p \mid m_2(L)$, we deduce that $p \mid m_{2p}(G)$, which is a contradiction. So Lemma 2.5 implies that $p \mid m_{tp}(G)$. Hence $p \mid t - 1$ and since $m_{tp}(G) < |G|$ we deduce $t - 1 \leq 6$. As a result $t \in \{2, 3, 5, 7\}$, where this is a contradiction. Let $r \in \pi_e(G) - \{1, p\}$. Since p is an isolated vertex of $\Gamma(G)$, it follows that $p \nmid r$ and $pr \notin \pi_e(G)$. Thus, G_p acts fixed point freely on the set of elements of order r by conjugation and hence $|G_p| \mid m_r(G)$. So, we conclude that $p \mid m_r(G)$. □

3. Proof of the Main Theorem

In this section, we prove the main theorem in the following lemmas. We denote the Projective specialy unitary group $L_2(q)$, where q is an prime number by L and prime number $\frac{q\pm1}{2}$ by p. Recall that G is a group with $|G| = |L|$ and $nse(G) = nse(L)$.

Lemma 3.1. *$m_2(G) = m_2(L)$, $m_p(G) = m_p(L)$, $n_p(G) = n_p(L)$, and $p \mid m_k(G)$ for every $k \in \pi_e(G) - \{1, p\}$.*

Proof. By Lemma 2.6, for every $1 \neq r \in \pi_e(G)$, $r = 2$ if and only if $m_r(G)$ is odd. Thus it follows that $m_2(G) = m_2(L)$. According to Lemma 2.6, $(m_p(G), p) = 1$. Thus $p \nmid m_p(G)$ and hence Lemma 2.9 implies that $m_p(G) \in \{m_1(L), m_2(L), m_p(L)\}$. Moreover, $m_p(G)$ is even, so we deduce that $m_p(G) = m_p(L)$. Since G_p and L_p are cyclic groups of order p and $m_p(G) = m_p(L)$, we deduce that $m_p(G) = \varphi(p)n_p(G) = \varphi(p)n_p(L) = m_p(L)$, so $n_p(G) = n_p(L)$. Let $k \in \pi_e(G) - \{1, p\}$. Since p is an isolated vertex of $\Gamma(G)$, $p \nmid k$ and $pk \notin \pi_e(G)$. Thus, G_p acts fixed point freely on the set of elements of order k by conjugation and hence $|G_p| \mid m_k(G)$. So, we conclude that $p \mid m_k(G)$. □

Lemma 3.2. *The group G is not a Frobenius group.*

Proof. Let G be a Frobenius group with kernel K and complement H. Then by Lemma 3.2, $t(G) = 2$ and $\pi(H)$ and $\pi(K)$ are vertex sets of the connected components of $\Gamma(G)$ and $|H|$ divides $|K| - 1$. Now, by Lemma 3.1, p is an isolated vertex of $\Gamma(G)$. It follows that (i) $|H| = p$ and $|K| = \frac{|G|}{p}$ or (ii) $|H| = \frac{|G|}{p}$ and $|K| = p$. Since $|H|$ divides $|K| - 1$, we deduce that the case (ii) cannot occur. So, $|H| = p$ and $|K| = \frac{|G|}{p}$. Hence, $\frac{q\pm1}{2} \mid \frac{q(q^2-1)}{q\pm1} - 1$. So, we have $q \pm 1 \mid 2q^2 \pm 2q - 2$. It follows that $p \mid 2$. Thus, $p \mid 2$, which is impossible. □

Lemma 3.3. *The group G is not soluble group.*

Proof. Let $r \neq 2$ and s be a prime divisor of $\frac{q+1}{2}$ and let t be a prime divosor $\frac{q-1}{2}$. If G were soluble, then there would exist a $\{2, r, s\}$-Hall subgroup H of G. Since H does not contain any elements of orders $2r$, $2s$, rs. Thus, all of elements of H would be of prime power order. But this contradicts Lemma 2.1. So, G is not soluble group. □

Lemma 3.4. *The group G is not a 2-Frobenius group.*

Proof. By previous lemma, we have that G is not soluble group. On the other hand by Lemma 2.3 since that every 2-Frobenius group is soluble group, so G is not a 2-Frobenius group. □

Lemma 3.5. *The group G is isomorphic to the group L.*

Proof. By Lemma 2.9, p is an isolated vertex of $\Gamma(G)$. Thus, $t(G) > 1$ and G satisfies one of the cases of Lemma 2.4. Now, Lemma 3.2 and Lemma 3.4 imply that G is neither a Frobenius group nor a 2-Frobenius group. Thus only the case (c) of Lemma 2.4 occurs. So, G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H and G/K are π_1-groups, K/H is a non-abelian simple group. Since p is an isolated vertex of $\Gamma(G)$, we have $p \mid |\frac{K}{H}|$. On the other hand, we know that $5 \nmid |G|$. Thus, $\frac{K}{H}$ is isomorphic to one of the groups in Lemma 2.7. Hence, we consider the following isomorphisms.

(1) If $\frac{K}{H} \cong G_2(q')$, where $q' \equiv \pm 2 \pmod 5$, then by [25], $p = q'^2 \pm q' + 1$. On the other hand, we know $|G_2(q')|$ divides $|G|$, in other words $q'^6(q'^6-1)(q'-1) \mid q(q^2-1)$. Thus, $\frac{q\pm1}{2} = q'^2 \pm q' + 1$ as a result $q \pm 1 = 2q'^2 + 2q' + 2$. So, we deduce $q = 2q'^2 + 2q' + 1$ and $q = 2q'^2 + 2q' + 3$.

(2) If $\frac{K}{H} \cong {}^2A_2(q')$, where $q' \equiv \pm 2 \pmod 5$, then by [25], $p = \frac{q'^2-q'+1}{(3,q'+1)}$. On the other hand, we know $|{}^2A_2(q')| \mid |G|$, in other words $\frac{q'^3(q'^3+1)(q'^2-1)}{(3,q'+1)} \mid \frac{q(q^2-1)}{2}$. First, if $(3, q'+1) = 1$, then similar to part (1) we deduce a contradiction. Now let $(3, q'+1) = 3$. Then, $\frac{q\pm1}{2} = \frac{q'^2-q'+1}{3}$. Thus $3q \pm 3 = 2q'^2 - 2q' + 2$. As a result $q = \frac{2q'^2-2q'-1}{3}$ and $q = \frac{2q'^2-2q'+5}{3}$. Since that $|{}^2A_2(q')| \nmid |G|$, so we have a contradiction.

(3) If $\frac{K}{H} \cong {}^2G_2(q')$, where $q' \equiv \pm 2 \pmod 5$, then by [25] $p = q' \pm \sqrt{3q'} + 1$. On the other hand, we know $|{}^2G_2(q')| \mid |G|$, in other words $q'^3(q'^3+1)(q'-1) \mid \frac{q(q^2-1)}{2}$. Now, we consider $\frac{q\pm1}{2} = q' \pm \sqrt{3q'} + 1$ as a result $q = 2(3^{2m+1}) + 2(3^{m+1} + 1)$ and $q = 2(3^{2m+1}) - 2(3^{m+1}) + 3$. Since that $|{}^2G_2(q')| \nmid |G|$, so we have a contradiction.

(4) If $\frac{K}{H} \cong L_3(q')$, where $q' \equiv \pm 2 \pmod 5$, then by [25], $p = \frac{q'^2+q'+1}{(3,q'-1)}$. On the other hand, we know $|L_3(q')| \mid |G|$, in other words $\frac{q'^2+q'+1}{(3,q'-1)} \mid \frac{q(q^2-1)}{2}$. First if $\frac{q\pm1}{2} = q'^2 + q' + 1$, then $q \pm 1 = 2q'^2 + 2q' + 2$. So, we deduce $q = 2q'^2 + 2q' + 1$and $q = 2q'^2 + 2q' + 3$. Now since $|L_3(q')| \nmid |G|$, which is a contradiction. Now, if $(3; q'-1) = 3$, then $\frac{q\pm1}{2} = \frac{q'^2+q'+1}{3}$, so $3q \pm 3 = 2q'^2 + 2q' + 2$. It follows that $q = \frac{2q'^2+2q'-1}{3}$ and $q = \frac{2q'^2+2q'+5}{3}$. But $|L_3(q')| \nmid |G|$, which is a contradiction.

Now, let $(3; q'+1) = 3$. So, $3q \pm 3 = 2q'^2 - 2q' + 2$. As a result $q = \frac{2q'^2-2q'-1}{3}$ and $q = \frac{2q'^2-2q'+5}{3}$. But $|L_3(q')| \nmid |G|$, which is a contradiction.

(5) If $\frac{K}{H} \cong {}^3D_4(q')$, where $q' \equiv \pm 2 \pmod 5$, then by [25], $p = q'^4 - q'^2 + 1$. On the other hand, we know $|{}^3D_4(q')| \mid |G|$, in other words $q'^{12}(q'^8 + q'^4 + 1)(q'^6 - 1)(q'^2 - 1) \mid \frac{q(q^2-1)}{2}$. As a result $\frac{q\pm1}{2} = q'^4 - q'^2 + 1$, so $q = 2q'^4 - 2q'^2 + 1$ and $q = 2q'^4 - 2q'^2 + 3$. Since $|{}^3D_4(q')| \nmid |G|$, which is a contradiction.

(6) Hence, $\frac{K}{H} \cong L_2(q')$. As a result $|\frac{K}{H}| = |L_2(q')|$. On the other hand, we have $p \mid \pi(\frac{K}{H})$, so $\frac{q\pm1}{2} = \frac{q'\pm1}{2}$. It follows that $q = q'$. Now, on the other hand, G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H and $\frac{G}{K}$ are π_1-groups, so $|H| = 1$ and $K = L_2(q') \cong G$. □

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