ON $\check{\phi}$-SEMISYMMETRIC LP-KENMOTSU MANIFOLDS WITH A QSNM-CONNECTION ADMITTING RICCI SOLITONS

RAJENDRA PRASAD[1], ABDUL HASEEB[2], AND UMESH KUMAR GAUTAM[1]

ABSTRACT. In the present work, we characterize Lorentzian para-Kenmotsu (briefly, LP-Kenmotsu) manifolds with a quarter-symmetric non-metric connection (briefly, QSNM-connection) $\widehat{\nabla}$ satisfying certain $\ddot{\phi}$-semisymmetric conditions admitting Ricci solitons. At the end of the paper, a 3-dimensional example of LP-Kenmotsu manifolds with a connection $\widehat{\nabla}$ is given to verify some results of the present paper.

1. Introduction

In a $(2n+1)$-dimensional connected and C^∞-smooth semi-Riemannian manifold $(M, \check{g})$, the Levi-Civita connection $\check{\nabla}$, the Riemannian-Christoffel curvature tensor $\check{R}$, the projective curvature tensor $\check{P}$, the concircular curvature tensor $\check{V}$, the conformal curvature tensor $\check{C}$ and the D-conformal curvature tensor $\check{B}$ are defined by [5,6]

(1.1)
$$\check{R}(\check{E}, \check{F})\check{W} = \check{\nabla}_{\check{E}} \check{\nabla}_{\check{F}} \check{W} - \check{\nabla}_{\check{F}} \check{\nabla}_{\check{E}} \check{W} - \check{\nabla}_{[\check{E}, \check{F}]} \check{W},$$

(1.2)
$$\check{P}(\check{E}, \check{F})\check{W} = \check{R}(\check{E}, \check{F})\check{W} - \frac{1}{2n}[\check{S}(\check{F}, \check{W})\check{E} - \check{S}(\check{E}, \check{W})\check{F}],$$

(1.3)
$$\check{V}(\check{E}, \check{F})\check{W} = \check{R}(\check{E}, \check{F})\check{W} - \frac{\check{r}}{2n(2n+1)}[\check{g}(\check{F}, \check{W})\check{E} - \check{g}(\check{E}, \check{W})\check{F}],$$

(1.4)
$$\check{C}(\check{E},\check{F})\check{W} = \check{R}(\check{E},\check{F})\check{W} - \frac{1}{(2n-1)}[\check{S}(\check{F},\check{W})\check{E} - \check{S}(\check{E},\check{W})\check{F}$$
$$+ \check{g}(\check{F},\check{W})\check{Q}\check{E} - \check{g}(\check{E},\check{W})\check{Q}\check{F}] + \frac{\check{r}}{2n(2n-1)}[\check{g}(\check{F},\check{W})\check{E} - \check{g}(\check{E},\check{W})\check{F}],$$

(1.5)
$$\check{B}(\check{E},\check{F})\check{W} = \check{R}(\check{E},\check{F})\check{W} + \frac{1}{2(n-1)}[\check{S}(\check{E},\check{W})\check{F} - \check{S}(\check{F},\check{W})\check{E} + \check{g}(\check{E},\check{W})\check{Q}\check{F}$$
$$- \check{g}(\check{F},\check{W})\check{Q}\check{E} - \check{S}(\check{E},\check{W})\check{\eta}(\check{F})\xi + \check{S}(\check{F},\check{W})\check{\eta}(\check{E})\xi - \check{\eta}(\check{E})\check{\eta}(\check{W})\check{Q}\check{F}$$
$$+ \check{\eta}(\check{F})\check{\eta}(\check{W})\check{Q}\check{E}] - \frac{k-2}{2(n-1)}[\check{g}(\check{E},\check{W})\check{F} - \check{g}(\check{F},\check{W})\check{E}] + \frac{k}{2(n-1)}$$
$$\times [\check{g}(\check{E},\check{W})\check{\eta}(\check{F})\xi - \check{g}(\check{F},\check{W})\check{\eta}(\check{E})\xi + \check{\eta}(\check{E})\check{\eta}(\check{W})\check{F} - \check{\eta}(\check{F})\check{\eta}(\check{W})\check{E}],$$

respectively, where $\check{r}$ is the scalar curvature, $\check{S}$ and $\check{Q}$ are the Ricci tensor and the Ricci operator, respectively such that $\check{S}(\check{E},\check{F}) = \check{g}(\check{Q}\check{E},\check{F})$ and $k = \frac{\check{r}+4n}{2n-1}$.

The connection $\widehat{\nabla}$ which is linear and defined on $(M,\check{g})$ is said to be a quarter-symmetric [11] if its torsion tensor $\check{T}$

(1.6) $$\check{T}(\check{E},\check{F}) = \widehat{\nabla}_{\check{E}}\check{F} - \widehat{\nabla}_{\check{F}}\check{E} - [\check{E},\check{F}] = \check{\eta}(\check{F})\check{\phi}\check{E} - \check{\eta}(\check{E})\check{\phi}\check{F},$$

where $\check{\phi}$ is a $(1,1)$-tensor field and $\check{\eta}$ is a 1-form. If moreover, $\widehat{\nabla}$ satisfies the condition

(1.7) $$(\widehat{\nabla}_{\check{E}}\check{g})(\check{F},\check{W}) = -\check{\eta}(\check{F})\check{g}(\check{\phi}\check{E},\check{W}) - \check{\eta}(\check{W})\check{g}(\check{F},\check{\phi}\check{E}),$$

where $\check{E},\check{F},\check{W} \in \chi(M)$ and $\chi(M)$ is the set of all differentiable vector fields on M, then connection $\widehat{\nabla}$ is called a QSNM-connection. The authors in [2, 3, 7, 12] have studied QSNM-connection in various manifolds.

In an LP-Kenmotsu manifold, a relation between the connections $\widehat{\nabla}$ and $\check{\nabla}$ is given by

(1.8) $$\widehat{\nabla}_{\check{E}}\check{F} = \check{\nabla}_{\check{E}}\check{F} + \check{\eta}(\check{F})\check{\phi}\check{E}.$$

On a Riemannian manifold $(M,\check{g})$, a Ricci soliton $(\check{g},U,\check{\lambda})$ is a generalization of an Einstein metric such that (see [9,10]) $\check{\mathcal{L}}_U\check{g} + 2\check{S} + 2\check{\lambda}\check{g} = 0$, where $\check{S}$, $\check{\mathcal{L}}_U$ and $\check{\lambda}$ are the Ricci tensor, the Lie derivative operator along the vector field U on M and a real constant, respectively. A Ricci soliton is said to be shrinking, steady or expanding according as $\check{\lambda} < 0$, $\check{\lambda} = 0$ or $\check{\lambda} > 0$, respectively.

The present work is arranged in the following manner. After Introduction, a brief introducton of LP-Kenmotsu manifolds is given in Section 2. In Section 3, we find the relation between the curvature tensors of an LP-Kenmotsu manifold with the connections $\check{\nabla}$ and $\widehat{\nabla}$. In Section 4, we study LP-Kenmotsu manifolds with a connection $\widehat{\nabla}$ admitting Ricci solitons. $\check{\phi}$-projectively semisymmetric, $\check{\phi}$-concircularly semisymmetric, $\check{\phi}$-conformally semisymmetric and $\check{\phi}$-D-conformally semisymmetric LP-Kenmotsu manifolds with a connection $\widehat{\nabla}$ admitting Ricci solitons have been

studied in Section 5. At the end of the paper, a 3-dimensional example of *LP*-Kenmotsu manifolds with a connection $\widehat{\nabla}$ is given to verify some results of the present paper.

2. Preliminaries

A $(2n + 1)$-dimensionsional differentiable manifold M with structure $(\breve{\phi}, \xi, \breve{\eta}, \breve{g})$ is said to be a Lorentzian almost paracontact metric manifold, if it admits $\breve{\phi}$: a tensor field of type $(1, 1)$, ξ: a contravariant vector field, $\breve{\eta}$: a 1-form and $\breve{g}$: a Lorentzian metric satisfying [8]

$$(2.1) \qquad \breve{\eta}(\xi) = -1,$$

$$(2.2) \qquad \breve{\phi}^2 \breve{E} = \breve{E} + \breve{\eta}(\breve{E})\xi,$$

$$(2.3) \qquad \breve{\phi}\xi = 0, \quad \breve{\eta}(\breve{\phi}Y) = 0,$$

$$\breve{g}(\breve{\phi}\breve{E}, \breve{\phi}\breve{F}) = \breve{g}(\breve{E}, \breve{F}) + \breve{\eta}(\breve{E})\breve{\eta}(\breve{F}),$$

$$(2.4) \qquad \breve{g}(\breve{E}, \xi) = \breve{\eta}(\breve{E}),$$

$$(2.5) \qquad \breve{\Phi}(\breve{F}, \breve{E}) = \breve{\Phi}(\breve{E}, \breve{F}) = \breve{g}(\breve{E}, \breve{\phi}\breve{F}),$$

for any $\breve{E}, \breve{F}$ on M.

For ξ: a killing vector field, the (para) contact structure is said to be a K-(para) contact. In this case, we have

$$(2.6) \qquad \breve{\nabla}_{\breve{E}}\xi = \breve{\phi}\breve{E}.$$

A Lorentzian almost paracontact manifold M is called an *LP*-Sasakian manifold if

$$(\breve{\nabla}_{\breve{E}}\breve{\phi})\breve{F} = \breve{g}(\breve{E}, \breve{F})\xi + \breve{\eta}(\breve{F})\breve{E} + 2\breve{\eta}(\breve{E})\breve{\eta}(\breve{F})\xi,$$

for any $\breve{E}, \breve{F}$ on M.

Now, we define a new manifold called a Lorentzian para-Kenmostu (briefly, *LP*-Kenmotsu) manifold:

Definition 2.1. A Lorentzian almost paracontact manifold is called Lorentzian para-Kenmostu (briefy, *LP*-Kenmostu) manifold if [1]

$$(2.7) \qquad (\breve{\nabla}_{\breve{E}}\breve{\phi})\breve{F} = -\breve{g}(\breve{\phi}\breve{E}, \breve{F})\xi - \breve{\eta}(\breve{F})\breve{\phi}\breve{E},$$

for any $\breve{E}, \breve{F}$ on M.

In the Lorentzian para-Kenmostu manifold, we have

$$\breve{\nabla}_{\breve{E}}\xi = -\breve{\phi}^2 \breve{E},$$

$$(\breve{\nabla}_{\breve{E}}\breve{\eta})\breve{F} = -\breve{g}(\breve{\phi}\breve{E}, \breve{\phi}\breve{F}).$$

Moreover, on an *LP*-Kenmotsu, the following relations hold [1]:

$$\breve{g}(\breve{R}(\breve{E}, \breve{F})\breve{W}, \xi) = \breve{\eta}(\breve{R}(\breve{E}, \breve{F})\breve{W}) = \breve{g}(\breve{F}, \breve{W})\breve{\eta}(\breve{E}) - \breve{g}(\breve{E}, \breve{W})\breve{\eta}(\breve{F}),$$

$$\check{R}(\xi, \check{E})\check{F} = - \check{R}(\check{E}, \xi)\check{F} = \check{g}(\check{E}, \check{F})\xi - \check{\eta}(\check{F})\check{E},$$
$$\check{R}(\check{E}, \check{F})\xi = \check{\eta}(\check{F})\check{E} - \check{\eta}(\check{E})\check{F},$$
$$\check{R}(\xi, \check{E})\xi = \check{E} + \check{\eta}(\check{E})\xi,$$
$$\check{S}(\check{E}, \xi) = (\dim M - 1)\check{\eta}(\check{E}), \quad \check{S}(\xi, \xi) = -(\dim M - 1),$$
$$\check{Q}\xi = (\dim M - 1)\xi,$$

for any $\check{E}, \check{F}, \check{W}$ on M.

Definition 2.2. An LP-Kenmotsu manifold is called an η-Einstein manifold if its Ricci tensor satisfies [4] $\check{S}(\check{E}, \check{F}) = a_1\check{g}(\check{E}, \check{F}) + a_2\check{\eta}(\check{E})\check{\eta}(\check{F})$, where a_1 and a_2 are smooth functions on M.

3. Curvature Tensor of LP-Kenmotsu Manifolds with a Connection $\widehat{\nabla}$

The curvature tensor $\widehat{R}$ of an LP-Kenmotsu manifold with a connection $\widehat{\nabla}$ is defined by

$$(3.1) \qquad \widehat{R}(\check{E}, \check{F})\check{W} = \widehat{\nabla}_{\check{E}}\widehat{\nabla}_{\check{F}}\check{W} - \widehat{\nabla}_{\check{F}}\widehat{\nabla}_{\check{E}}\check{W} - \widehat{\nabla}_{[\check{E}, \check{F}]}\check{W}.$$

From (1.8), (2.1), (2.4), (2.6), (2.7) and (3.1), we obtain

$$(3.2) \qquad \widehat{R}(\check{E}, \check{F})\check{W} = \check{R}(\check{E}, \check{F})\check{W} - \check{g}(\check{E}, \check{W})\check{\phi}\check{F} + \check{g}(\check{F}, \check{W})\check{\phi}\check{E},$$

where $\check{R}(\check{E}, \check{F})\check{W}$ is given by (1.1). Contracting $\check{E}$ in (3.2), we get

$$(3.3) \qquad \widehat{S}(\check{F}, \check{W}) = \check{S}(\check{F}, \check{W}) + \check{g}(\check{F}, \check{W})\check{\psi} - \check{g}(\check{\phi}\check{F}, \check{W}).$$

From (3.3), it follows that

$$\widehat{Q}\check{F} = \check{Q}\check{F} + \check{\psi}\check{F} - \check{\phi}\check{F},$$

Contracting again $\check{F}$ and $\check{W}$ in (3.3), we obtain

$$(3.4) \qquad \widehat{r} = \check{r} + 2n\check{\psi},$$

where $\widehat{Q}$ is the Ricci operator, $\widehat{S}$ is the Ricci tensor and $\widehat{r}$ is the scalar curvature with respect to $\widehat{\nabla}$.

Lemma 3.1. *In a $(2n + 1)$-dimensional LP-Kenmotsu manifold with a connection $\widehat{\nabla}$, we have*

$$\widehat{R}(\check{E}, \check{F})\xi = \check{\eta}(\check{F})\check{E} - \check{\eta}(\check{E})\check{F} + \check{\eta}(\check{F})\check{\phi}\check{E} - \check{\eta}(\check{E})\check{\phi}\check{F},$$
$$(3.5) \qquad \widehat{R}(\xi, \check{E})\check{F} = - \widehat{R}(\check{E}, \xi)\check{F} = \check{g}(\check{E}, \check{F})\xi - \check{\eta}(\check{F})\check{E} - \check{\eta}(\check{F})\check{\phi}\check{E},$$
$$\widehat{R}(\xi, \check{E})\xi = \check{\eta}(\check{E})\check{\xi} + \check{E} + \check{\phi}\check{E},$$
$$(3.6) \qquad \widehat{S}(\check{E}, \xi) = (2n + \check{\psi})\check{\eta}(\check{E}), \quad \widehat{S}(\xi, \xi) = -(2n + \check{\psi}),$$
$$(3.7) \qquad \widehat{\nabla}_{\check{E}}\xi = - \check{E} - \check{\eta}(\check{E})\xi - \check{\phi}\check{E},$$
$$(3.8) \qquad \widehat{Q}\xi = (2n + \check{\psi})\xi,$$

for any $\check{E}, \check{F}$ on M.

4. RICCI SOLITON ON *LP*-KENMOTSU MANIFOLDS WITH A CONNECTION $\widehat{\nabla}$

Suppose that an *LP*-Kenmotsu manifold with a connection $\widehat{\nabla}$ admits a Ricci soliton $(\check{g}, \xi, \check{\lambda})$. Then in view of (1.9), we have

$$(4.1) \qquad (\widehat{\check{\mathcal{L}}}_\xi \check{g})(\check{F}, \check{W}) + 2\widehat{S}(\check{F}, \check{W}) + 2\check{\lambda}\check{g}(\check{F}, \check{W}) = 0.$$

By using (3.7) and (1.6), we find

$$(4.2) \qquad (\widehat{\check{\mathcal{L}}}_\xi \check{g})(\check{F}, \check{W}) = -2[\check{g}(\check{F}, \check{W}) + \check{\eta}(\check{F})\check{\eta}(\check{W})].$$

Combining (4.1) and (4.2), we obtain

$$(4.3) \qquad \widehat{S}(\check{F}, \check{W}) = (1 - \check{\lambda})\check{g}(\check{F}, \check{W}) + \check{\eta}(\check{F})\check{\eta}(\check{W}).$$

Taking $\check{W} = \xi$ in (4.3) and then using (2.1), (2.3), we get

$$(4.4) \qquad \widehat{S}(\check{F}, \xi) = -\check{\lambda}\check{\eta}(\check{F}).$$

Thus from (3.6) and (4.4), it follows that

$$(4.5) \qquad \check{\lambda} = -(2n + \check{\psi}).$$

Hence, (4.3) together with (4.5) leads to the following theorem.

Theorem 4.1. *If an LP-Kenmotsu manifold M with a connection $\widehat{\nabla}$ admits a Ricci soliton $(\check{g}, \xi, \check{\lambda})$, then M is an η-Einstein manifold and its Ricci solition will be expanding, shrinking or steady according to $\check{\psi} < -2n$, $\check{\psi} > -2n$ or $\check{\psi} = -2n$.*

Now, assumig that $(\check{g}, U, \check{\lambda})$ is a Ricci soliton on an *LP*-Kenmotsu manifold with a connection $\widehat{\nabla}$ such that U is pointwise collinear with ξ, i.e., $U = \beta\xi$, where β is a function. Then (1.9) holds and we have

$$\beta\check{g}(\widehat{\nabla}_{\check{E}}\xi, \check{F}) + (\check{E}\beta)\check{\eta}(\check{F}) + \beta\check{g}(\check{E}, \widehat{\nabla}_{\check{F}}\xi) + (\check{F}\beta)\check{\eta}(\check{E}) + 2\widehat{S}(\check{E}, \check{F}) + 2\check{\lambda}\check{g}(\check{E}, \check{F}) = 0,$$

which in view of (3.7) and (1.6) becomes

$$(4.6) \quad -2\beta[\check{g}(\check{E}, \check{F}) + \check{\eta}(\check{E})\check{\eta}(\check{F})] + (\check{E}\beta)\check{\eta}(\check{F}) + (\check{F}\beta)\check{\eta}(\check{E}) + 2\widehat{S}(\check{E}, \check{F}) + 2\check{\lambda}\check{g}(\check{E}, \check{F}) = 0.$$

Replacing $\check{F}$ by ξ in (4.6) and using (2.1), (2.4) and (3.6), we find

$$(4.7) \qquad -(\check{E}\beta) + [(\xi\beta) + 2(2n + \check{\psi}) + 2\check{\lambda}]\check{\eta}(\check{E}) = 0,$$

which by taking $\check{E} = \xi$ and using (2.1) yields

$$(4.8) \qquad (\xi\beta) + (2n + \check{\psi}) + \check{\lambda} = 0.$$

Combining the equations (4.7) and (4.8), we find

$$(4.9) \qquad d\beta = [(2n + \check{\psi}) + \check{\lambda}]\check{\eta}.$$

Now, applying d on (4.9), we get

$$(4.10) \qquad [(2n + \check{\psi}) + \check{\lambda}]\check{\eta} = 0 \implies \check{\lambda} = -(2n + \check{\psi}), \quad d\check{\eta} \neq 0.$$

Thus, from (4.9) and (4.10), we obtain $d\beta = 0$, i.e., β is a constant. Therefore, (4.6) reduces to

$$(4.11) \qquad \widehat{S}(\check{E}, \check{F}) = (\beta - \check{\lambda})\check{g}(\check{E}, \check{F}) + \beta\check{\eta}(\check{E})\check{\eta}(\check{F}).$$

Hence, (4.10) together with (4.11) leads the following theorem.

Theorem 4.2. *If an LP-Kenmotsu manifold M with a connection $\widehat{\nabla}$ admits a Ricci soliton $(\check{g}, U, \check{\lambda})$ such that U is pointwise collinear with ξ, then U is a constant multiple of ξ and M is an η-Einstein manifold and its Ricci solition will be expanding, shrinking or steady according to $\check{\psi} < -2n$, $\check{\psi} > -2n$ or $\check{\psi} = -2n$.*

5. Ricci soliton on $\check{\phi}$-semisymmetric LP-Kenmotsu manifolds with a connection $\widehat{\nabla}$

Definition 5.1. An LP-Kenmotsu manifold with a connection $\widehat{\nabla}$ is called $\check{\phi}$-projectively semisymmetric if (see [13]) $\widehat{P}(\check{E}, \check{F}) \cdot \check{\phi} = 0$ for all $\check{E}, \check{F}$ on M.

Analogous to the equation (1.2), the projective curvature tensor with a connection $\bar{\nabla}$ is given by

$$(5.1) \qquad \widehat{P}(\check{E}, \check{F})\check{W} = \widehat{R}(\check{E}, \check{F})\check{W} - \frac{1}{2n}[\widehat{S}(\check{F}, \check{W})\check{E} - \widehat{S}(\check{E}, \check{W})\check{F}].$$

Suppose that a $(2n + 1)$-dimensional LP-Kenmotsu manifold with a connection $\widehat{\nabla}$ is $\check{\phi}$-projectively semisymmetric, therefore

$$(5.2) \qquad (\widehat{P}(\check{E}, \check{F}) \cdot \check{\phi})\check{W} = \widehat{P}(\check{E}, \check{F})\check{\phi}\check{W} - \check{\phi}\widehat{P}(\check{E}, \check{F})\check{W} = 0,$$

for all $\check{E}, \check{F}, \check{W}$ on M. From (5.1), we find

$$(5.3) \qquad \widehat{P}(\check{E}, \check{F})\check{\phi}\check{W} = \widehat{R}(\check{E}, \check{F})\check{\phi}\check{W} - \frac{1}{2n}[\widehat{S}(\check{F}, \check{\phi}\check{W})\check{E} - \widehat{S}(\check{E}, \check{\phi}\check{W})\check{F}],$$

$$(5.4) \qquad \check{\phi}\widehat{P}(\check{E}, \check{F})\check{W} = \check{\phi}\widehat{R}(\check{E}, \check{F})\check{W} - \frac{1}{2n}[\widehat{S}(\check{F}, \check{W})\check{\phi}\check{E} - \widehat{S}(\check{E}, \check{W})\check{\phi}\check{F}].$$

By combining (5.2), (5.3) and (5.4), we have

$$(5.5) \qquad \widehat{R}(\check{E}, \check{F})\check{\phi}\check{W} - \check{\phi}\widehat{R}(\check{E}, \check{F})\check{W} - \frac{1}{2n}[\widehat{S}(\check{F}, \check{\phi}\check{W})\check{E} - \widehat{S}(\check{E}, \check{\phi}\check{W})\check{F}]$$

$$+ \frac{1}{2n}[\widehat{S}(\check{F}, \check{W})\check{\phi}\check{E} - \widehat{S}(\check{E}, \check{W})\check{\phi}\check{F}] = 0.$$

Taking $\check{F} = \xi$ in (5.5) and using (2.3), (3.5) and (3.6), we find

$$- \check{g}(\check{E}, \check{\phi}\check{W})\xi + \frac{1}{2n}\widehat{S}(\check{E}, \check{\phi}\check{W})\xi - \check{\eta}(\check{W})\check{\phi}\check{E} - \check{\eta}(\check{W})\check{\phi}^2\check{E} + \frac{(2n + \check{\psi})}{2n}\check{\eta}(\check{W})\check{\phi}\check{E} = 0.$$

Taking inner product of the above equation with ξ and making use of (2.1) and (2.3) yields $\widehat{S}(\check{E}, \check{\phi}\check{W}) = 2n\check{g}(\check{E}, \check{\phi}\check{W})$, which by setting $\check{W} = \check{\phi}\check{W}$ and using (2.2) gives

$$(5.6) \qquad \widehat{S}(\check{E}, \check{W}) = 2n\check{g}(\check{E}, \check{W}) - \check{\psi}\check{\eta}(\check{E})\check{\eta}(\check{W}).$$

Now, taking $\check{W} = \xi$ in (5.6), we find

$$(5.7) \qquad \widehat{S}(\check{E}, \xi) = (2n + \check{\psi})\check{\eta}(\check{E}).$$

Thus, from (4.4) and (5.7), we obtain

$$(5.8) \qquad \check{\lambda} = -(2n + \check{\psi}).$$

Hence, (5.6) together with (5.8) leads to the following theorem.

Theorem 5.1. *If a $(2n+1)$-dimensional LP-Kenmotsu manifold M with a connection $\widehat{\nabla}$ admitting Ricci soliton is $\check{\phi}$-projectively semisymmetric, then M is an $\eta-Einstein$ manifold and its Ricci solition will be expanding, shrinking or steady according to $\check{\psi} < -2n$, $\check{\psi} > -2n$ or $\check{\psi} = -2n$.*

Definition 5.2. An *LP*-Kenmotsu manifold with a connection $\widehat{\nabla}$ is called $\check{\phi}$-concircularly semisymmetric if $\widehat{V}(\check{E}, \check{F}) \cdot \check{\phi} = 0$ for all $\check{E}, \check{F}$ on M.

Analogous to the equation (1.3), the concircular curvature tensor with a connection $\widehat{\nabla}$ is given by

$$(5.9) \qquad \widehat{V}(\check{E}, \check{F})\check{W} = \widehat{R}(\check{E}, \check{F})\check{W} - \frac{\widehat{r}}{2n(2n+1)}[\check{g}(\check{F}, \check{W})\check{E} - \check{g}(\check{E}, \check{W})\check{F}].$$

Suppose that a $(2n + 1)$-dimensional *LP*-Kenmotsu manifold with a connection $\widehat{\nabla}$ is $\check{\phi}$-concircularly semisymmetric, therefore

$$(5.10) \qquad (\widehat{V}(\check{E}, \check{F}) \cdot \check{\phi})\check{W} = \widehat{V}(\check{E}, \check{F})\check{\phi}\check{W} - \check{\phi}\widehat{V}(\check{E}, \check{F})\check{W} = 0,$$

for all $\check{E}, \check{F}, \check{W}$ on M. From (5.9), it follows that

$$(5.11) \qquad \widehat{V}(\check{E}, \check{F})\check{\phi}\check{W} = \widehat{R}(\check{E}, \check{F})\check{\phi}\check{W} - \frac{\widehat{r}}{2n(2n+1)}[\check{g}(\check{F}, \check{\phi}\check{W})\check{E} - \check{g}(\check{E}, \check{\phi}\check{W})\check{F}],$$

$$(5.12) \qquad \check{\phi}\widehat{V}(\check{E}, \check{F})\check{W} = \check{\phi}\widehat{R}(\check{E}, \check{F})\check{W} - \frac{\widehat{r}}{2n(2n+1)}[\check{g}(\check{F}, \check{W})\check{\phi}\check{E} - \check{g}(\check{E}, \check{W})\check{\phi}\check{F}].$$

Combining (5.10), (5.11) and (5.12), we have

$$\widehat{R}(\check{E}, \check{F})\check{\phi}\check{W} - \check{\phi}\widehat{R}(\check{E}, \check{F})\check{W} - \frac{\widehat{r}}{2n(2n+1)}[\check{g}(\check{F}, \check{\phi}\check{W})\check{E} - \check{g}(\check{E}, \check{\phi}\check{W})\check{F}]$$

$$+ \frac{\widehat{r}}{2n(2n+1)}[\check{g}(\check{F}, \check{W})\check{\phi}\check{E} - \check{g}(\check{E}, \check{W})\check{\phi}\check{F}] = 0,$$

which, by taking $\check{F} = \xi$ and using (2.3), (2.4) and (3.5), takes the form

$$(5.13) \qquad \left[\frac{\widehat{r}}{2n(2n+1)} - 1\right]\check{g}(\check{E}, \check{\phi}\check{W})\xi + \left[\frac{\widehat{r}}{2n(2n+1)} - 1\right]\check{\eta}(\check{W})\check{\phi}\check{E} + \check{\eta}(\check{W})\check{\phi}^2\check{E} = 0.$$

Taking inner product of (5.13) with ξ and making use of (2.1) and (2.3), we get

$$\widehat{r} = 2n(2n + 1), \qquad \check{g}(\check{E}, \check{\phi}\check{W}) \neq 0.$$

This leads to the following theorem.

Theorem 5.2. *If a $(2n+1)$-dimensional LP-Kenmotsu manifold with a connection $\widehat{\nabla}$ is $\check{\phi}$-concirculary semisymmetric, then the scalar curvature is constant.*

Definition 5.3. An *LP*-Kenmotsu manifold with a connection $\widehat{\nabla}$ is called $\check{\phi}$-conformally semisymmetric if $\widehat{C}(\check{E}, \check{F}) \cdot \check{\phi} = 0$ for all $\check{E}, \check{F}$ on M.

Analogous to the equation (1.4), the conformal curvature tensor with a connection $\widehat{\nabla}$ is given by

(5.14)

$$
\widehat{C}(\check{E}, \check{F})\check{W} = \widehat{R}(\check{E}, \check{F})\check{W} - \frac{1}{(2n-1)}[\widehat{S}(\check{F}, \check{W})\check{E} - \widehat{S}(\check{E}, \check{W})\check{F}
$$
$$
+ \check{g}(\check{F}, \check{W})\widehat{Q}\check{E} - \check{g}(\check{E}, \check{W})\widehat{Q}\check{F}] + \frac{\widehat{r}}{2n(2n-1)}[\check{g}(\check{F}, \check{W})\check{E} - \check{g}(\check{E}, \check{W})\check{F}].
$$

Suppose that a $(2n+1)$-dimensional *LP*-Kenmotsu manifold with a connection $\widehat{\nabla}$ is $\check{\phi}$-conformally semisymmetric, therefore

(5.15) $(\widehat{C}(\check{E}, \check{F}) \cdot \check{\phi})\check{W} = \widehat{C}(\check{E}, \check{F})\check{\phi}\check{W} - \check{\phi}\widehat{C}(\check{E}, \check{F})\check{W} = 0,$

for all $\check{E}, \check{F}, \check{W}$ on M. From (5.14), it follows that

(5.16) $\widehat{C}(\check{E}, \check{F})\check{\phi}\check{W} = \widehat{R}(\check{E}, \check{F})\check{\phi}\check{W} - \dfrac{1}{(2n-1)}[\widehat{S}(\check{F}, \check{\phi}\check{W})E$
$$
- \widehat{S}(\check{E}, \check{\phi}\check{W})\check{F} + \check{g}(\check{F}, \check{\phi}\check{W})\widehat{Q}\check{E} - \check{g}(\check{E}, \check{\phi}\check{W})\widehat{Q}\check{F}]
$$
$$
+ \frac{\widehat{r}}{2n(2n-1)}[\check{g}(\check{F}, \check{\phi}\check{W})\check{E} - \check{g}(\check{E}, \check{\phi}\check{W})\check{F}],
$$

(5.17) $\check{\phi}\widehat{C}(\check{E}, \check{F})\check{W} = \check{\phi}\widehat{R}(\check{E}, \check{F})\check{W} - \dfrac{1}{(2n-1)}[\widehat{S}(\check{F}, \check{W})\check{\phi}\check{E}$
$$
- \widehat{S}(\check{E}, \check{W})\check{\phi}\check{F} + \check{g}(\check{F}, \check{W})\check{\phi}\widehat{Q}\check{E} - \check{g}(\check{E}, \check{W})\check{\phi}\widehat{Q}\check{F}]
$$
$$
+ \frac{\widehat{r}}{2n(2n-1)}[\check{g}(\check{F}, \check{W})\check{\phi}\check{E} - \check{g}(\check{E}, \check{W})\check{\phi}\check{F}].
$$

Combining (5.15), (5.16) and (5.17), we have

$$
\widehat{R}(\check{E}, \check{F})\check{\phi}\check{W} - \check{\phi}\widehat{R}(\check{E}, \check{F})\check{W} - \frac{1}{(2n-1)}[\widehat{S}(\check{F}, \check{\phi}\check{W})\check{E} - \widehat{S}(\check{E}, \check{\phi}\check{W})\check{F}
$$
$$
+ \check{g}(\check{F}, \check{\phi}\check{W})\widehat{Q}\check{E} - \check{g}(\check{E}, \check{\phi}\check{W})\widehat{Q}\check{F}] + \frac{1}{(2n-1)}[\widehat{S}(\check{F}, \check{W})\check{\phi}\check{E} - \widehat{S}(\check{E}, \check{W})\check{\phi}\check{F}
$$
$$
+ \check{g}(\check{F}, \check{W})\check{\phi}\widehat{Q}\check{E} - \check{g}(\check{E}, \check{W})\check{\phi}\widehat{Q}\check{F}] + \frac{\widehat{r}}{2n(2n-1)}[\check{g}(\check{F}, \check{\phi}\check{W})\check{E} - \check{g}(\check{E}, \check{\phi}\check{W})\check{F}]
$$
$$
- \frac{\widehat{r}}{2n(2n-1)}[\check{g}(\check{F}, \check{W})\check{\phi}\check{E} - \check{g}(\check{E}, \check{W})\check{\phi}\check{F}] = 0,
$$

which by replacing $\check{E} = \xi$ and making use of (2.3), (2.4), (3.5), (3.6) and (3.8) takes the form

$$(5.18) \quad \left[\frac{2n + \check{\psi}}{2n - 1} - \frac{\widehat{r}}{2n(2n - 1)} - 1\right](\check{g}(\check{E}, \check{\phi}\check{W})\xi + \check{\eta}(\check{W})\check{\phi}E) + \frac{1}{(2n - 1)}\widehat{S}(\check{E}, \check{\phi}\check{W})\xi$$

$$+ \frac{1}{(2n - 1)}\check{\eta}(\check{W})\check{\phi}\widehat{Q}\check{E} - \check{\eta}(\check{W})\check{\phi}^2\check{E} = 0.$$

Now, taking inner product of (5.18) with ξ and making use of (2.1) and (2.3), we obtain

$$(5.19) \quad \widehat{S}(\check{E}, \check{\phi}\check{W}) = \left[(2n - 1) + \frac{\widehat{r}}{2n} - (2n + \check{\psi})\right]\check{g}(\check{E}, \check{\phi}\check{W}).$$

By replacing $\check{W}$ by $\check{\phi}\check{W}$ in (5.19) and then using (2.2), (2.4), (3.6), we get

$$(5.20) \quad \widehat{S}(\check{E}, \check{W}) = \left[(2n - 1) + \frac{\widehat{r}}{2n} - (2n + \check{\psi})\right]\check{g}(\check{E}, \check{W})$$

$$+ \left[(2n - 1) + \frac{\widehat{r}}{2n} - 2(2n + \check{\psi})\right]\check{\eta}(\check{E})\check{\eta}(\check{W}).$$

Taking $\check{W} = \xi$ in (5.20), we find

$$(5.21) \quad \widehat{S}(\check{E}, \xi) = (2n + \check{\psi})\check{\eta}(\check{E}).$$

Thus, from (4.4) and (5.21), we obtain

$$(5.22) \quad \check{\lambda} = -(2n + \check{\psi}).$$

Hence, (5.20) together with (5.22) leads to the following theorem.

Theorem 5.3. *If a $(2n+1)$-dimensional LP-Kenmotsu manifold M with a connection $\widehat{\nabla}$ admitting Ricci soliton is $\check{\phi}$-conformally semisymmetric, then M is an $\eta-$Einstein manifold and its Ricci solition will be expanding, shrinking or steady according to $\check{\psi} < -2n$, $\check{\psi} > -2n$ or $\check{\psi} = -2n$.*

Definition 5.4. An *LP*-Kenmotsu manifold with a connection $\widehat{\nabla}$ is called $\check{\phi}$-*D*-conformally semisymmetric if $\widehat{B}(\check{E}, \check{F}) \cdot \check{\phi} = 0$ for all $\check{E}, \check{F}$ on M.

Analogous to the equation (1.5), the $D-$conformal curvature tensor with a connection $\widehat{\nabla}$ is given by

$$(5.23) \quad \widehat{B}(\check{E}, \check{F})\check{W} = \widehat{R}(\check{E}, \check{F})\check{W} + \frac{1}{2(n - 1)}[\widehat{S}(\check{E}, \check{W})\check{F} - \widehat{S}(\check{F}, \check{W})\check{E}$$

$$+ \check{g}(\check{E}, \check{W})\widehat{Q}\check{F} - \check{g}(\check{F}, \check{W})\widehat{Q}\check{E} - \widehat{S}(\check{E}, \check{W})\check{\eta}(\check{F})\xi$$

$$+ \widehat{S}(\check{F}, \check{W})\check{\eta}(\check{E})\xi - \check{\eta}(\check{E})\check{\eta}(\check{W})\widehat{Q}\check{F} + \check{\eta}(\check{F})\check{\eta}(\check{W})\widehat{Q}\check{E}]$$

$$- \frac{\widehat{k} - 2}{2(n - 1)}[\check{g}(\check{E}, \check{W})\check{F} - \check{g}(\check{F}, \check{W})\check{E}] + \frac{\widehat{k}}{2(n - 1)}[\check{g}(\check{E}, \check{W})\check{\eta}(\check{F})\xi$$

$$- \check{g}(\check{F}, \check{W})\check{\eta}(\check{E})\xi + \check{\eta}(\check{E})\check{\eta}(\check{W})\check{F} - \check{\eta}(\check{F})\check{\eta}(\check{W})\check{E}],$$

where $\widehat{k} = \frac{\widehat{r}+4n}{(2n-1)}$.

Suppose that a $(2n+1)$-dimensional LP-Kenmotsu manifold with a connection $\widehat{\nabla}$ is $\check{\phi}$-D-conformally semisymmetric, therefore

$$(5.24) \qquad (\widehat{B}(\check{E}, \check{F}) \cdot \check{\phi})\check{W} = \widehat{B}(\check{E}, \check{F})\check{\phi}\check{W} - \check{\phi}\widehat{B}(\check{E}, \check{F})\check{W} = 0,$$

for all $\check{E}, \check{F}, \check{W}$ on M. From (5.23), it follows that

(5.25)

$$\begin{aligned}
\widehat{B}(\check{E}, \check{F})\check{\phi}\check{W} = &\widehat{R}(\check{E}, \check{F})\check{\phi}\check{W} + \frac{1}{2(n-1)}[\widehat{S}(\check{E}, \check{\phi}\check{W})\check{F} - \widehat{S}(\check{F}, \check{\phi}\check{W})\check{E} + \check{g}(\check{E}, \check{\phi}\check{W})\widehat{Q}\check{F} \\
&- \check{g}(\check{F}, \check{\phi}\check{W})\widehat{Q}\check{E} - \widehat{S}(\check{E}, \check{\phi}\check{W})\check{\eta}(\check{F})\xi + \widehat{S}(\check{F}, \check{\phi}\check{W})\check{\eta}(\check{E})\xi] \\
&- \frac{\widehat{k}-2}{2(n-1)}[\check{g}(\check{E}, \check{\phi}\check{W})\check{F} - \check{g}(\check{F}, \check{\phi}\check{W})\check{E}] + \frac{\widehat{k}}{2(n-1)}[\check{g}(\check{E}, \check{\phi}\check{W})\check{\eta}(\check{F})\xi \\
&- \check{g}(\check{F}, \check{\phi}\check{W})\check{\eta}(\check{E})\xi],
\end{aligned}$$

(5.26)

$$\begin{aligned}
\check{\phi}\widehat{B}(\check{E}, \check{F})W = &\check{\phi}\widehat{R}(\check{E}, \check{F})\check{W} + \frac{1}{2(n-1)}[\widehat{S}(\check{E}, \check{W})\check{\phi}\check{F} - \widehat{S}(\check{F}, \check{W})\check{\phi}\check{E} \\
&+ \check{g}(\check{E}, \check{W})\check{\phi}\widehat{Q}\check{F} - \check{g}(\check{F}, \check{W})\check{\phi}\widehat{Q}\check{E} - \check{\eta}(\check{E})\check{\eta}(\check{W})\check{\phi}\widehat{Q}\check{F} \\
&+ \check{\eta}(\check{F})\check{\eta}(\check{W})\check{\phi}\widehat{Q}\check{E}] - \frac{\widehat{k}-2}{2(n-1)}[\check{g}(\check{E}, \check{W})\check{\phi}\check{F} - \check{g}(\check{F}, \check{W})\check{\phi}\check{E}] \\
&+ \frac{\widehat{k}}{2(n-1)}[\check{\eta}(\check{E})\check{\eta}(\check{W})\check{\phi}\check{F} - \check{\eta}(\check{F})\check{\eta}(\check{W})\check{\phi}\check{E}].
\end{aligned}$$

Combining (5.24), (5.25) and (5.26), we have

$$\begin{aligned}
(5.27) \quad &\widehat{R}(\check{E}, \check{F})\check{\phi}\check{W} - \check{\phi}\widehat{R}(\check{E}, \check{F})\check{W} + \frac{1}{2(n-1)}[\widehat{S}(\check{E}, \check{\phi}\check{W})\check{F} - \widehat{S}(\check{F}, \check{\phi}\check{W})\check{E} \\
&+ \check{g}(\check{E}, \check{\phi}\check{W})\widehat{Q}\check{F} - \check{g}(\check{F}, \check{\phi}\check{W})\widehat{Q}\check{E} - \widehat{S}(\check{E}, \check{\phi}\check{W})\check{\eta}(\check{F})\xi + \widehat{S}(\check{F}, \check{\phi}\check{W})\check{\eta}(\check{E})\xi] \\
&- \frac{1}{2(n-1)}[\widehat{S}(\check{E}, \check{W})\check{\phi}\check{F} - \widehat{S}(\check{F}, \check{W})\check{\phi}\check{E} + \check{g}(\check{E}, \check{W})\check{\phi}\widehat{Q}\check{F} - \check{g}(\check{F}, \check{W})\check{\phi}\widehat{Q}\check{E} \\
&- \check{\eta}(\check{E})\check{\eta}(\check{W})\check{\phi}\widehat{Q}\check{F} + \check{\eta}(\check{F})\check{\eta}(\check{W})\check{\phi}\widehat{Q}\check{E}] - \frac{\widehat{k}-2}{2(n-1)}[\check{g}(\check{E}, \check{\phi}\check{W})\check{F} - \check{g}(\check{F}, \check{\phi}\check{W})\check{E}] \\
&+ \frac{\widehat{k}-2}{2(n-1)}[\check{g}(\check{E}, \check{W})\check{\phi}\check{F} - \check{g}(\check{F}, \check{W})\check{\phi}\check{E}] + \frac{\widehat{k}}{2(n-1)}[\check{g}(\check{E}, \check{\phi}\check{W})\check{\eta}(\check{F}) \\
&- \check{g}(\check{F}, \check{\phi}\check{W})\check{\eta}(\check{E})]\xi - \frac{\widehat{k}}{2(n-1)}[\check{\eta}(\check{E})\check{\eta}(\check{W})\check{\phi}\check{F} - \check{\eta}(\check{F})\check{\eta}(\check{W})\check{\phi}\check{E}] = 0.
\end{aligned}$$

By taking $\check{F} = \xi$ in (5.27) and then using (2.1), (2.3), (3.5), (3.6) and (3.8) takes the form

(5.28)
$$\frac{4 + \check{\psi} - 2\widehat{k}}{2(n-1)}[\check{g}(\check{E}, \check{\phi}\check{W})\xi + \check{\eta}(\check{W})\check{\phi}\check{E}] + \frac{1}{n-1}\widehat{S}(\check{E}, \check{\phi}\check{W})\xi - \check{\eta}(\check{W})\check{\phi}^2\check{E}$$
$$+ \frac{1}{n-1}\check{\eta}(\check{W})\check{\phi}\widehat{Q}\check{E} = 0.$$

Inner product of (5.28) with ξ and making use of (2.1) and (2.3) gives

(5.29)
$$\widehat{S}(\check{E}, \check{\phi}\check{W}) = \left[\frac{\check{\psi}}{2} + 2 - \widehat{k}\right]\check{g}(\check{E}, \check{\phi}\check{W}).$$

Now, we replace $\check{W}$ by $\check{\phi}\check{W}$ in (5.29) and using (2.2), (2.4) and (3.6), we get

(5.30)
$$\widehat{S}(\check{E}, \check{W}) = \left[\frac{\check{\psi}}{2} + 2 - \widehat{k}\right]\check{g}(\check{E}, \check{W}) - \left[\frac{\check{\psi}}{2} + \widehat{k} + 2n - 2\right]\check{\eta}(\check{E})\check{\eta}(\check{W}).$$

Taking $\check{W} = \xi$ in (5.30), we find

(5.31)
$$\widehat{S}(\check{E}, \xi) = (2n + \check{\psi})\check{\eta}(\check{E}).$$

Thus, from (4.4) and (5.31), we obtain

(5.32)
$$\check{\lambda} = -(2n + \check{\psi}).$$

Hence, (5.30) together with (5.32) leads to the following theorem.

Theorem 5.4. *If a $(2n+1)$-dimensional LP-Kenmotsu manifold M with a connection $\widehat{\nabla}$ admitting Ricci soliton is $\check{\phi}$-D-conformally semisymmetric, then M is an $\eta-$Einstein manifold and its Ricci solition will be expanding, shrinking or steady according to $\check{\psi} < -2n$, $\check{\psi} > -2n$ or $\check{\psi} = -2n$.*

Example 5.1. Let on a 3-dimensional manifold $M = \{(\check{w}_1, \check{w}_2, \check{w}_3) \in R^3 : w > 0\}$, where $(\check{w}_1, \check{w}_2, \check{w}_3)$ are the standard coordinates of R^3, the linearly independent vector fields that at each point of M are given by

$$v^1 = \frac{\check{w}_3\partial}{\partial\check{w}_1}, \quad v^2 = \frac{w\partial}{\partial\check{w}_2}, \quad v^3 = \frac{w\partial}{\partial\check{w}_3} = \xi.$$

Suppose the Lorentzian metric $\check{g}$ is defined by

$$\check{g}(v^1, v^1) = \check{g}(v^2, v^2) = 1, \quad \check{g}(v^3, v^3) = -1, \quad \check{g}(v^1, v^2) = \check{g}(v^2, v^3) = \check{g}(v^1, v^3) = 0.$$

Suppose the 1-form $\check{\eta}$ is defined by $\check{\eta}(\check{E}) = \check{g}(\check{E}, v^3) = \check{g}(\check{E}, \xi)$ for all $\check{E}$ on M, and the $(1,1)$-tensor field $\check{\phi}$ is defined by

$$\check{\phi}v^1 = -v^1, \quad \check{\phi}v^2 = -v^2, \quad \check{\phi}v^3 = 0.$$

Then, using the linearity of $\check{g}$ and $\check{\phi}$, we have

$$\check{\eta}(\xi) = -1, \quad \check{\phi}^2\check{E} = \check{E} + \check{\eta}(\check{E})\xi, \quad \check{g}(\check{\phi}\check{E}, \check{\phi}\check{F}) = \check{g}(\check{E}, \check{F}) + \check{\eta}(\check{E})\check{\eta}(\check{F}),$$

for all $\check{E}, \check{F}$ on M. Thus, $(\check{\phi}, \xi, \check{\eta}, \check{g})$ defines a Lorentzian almost paracontact metric structure on M. Also, we have

$$(5.33) \qquad [v^1, v^2] = 0, \quad [v^1, v^3] = -v^1, \quad [v^2, v^3] = -v^2.$$

From the Koszul's formula for $\check{g}$, we calculate

$$(5.34) \qquad \check{\nabla}_{v^1} v^1 = -v^3, \quad \check{\nabla}_{v^1} v^2 = 0, \quad \check{\nabla}_{v^1} v^3 = -v^1, \quad \check{\nabla}_{v^2} v^1 = 0,$$
$$\check{\nabla}_{v^2} v^2 = -v^3, \quad \check{\nabla}_{v^2} v^3 = -v^2, \quad \check{\nabla}_{v^3} v^1 = 0, \quad \check{\nabla}_{v^3} v^2 = 0, \quad \check{\nabla}_{v^3} v^3 = 0.$$

Also, one can easily verify that

$$\check{\nabla}_{\check{E}} \xi = -\check{E} - \check{\eta}(\check{E})\xi \text{ and } (\check{\nabla}_{\check{E}} \check{\phi})\check{F} = -\check{g}(\check{\phi}\check{E}, \check{F})\xi - \check{\eta}(\check{F})\check{\phi}\check{E}.$$

Therefore, M is an LP-Kenmotsu manifold. From (1.1), (5.33) and (5.34), we obtain

$$(5.35) \qquad \check{R}(v^1, v^2)v^1 = -v^2, \quad \check{R}(v^2, v^3)v^1 = 0, \quad \check{R}(v^1, v^3)v^1 = -v^3,$$
$$\check{R}(v^1, v^2)v^2 = v^1, \quad \check{R}(v^1, v^3)v^2 = 0, \quad \check{R}(v^2, v^3)v^2 = -v^3,$$
$$\check{R}(v^1, v^2)v^3 = 0, \quad \check{R}(v^1, v^3)v^3 = -v^1, \quad \check{R}(v^2, v^3)v^3 = -v^2,$$

from which it is clear that $\check{R}(\check{E}, \check{F})\check{W} = \check{g}(\check{F}, \check{W})\check{E} - \check{g}(\check{E}, \check{W})\check{F}$. Hence, $(M, \check{\phi}, \xi, \check{\eta}, \check{g})$ is an LP-Kenmotsu manifold of unit constant curvature. By virtue of (1.8) and (5.35), we obtain

$$\widehat{\nabla}_{v^1} v^1 = -v^3, \quad \widehat{\nabla}_{v^2} v^1 = 0, \quad \widehat{\nabla}_{v^3} v^1 = 0, \quad \widehat{\nabla}_{v^1} v^2 = 0, \quad \widehat{\nabla}_{v^2} v^2 = -v^3,$$
$$\widehat{\nabla}_{v^3} v^2 = 0, \quad \widehat{\nabla}_{v^1} v^3 = 0, \quad \widehat{\nabla}_{v^2} v^3 = 0, \quad \widehat{\nabla}_{v^3} v^3 = 0.$$

From (3.2) and (5.35), we can easily obtain

$$(5.36) \qquad \widehat{R}(v^1, v^2)v^1 = 0, \quad \widehat{R}(v^1, v^3)v^1 = -v^3, \quad \widehat{R}(v^2, v^3)v^1 = 0,$$
$$\widehat{R}(v^1, v^2)v^2 = 0, \quad \widehat{R}(v^1, v^3)v^2 = 0, \quad \widehat{R}(v^2, v^3)v^2 = -v^3,$$
$$\widehat{R}(v^1, v^2)v^3 = 0, \quad \widehat{R}(v^1, v^3)v^3 = 0, \quad \widehat{R}(v^2, v^3)v^3 = 0.$$

From (5.35) and (5.36), we calculate the Ricci tensors as follows:

$$\check{S}(v^1, v^1) = \check{S}(v^2, v^2) = 2, \quad \check{S}(v^3, v^3) = -2,$$

and

$$\widehat{S}(v^1, v^1) = \widehat{S}(v^2, v^2) = 1, \quad \widehat{S}(v^3, v^3) = 0.$$

Therefore, we find $\check{r} = 6$ and $\hat{r} = 2$, where $\check{\psi} = -2$. Hence, (3.4) is satisfied. From (2.5), (1.6) and (1.7), we find

$$\check{\Phi}(v^1, v^1) = \check{\Phi}(v^2, v^2) = -1, \quad \check{\Phi}(v^3, v^3) = 0,$$
$$\check{T}(v^i, v^j) = 0, \quad i = j = 1, 2, 3,$$
$$\check{T}(v^1, v^2) = 0, \quad \check{T}(v^1, v^3) = v^1, \quad \check{T}(v^2, v^3) = v^2,$$
$$(\widehat{\nabla}_{v^1} \check{g})(v^1, v^3) = (\widehat{\nabla}_{v^2} \check{g})(v^2, v^3) = -1, \quad (\widehat{\nabla}_{v^3} \check{g})(v^1, v^2) = 0,$$

respectively. Thus, the connection $\widehat{\nabla}$ defined on M is a QSNM. Now, by putting $F = W = v^i$ in (4.3) and summing up, we find $2 = 3(1 - \check{\lambda}) - 1$ implies $\check{\lambda} = 0$. Thus, a Ricci soliton on an *LP*-Kenmotsu manifold with a connection $\widehat{\nabla}$ is steady for $\check{\psi} = -2n = -2$.

References

[1] A. Haseeb and R. Prasad, *Certain results on Lorentzian para-Kenmotsu manifolds*, Bol. Soc. Parana. Mat. DOI 10.5269/bspm.40607.

[2] A. Prakash and D. Narain, *On a quarter-symmetric non-metric connection in a Lorentzian para-Sasakian manifold*, Int. Electron. J. Geom. **4** (2011), 129–137.

[3] A. K. Mondal and U. C. De, *Quarter-symmetric non-metric connection on P-Sasakian manifolds*, ISRN Geometry (2012), Article ID 659430, 14 pages.

[4] D. E. Blair, *Contact Manifolds in Riemannian Geometry*, Lecture Notes in Mathematics **509**, Springer-Verlag, Berlin, New York, 1976.

[5] G. Chuman, *On D-conformal curvature tensor*, Tensor (N. S.) **46** (1983), 125–129.

[6] K. Yano and M. Kon, *Structures on Manifolds*, Series in Pure Math. **3**, World Science, Singapore, 1984.

[7] M. Ahmad, C. Özgür and A. Haseeb, *Hypersurfaces of an almost r-paracontact Riemannian manifold endowed with a quarter symmetric non-metric connection*, Kyungpook Math. J. **49** (2009), 533–543.

[8] P. Alegre, *Slant submanifolds of Lorentzian Sasakian and Para Sasakian manifolds*, Taiwanese J. Math. **17** (2013), 897–910.

[9] R. S. Hamilton, *The Ricci flow on surfaces*, in: *Mathematics and General Relativity*, Contemp. Math. **71**, American Math. Soc., Providence, 1988, 237–262.

[10] R. S. Hamilton, *Three-manifolds with positive Ricci curvature*, J. Differential Geom. **17** (1982), 255–306.

[11] S. Golab, *On semi-symmetric and quarter-symmetric linear connections*, Tensor (N. S.) **29** (1975), 249–254.

[12] U. C. De and A. K. Mondal, *Hypersurfaces of Kenmotsu manifolds endowed with a quarter symmetric non-metric connection*, Kuwait J. Sci. **39** (2012), 43–56.

[13] U. C. De and P. Majhi, *φ-semisymmetric generalized Sasakian space-forms*, Arab J. Math. Sci. **21** (2015), 170–178.

[1]DEPARTMENT OF MATHEMATICS AND ASTRONOMY,
UNIVERSITY OF LUCKNOW,
LUCKNOW-226007, INDIA.
Email address: rp.manpur@rediffmail.com,
Email address: ugautamarya@gmail.com[3]

[2]DEPARTMENT OF MATHEMATICS,
FACULTY OF SCIENCE, JAZAN UNIVERSITY,
JAZAN-2097, KINGDOM OF SAUDI ARABIA.
Email address: malikhaseeb80@gmail.com, haseeb@jazanu.edu.sa