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(m, n)-HYPERFILTERS IN ORDERED SEMIHYPERGROUPS

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Abstract. First, we generalize concepts of left hyperfilters, right hyperfilters and hyperfilters of an ordered semihypergroup by introducing concepts of left-m-hyperfilters, right-n-hyperfilters and (m, n)-hyperfilters of an ordered semihypergroup. Then, some properties of these generalized hyperfilters have been studied. Finally, left-m-hyperfilters (resp. right-n-hyperfilters, (m, n)-hyperfilters) of $(m, 0)$-regular (resp. $(0, n)$-regular, (m, n)-regular) ordered semihypergroups characterize in terms of their completely prime generalized $(m, 0)$-hyperideals (resp. $(0, n)$-hyperideals, (m, n)-hyperideals).

1. Introduction and Preliminaries

In 1934, Marty [12] introduced the concept of hyperstructure and defined the notion of hypergroup. The beauty of hyperstructure is that in hyperstructures multiplication of two elements is a set while in classical algebraic structures, the multiplication of two elements is an element which is the main reason for the researcher to attract towards such type of algebraic structures. Thus, the notion of algebraic hyperstructures is a generalization of classical notion of algebraic structures. The concept of ordered semihypergroup is a generalization of the concept of ordered semigroup and was introduced by Heidari and Davvaz in [6]. Thereafter it was studied by several authors. Davvaz et al. [1, 2, 6, 13] studied some properties of hyperideals, bi-hyperideals and quasi-hyperideals in ordered semihypergroups. The notion of (m, n)-ideals of semigroups was introduced by Lajos [10] as a generalization of the notion of bi-ideals in semigroups. In [9], authors introduced the notion of an (m, n)-quasi-hyperideal

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and proved different characterizations of (m,n)-quasi-hyperideals and minimal (m,n)-quasi-hyperideals in semihypergroups.

In 1987, Kehayopulu [7] introduced the concept of filter on *poe*-semigroups. Later on in 1990, Kehayopulu [8] defined the relation $\mathcal{N}$ on *po*-semigroup. The study of left(right)-filter on *po*-semigroup initiated by S. K. Lee and S. S. Lee [11], and gave some characterizations of the left(right)-filter of *po*-semigroup in term of the right(left) prime ideals. In 2015, the notion of left hyperfilters, right hyperfilters and hyperfilters of ordered semihypergroups introduced by Tang et al. [14] and also investigated their related properties and characterized hyperfilters in terms of completely prime hyperideals in ordered semihypergroups. In 2016, Omidi and Davvaz [13] defined an equivalence relation $\mathcal{N}$ as follows. Let H be an ordered semihypergroup. Then, $\mathcal{N} = \{(a,b) \in H \times H \mid N(a) = N(b)\}$, where $N(a)$ denote the hyperfiter of H generated by an element a of H, and also shown that $\mathcal{N}$ is the intersection of the semilattice equivalence relation $\sigma_P = \{(a,b) \in H \times H \mid a,b \in P \text{ or } a,b \notin P\}$, where P is completely prime hyperideal of H. Recently, Gu and Tang [4] constructed a strongly ordered regular equivalence relation on an ordered semihypergroup by using the concept of hyperfilter and shown that the corresponding quotient structure is a semilattice.

A hyperoperation on a non-empty set H is a map $\circ : H \times H \to \mathcal{P}^*(H)$, where $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\emptyset\}$ (the set of all non-empty subsets of H). In such case, the H is called a hypergroupoid. Let H be a hypergroupoid, A and B be any non-empty subsets of H. Then

$$A \circ B = \bigcup_{a\in A, b\in B} a \circ b.$$

We shall write, in whatever follows, $A \circ x$ instead of $A \circ \{x\}$ and $x \circ A$ instead of $\{x\} \circ A$ for any $x \in H$. Also, for simplicity, throughout the paper, we denote $a \circ a \circ \cdots \circ a$ (m-copies of a) with a^m for all $a \in H$ and $m \in \mathbb{Z}$. Moreover, the hypergroupoid H is called a semihypergroup if, for all $x,y,z \in H$,

$$(x \circ y) \circ z = x \circ (y \circ z),$$

i.e.,

$$\bigcup_{u\in x\circ y} u \circ z = \bigcup_{v\in y\circ z} x \circ v.$$

A non-empty subset T of semihypergroup H is called a subsemihypergroup of H if $T \circ T \subseteq T$.

Definition 1.1 ([14])**.** Let H be a non-empty set. The triplet $(H, \circ, \leq)$ is called an ordered semihypergroup if $(H, \circ)$ is a semihypergroup and $(H, \leq)$ is a partially ordered set such that

$$x \leq y \Rightarrow x \circ z \leq y \circ z \quad \text{and} \quad z \circ x \leq z \circ y,$$

for all $x, y, z \in H$. Here, if A and B are non-empty subsets of H, then we say that $A \leq B$ if for every $a \in A$ there exists $b \in B$ such that $a \leq b$.

Let H be an ordered semihypergroup. For a non-empty subset T of H, we denote $(T] = \{x \in H \mid x \leq a \text{ for some } a \in T\}$.

Definition 1.2. Let H be an ordered semihypergroup and A be a non-empty subset of H. Then, A is called a left (resp. right) hyperideal [2] of H if

(1) $H \circ A \subseteq A$ (resp. $A \circ H \subseteq A$);
(2) $(A] \subseteq A$.

A is called hyperideal of H if A is both left hyperideal and right hyperideal of H.

A subsemihypergroup F of ordered semihypergroup H is called left hyperflter (resp. right hyperfilter) [14] if for any $a, b \in H$, $a \circ b \cap F \neq \emptyset$ implies $a \in F$ (resp. $b \in F$) and for any $a \in F, b \in H$ such that $a \leq b$ implies $b \in F$. If F is both left-hyperflter and right-hyperflter of H, then F is said to be hyperfilter of H.

An ordered semihypergroup H is called regular (left regular, right regular) [2] if for each $x \in H$, $x \in (x \circ H \circ x]$ ($x \in (H \circ x \circ x]$, $x \in (x \circ x \circ H]$).

Lemma 1.1 ([2]). *Let H be an ordered semihypergroup and A, B be any non-empty subsets of H. Then the following hold:*

(1) $A \subseteq (A]$*;*
(2) $A \subseteq B$ $(A] \subseteq (B]$*;*
(3) $(A] \circ (B] \subseteq (A \circ B]$*;*
(4) $((A] \circ (B]] = (A \circ B]$*;*
(5) $(A] \cup (B] = (A \cup B]$*.*

Throughout this paper, H always denotes an ordered semihypergroup and m, n denote positive integers, unless otherwise specified.

2. Main Results

Definition 2.1. A subsemihypergroup F of ordered semihypergroup H is called left-m-hyperfilter (resp. right-n-hyperfilter) if

(1) for any $a, b \in S$, $a \circ b \cap F \neq \emptyset$ implies $a^m \subseteq F$ (resp. $b^n \subseteq F$);
(2) $a \in F$, $a \leq b \in S$ implies $b \in F$.

If F is both left-m-hyperfilter and right-n-hyperfilter of H, then F is called (m, n)-hyperfilter.

Remark 2.1. In particular for $m = 1$ (resp. $n = 1$), F is a left hyperfilter (resp. right hyperfilter). Clearly, each left hyperfilter (resp. right hyperfilter, hyperfilter) of an ordered semihypergroup H is left-m-hyperfilter (resp. right-n-hyperfilter, (m, n)-hyperfilter) for each positive integers m and n. Indeed let F be any hyperfilter of H and $a, b \in H$ such that $a \circ b \cap F \neq \emptyset$. As F is left hyperfilter, $a \in F$. Since F is left hyperfilter, F is subsemihypergroup, and thus $a^m \subseteq F$. Therefore, the concept of a left-m-hyperfilter (resp. right-n-hyperfilter, (m, n)-hyperfilter) is the generalization of a left hyperfilter (resp. right hyperfilter, hyperfilter). Conversely, each left-m-hyperfilter (resp. right-n-hyperfilter, (m,n)-hyperfilter) need not be a left hyperfilter (resp. right hyperfilter, hyperfilter).

Example 2.1. Let $H = \{a,b,c,d\}$. Define hyperoperation $\circ$ and order $\leq$ on H as follows:

$\circ$	a	b	c	d
a	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$
b	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$
c	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{b\}$
d	$\{a,b\}$	$\{a,b\}$	$\{b\}$	$\{c\}$

$$\leq := \{(a,a),(b,b),(c,c),(d,d),(a,b)\}.$$

Then H is an ordered semihypergroup. Let $F = \{a,b\}$. Since $d \circ c \cap F \neq \emptyset$ but $d \notin F$ while $d^3 \subseteq F$. Therefore, F is a left-3-hyperfilter of H but not a left hyperfilter of H.

Lemma 2.1. *Let H be an ordered semihypergroup and T be a subsemihypergroup of H. Then, for every left-m-hyperfilter (resp. right-n-hyperfilter) F of H, either $F \cap T = \emptyset$ or $F \cap T$ is a left-m-hyperfilter (resp. right-n-hyperfilter) of T.*

Proof. Let $F \cap T \neq \emptyset$ and $x, y \in F \cap T$. Then, $x, y \in F, T$. As F and T are left-m-hyperfilter and subsemihypergroup of H, respectively. So $x \circ y \subseteq F$ and $x \circ y \subseteq T$. Thus, $x \circ y \subseteq F \cap T$. Next, we assume that for any $x, y \in T$, $x \circ y \cap (F \cap T) \neq \emptyset$. Therefore, $x \circ y \cap F \neq \emptyset$. Since $x, y \in H$ and F is left-m-hyperfilter of H, $x^m \subseteq F$. Also $x^m \subseteq T$. Thus, $x^m \subseteq (F \cap T)$. Finally, take an element $x \in T \cap F$ and $y \in T$ such that $x \leq y$. As F is left-m-hyperfilter of H and $F \ni x \leq y \in H$, $y \in F$. Therefore, $y \in T \cap F$. Hence, $F \cap T$ is a left-m-hyperfilter of T. ☐

Corollary 2.1. *Let H be an ordered semihypergroup and T be a subsemihypergroup of H. Then for every (m,n)-hyperfilter F of H, either $F \cap T = \emptyset$ or $F \cap T$ is an (m,n)-hyperfilter of T.*

Lemma 2.2. *Let H be an ordered semihypergroup and $\{F_i \mid i \in I\}$ be a family of left-m-hyperfilters (resp. right-n-hyperfilters) of H. If $\bigcap_{i\in I} F_i \neq \emptyset$, then $\bigcap_{i\in I} F_i$ is a left-m-hyperfilter (resp. right-n-hyperfilter) of H.*

Proof. Assume that $\bigcap_{i\in I} F_i \neq \emptyset$ and $x, y \in \bigcap_{i\in I} F_i$. Then $x, y \in F_i$ for each $i \in I$. As for each $i \in I$, F_i is left-m-hyperfilter, $x \circ y \subseteq F_i$. Therefore, $x \circ y \subseteq \bigcap_{i\in I} F_i$. Thus, $\bigcap_{i\in I} F_i$ is a subsemihypergroup of H. Now, let $x, y \in H$ and $x \circ y \subseteq \bigcap_{i\in I} F_i$. Therefore, $x \circ y \subseteq F_i$ for each $i \in I$. As F_i's are left-m-hyperfilters, $x^m \subseteq F_i$ for each $i \in I$. So, $x^m \subseteq \bigcap_{i\in I} F_i$. Now take an element $a \in \bigcap_{i\in I} F_i$ and $b \in H$ such that $a \leq b$. Then $a \in F_i$ for each $i \in I$. Since F_i's are left-m-hyperfilters, $b \in \bigcap_{i\in I} F_i$. Hence, $\bigcap_{i\in I} F_i$ is a left-m-hyperfilter. ☐

Corollary 2.2. *Let H be an ordered semihypergroup and $\{F_i \mid i \in I\}$ be a family of (m,n)-hyperfilters of H. If $\bigcap_{i\in I} F_i \neq \emptyset$, then $\bigcap_{i\in I} F_i$ is an (m,n)-hyperfilter of H.*

Remark 2.2. Union of any family of left-m-hyperfilters (resp. right-n-hyperfilters, (m,n)-hyperfilters) of ordered semihypergroup H is not a left-m-hyperfilter (resp. right-n-hyperfilter, (m,n)-hyperfilter) in general.

Following example shows that in general union of any family of left-m-hyperfilters (resp. right-n-hyperfilters, (m,n)-hyperfilters) of ordered semihypergroup H is not a left-m-hyperfilter (resp. right-n-hyperfilter, (m,n)-hyperfilter).

Example 2.2. Let $H = \{a,b,c,d,e\}$. Define hyperoperation $\circ$ and an order $\leq$ on H as follows:

$$\begin{array}{c|ccccc} \circ & a & b & c & d & e \\ \hline a & \{b\} & \{b\} & \{d\} & \{d\} & \{d\} \\ b & \{b\} & \{b\} & \{d\} & \{d\} & \{d\} \\ c & \{d\} & \{d\} & \{c,e\} & \{d\} & \{c,e\} \\ d & \{d\} & \{d\} & \{d\} & \{d\} & \{d\} \\ e & \{d\} & \{d\} & \{c,e\} & \{d\} & \{c,e\} \end{array},$$

$$\leq := \{(a,a),(b,b),(c,c),(d,d),(a,b),(c,e)\}.$$

Then H is an ordered semihypergroup. Here $F_1 = \{b\}$ is left-2-hyperfilter because $F_1 \circ F_1 \subseteq F_1$ and $a \circ a \cap F_1 \neq \emptyset$ implies $a^2 \subseteq F_1$. Thus F_1 is left-2-hyperfilter but not a hyperfilter. Similarly, $F_2 = \{c,e\}$ is left-2-hyperfilter. Now $F_1 \cup F_2 = \{b,c,e\}$, since $b \circ c = \{d\} \nsubseteq F_1 \cup F_2$, therefore $F_1 \cup F_2$ is not a subsemihypergroup of H, and hence $F_1 \cup F_2$ is not a left-2-hyperfilter.

Let $(H, \bullet, \leq_H)$ and $(T, \circ, \leq_T)$ be two ordered semihypergroups. Under the coordinatewise multiplication

$$(h_1,t_1) \diamond (h_2,t_2) = h_1 \bullet h_2 \times t_1 \circ t_2,$$

where $(s_1,t_1),(s_2,t_2) \in H \times T$ the cartesian product $H \times T$ of H and T forms a semihypergroup. Define a partial order $\leq$ on $H \times T$ by $(h_1,t_1) \leq (h_2,t_2)$ if and only if $h_1 \leq_H h_2$ and $t_1 \leq_T t_2$, where $(h_1,t_1),(h_2,t_2) \in H \times T$. Then, $(H \times T, \diamond, \leq)$ is an ordered semihypergroup [4].

Lemma 2.3. *Let $(H, \bullet, \leq_H)$ and $(T, \circ, \leq_T)$ be two ordered semihypergroups, F_1 and F_2 be two left-m-hyperfilter (resp. right-n-hyperfilter) of H and T, respectively. Then $F_1 \times F_2$ is a left-m-hyperfilter (resp. right-n-hyperfilter) of $H \times T$.*

Proof. Let $(a,b),(c,d) \in F_1 \times F_2$. Now $(a,b) \diamond (c,d) = a \bullet c \times b \circ d$. As $a,c \in F_1$, $b,d \in F_2$ and F_1, F_2 are left-m-hyperfilters of H and T respectively, $a \bullet c \subseteq F_1, b \circ d \subseteq F_2$. Therefore, $a \bullet c \times b \circ d \subseteq F_1 \times F_2$, it follows that $F_1 \times F_2$ is a subsemihypergroup of $H_1 \times H_2$. Next, we assume that $(a,b),(c,d) \in H_1 \times H_2$ such that $(a,b) \diamond (c,d) \cap F_1 \times F_2 \neq \emptyset$.

Now, we have

$$\begin{aligned}
&(a,b)\diamond(c,d)\cap F_1\times F_2\neq\emptyset\\
\Rightarrow& a\bullet c\times b\circ d\cap F_1\times F_2\neq\emptyset\\
\Rightarrow& a\bullet c\cap F_1\neq\emptyset \text{ and } b\circ d\cap F_2\neq\emptyset\\
\Rightarrow& a^m\subseteq F_1 \text{ and } b^m\subseteq F_2\\
\Rightarrow&(a^m,b^m)\subseteq F_1\times F_2\\
\Rightarrow&(a,b)^m\subseteq F_1\times F_2.
\end{aligned}$$

Finally, we consider an element $(a,b)\in F_1\times F_2$ and $(c,d)\in H\times T$ such that $(a,b)\leq(c,d)$. Therefore, $a\leq_H c$ and $b\leq_T d$. Since F_1 and F_2 are left-m-hyperfilters of H and T, $c\in F_1$ and $d\in F_2$. Thus, $(c,d)\in F_1\times F_2$. Hence, $F_1\times F_2$ is a left-m-hyperfilter of $H\times T$. $\square$

Corollary 2.3. *Let $(H,\bullet,\leq_H)$ and $(T,\circ,\leq_T)$ be two ordered semihypergroups, F_1 and F_2 be two (m,n)-hyperfilters of H and T, respectively. Then $F_1\times F_2$ is an (m,n)-hyperfilter of $H\times T$.*

Definition 2.2 ([9])**.** Let H be an ordered semihypergroup, m and n be the positive integers. Then, a subsemihypergroup A of H is called an (m,n)-hyperideal of H if

(1) $A^m\circ H\circ A^n\subseteq A$ and
(2) $(A]\subseteq A$.

Dually, we may define $(m,0)$-hyperideal and $(0,n)$-hyperideal of H.

If we drop the subsemihypergroup condition from the above definition, then A is called a generalized (m,n)-hyperideal of H. Similarly, a generalized $(m,0)$-hyperideal and a generalized $(0,n)$-hyperideal are defined.

Remark 2.3. It is easy to check that each (m,n)-hyperideal (resp. $(m,0)$-hyperideal, $(0,n)$-hyperideal) of any ordered semihypergroup is always a generalized (m,n)-hyperideal (resp. $(m,0)$-hyerideal, $(0,n)$-hyperideal), but the converse is not true in general. This has been shown by the following example.

Example 2.3. Let $H=\{a,b,c,d\}$. Define hyperoperation $\circ$ and order $\leq$ on H as follows:

$\circ$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
c	$\{a\}$	$\{a\}$	$\{a\}$	$\{a,b\}$
d	$\{a\}$	$\{a\}$	$\{a,b\}$	$\{a,b,c\}$

,

$$\leq:=\{(a,a),(b,b),(c,c),(d,d),(a,b)\}.$$

Then H is an ordered semihypergroup. The subset $\{a,d\}$ of H is a generalized (m,n)-hyperideal of H, for all integers $m,n\geq 2$, which is not an (m,n)-hyperideal of H.

A generalized $(m,0)$-hyperideal (resp. generalized $(0,n)$-hyperideal, generalized (m,n)-hyperideal) A of an ordered semihypergroup H is called completely prime if for any two elements $a,b \in H$ such that $a \circ b \cap A \neq \emptyset$, then $a \in A$ or $b \in A$.

Let H be an ordered semihypergroup and m,n positive integers. Then H is called an (m,n)-regular (resp. $(m,0)$-regular, $(0,n)$-regular) if for any $a \in H$ there exists $x \in H$ such that $a \leq a^m \circ x \circ a^n$ (resp. $a \leq a^m \circ x, a \leq x \circ a^n$) i.e., if $a \in (a^m \circ H \circ a^n]$ (resp. $a \in (a^m \circ H], a \in (H \circ a^n]$) equivalently for each non-empty subset A of H, $A \subseteq (A^m \circ H \circ A^n]$ (resp. $A \subseteq (A^m \circ H]$, $A \subseteq (H \circ A^n]$).

Lemma 2.4. *Let H be an $(m,0)$-regular (resp. $(0,n)$-regular) ordered semihypergroup and F be a non-empty subset of H. Then the following statements are equivalent:*

(1) *F is left-m-hyperfilter (resp. right-n-hyperfilter) of H;*
(2) *$H \setminus F = \emptyset$ or $H \setminus F$ is completely prime generalized $(m,0)$-hyperideal ($(0,n)$ -hyperideal) of H, where $H \setminus F$ is the complement of F in H.*

Proof. (1) $\Rightarrow$ (2). Assume that $H \setminus F \neq \emptyset$. If $(H \setminus F)^m \circ H \subseteq F$, then $H \setminus F \subseteq (H \setminus F)^m \circ H \subseteq F$, which is a contradiction. Therefore, $(H \setminus F)^m \circ H \subseteq H \setminus F$. Let $H \ni a \leq b \in H \setminus F$. If $a \in F$, then, as F is a left-m-hyperfilter, we have $b \in F$, which is a contradiction. Thus, $a \in H \setminus F$. To show that $H \setminus F$ is completely prime $(m,0)$-hyperideal of H, let $a,b \in H$, $a \circ b \cap H \setminus F \neq \emptyset$. If $a \in F$ and $b \in F$, $a \circ b \subseteq F$. Thus, either $a \in H \setminus F$ or $b \in H \setminus F$.

(2) $\Rightarrow$ (1). Let $H \setminus F$ is completely prime generalized $(m,0)$-hyperideal of H. Let $a,b \in F$. If $a \circ b \subseteq H \setminus F$, by hypothesis $a \in H \setminus F$ or $b \in H \setminus F$, a contradiction. Thus $a \circ b \subseteq F$ it follows that F is subsemihypergroup. Now consider for any $a,b \in H, a \circ b \cap F \neq \emptyset$. If $a^m \subseteq H \setminus F$, then since H is $(m,0)$-regular there exist $s_1, s_2 \in H$ such that $a \circ b \leq a^m \circ s_1 \circ b \leq (a^m)^m \circ s_2 \circ s_1 \circ b \subseteq (H \setminus F)^m \circ H \subseteq H \setminus F$. So, $a \circ b \subseteq H \setminus F$, a contradiction. Therefore, $a^m \subseteq F$. Now take any element $a \in F$ and $b \in H$ such that $a \leq b$. If $b \in H \setminus F$, then $a \in H \setminus F$ which is a contradiction. Thus, $b \in F$. Hence, F is a left-m-filter of H. □

Corollary 2.4. *Let H be an (m,n)-regular ordered semihypergroup and F be a non-empty subset of H. Then the following statements are equivalent:*

(1) *F is (m,n)-hyperfilter of H;*
(2) *$H \setminus F = \emptyset$ or $H \setminus F$ is completely prime generalized (m,n)-hyperideal of H, where $H \setminus F$ is the complement of F in H.*

Lemma 2.5. *An $(m,0)$-regular ($(0,n)$-regular) ordered semihypergroup H does not contain proper left-m-hyperfilters (right-n-hyperfilters) if and only if H does not contain proper completely prime generalized $(m,0)$-hyperideals ($(0,n)$-hyperideals).*

Proof. Assume that H does not contain a proper left-m-hyperfilter. Let A be any proper completely prime generalized $(m,0)$-hyperideal of H. Then, by Lemma 2.4, $H \setminus A$ is proper left-m-hyperfilter of H which is a contradiction. Therefore, H does not contain any left-m-hyperfilter.

Conversely, assume that H does not contain proper completely prime $(m,0)$-hyperideals. Let F be any proper left-m-hyperfilter of H. Then by Lemma 2.4, $H \setminus F$ is a proper completely prime generalized $(m,0)$-hyperideal of H which is a contradiction. Hence, H does not contain proper left-m-hyperfilters. □

Corollary 2.5. *An (m,n)-regular ordered semihypergroup H does not contain proper (m,n)-hyperfilters if and only if H does not contain proper completely prime generalized (m,n)-hyperideals.*

Let $(H,\diamond,\leq_H)$ and $(T,\star,\leq_T)$ be two ordered semihypergroups. A mapping $\phi : H \to T$ is called a normal homomorphism if for each $a,b \in H$, $\phi(x\diamond y)=\phi(x)\star\phi(y)$ and ϕ is isotone, i.e., for each $x,y\in H$, $x\leq_H y$ implies $\phi(x)\leq_T\phi(y)$. Further, ϕ is called reverse isotone if for all $x,y\in H$, $\phi(x)\leq_T\phi(y)$ implies $x\leq_H y$.

Lemma 2.6. *Let $(H,\star,\leq_H)$ and $(T,\diamond,\leq_T)$ be two ordered semihypergroups and $\phi : H \to T$ normal homomorphism. If F a left-m-hyperfilter (right-n-hyperfilter) of T, then $\phi^{-1}(F)$ is a left-m-hyperfilter (right-n-hyperfilter) of H.*

Proof. First, we show that $\phi^{-1}(F)$ is a subsemihypergroup of H. Let $a,b\in\phi^{-1}(F)$, then $\phi(a),\phi(b)\in F$. As ϕ is normal homomorphism and F is left-m-hyperfilter of T, $\phi(a\star b)=\phi(a)\diamond\phi(b)\subseteq F$. So $a\star b\subseteq\phi^{-1}(F)$. Next, take any $a,b\in H$ such that

$$\begin{aligned}
(a\star b)\cap\phi^{-1}(F)\neq\emptyset &\Rightarrow \phi(a\star b)\cap F\neq\emptyset\\
&\Rightarrow (\phi(a)\diamond\phi(b))\cap F\neq\emptyset\\
&\Rightarrow (\phi(a))^m\subseteq F\\
&\Rightarrow \phi(a)\diamond\phi(a)\diamond\cdots\diamond\phi(a)\subseteq F\\
&\Rightarrow \phi(a\star a\star\cdots\star a)\subseteq F\\
&\Rightarrow a^m\subseteq\phi^{-1}(F).
\end{aligned}$$

If $a\in\phi^{-1}(F), b\in H$ such that $a\leq_H b$, then $\phi(a)\in F$ and $\phi(a)\leq_T\phi(b)$. Therefore, $\phi(b)\in F$ implies $b\in\phi^{-1}(F)$. Hence, $\phi^{-1}(F)$ is an left-m-hyperfilter of H. □

Corollary 2.6. *Let $(H,\star,\leq_H)$ and $(T,\diamond,\leq_T)$ be two ordered semihypergroups and $\phi : H\to T$ normal homomorphism. If F an (m,n)-hyperfilter of T, then $\phi^{-1}(F)$ is an (m,n)-hyperfilter of H.*

3. Conclusion

When we take $m=1=n$, in all results of this paper, then we obtain all results for left hyperfilters, right hyprerfilters and hyperfilters in an ordered semihypergroup and some characterizations of regular ordered semihypergroups which is the main application of results presented in this paper. Also we can extend all the results of this paper in the setting of ordered Γ-semihypergroup.

4. Problems

(1) Under what condition a left-m-hyperfilter (right-n-hyperfilter, (m,n)-hyperfilter) of an ordered semihypergroup coincides with a left hyperfilter (right hyperfilter, hyperfilter)?

(2) Under what conditions arbitrary union of left-m-hyperfilters (right-n-hyperfilters, (m,n)-hyperfilters) of an ordered semihypergroup is a left-m-hyperfilter (right-n-hyperfilter, (m,n)-hyperfilter)?

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