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A SURVEY ON STRONGLY REGULAR GRAPHS WITH $m_2 = qm_3$ AND $m_3 = qm_2$

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Abstract. We say that a regular graph G of order n and degree $r \geq 1$ (which is not the complete graph) is strongly regular if there exist non-negative integers τ and θ such that $|S_i \cap S_j| = \tau$ for any two adjacent vertices i and j, and $|S_i \cap S_j| = \theta$ for any two distinct non-adjacent vertices i and j, where S_k denotes the neighborhood of the vertex k. Let $\lambda_1 = r$, λ_2 and λ_3 be the distinct eigenvalues of a connected strongly regular graph. Let $m_1 = 1$, m_2 and m_3 denote the multiplicity of r, λ_2 and λ_3, respectively. We here survey results related to the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 2, 3, \dots, 12$.

1. Introduction

Let G be a simple graph of order n with vertex set $V(G) = \{1, 2, \dots, n\}$. The spectrum of G consists of the eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ of its (0,1) adjacency matrix A and is denoted by $\sigma(G)$. We say that a regular graph G of order n and degree $r \geq 1$ (which is not the complete graph K_n) is strongly regular if there exist non-negative integers τ and θ such that $|S_i \cap S_j| = \tau$ for any two adjacent vertices i and j, and $|S_i \cap S_j| = \theta$ for any two distinct non-adjacent vertices i and j, where $S_k \subseteq V(G)$ denotes the neighborhood of the vertex k. We know that a regular connected graph G is strongly regular if and only if it has exactly three distinct eigenvalues [1] (see also [3]). Let $\lambda_1 = r$, λ_2 and λ_3 denote the distinct eigenvalues of a connected strongly regular graph G. Let $m_1 = 1$, m_2 and m_3 denote the multiplicity of r, λ_2 and λ_3. Further, let $\overline{r} = (n-1) - r$, $\overline{\lambda}_2 = -\lambda_3 - 1$ and $\overline{\lambda}_3 = -\lambda_2 - 1$ denote the distinct

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eigenvalues of the strongly regular graph $\overline{G}$, where $\overline{G}$ denotes the complement of G. Then $\overline{\tau} = n - 2r - 2 + \theta$ and $\overline{\theta} = n - 2r + \tau$, where $\overline{\tau} = \tau(\overline{G})$ and $\overline{\theta} = \theta(\overline{G})$.

Remark 1.1. (i) If G is a disconnected strongly regular graph of degree r, then $G = mK_{r+1}$, where mH denotes the m-fold union of the graph H.

(ii) G is a disconnected strongly regular graph if and only if $\theta = 0$.

Remark 1.2. (i) A strongly regular graph G of order $n = 4k + 1$ and degree $r = 2k$ with $\tau = k - 1$ and $\theta = k$ is called a conference graph.

(ii) A strongly regular graph is a conference graph if and only if $m_2 = m_3$.

(iii) If $m_2 \neq m_3$, then G is an integral graph.

Note. We say that a connected or disconnected graph G is integral if its spectrum $\sigma(G)$ consists only of integral values.

We have recently started to investigate strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, where q is a positive integer [4]. In the same work we have described the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 2, 3, 4$. In particular, we have described in [5], [6] and [7] the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 5, 6, \ldots, 12$. We here survey results related to the parameters of strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 2, 3, \ldots, 12$, as follows.

Proposition 1.1 ([2]). *Let G be a connected or disconnected strongly regular graph of order n and degree r. Then,*

$$r^2 - (\tau - \theta + 1)r - (n - 1)\theta = 0. \tag{1.1}$$

Proposition 1.2 ([2]). *Let G be a connected strongly regular graph of order n and degree r. Then,*

$$2r + (\tau - \theta)(m_2 + m_3) + \delta(m_2 - m_3) = 0, \tag{1.2}$$

where $\delta = \lambda_2 - \lambda_3$.

Remark 1.3 ([4]). Using the same procedure applied in [4] we can establish the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for any fixed value $q \in \mathbb{N}$, as follows. First, let $m_3 = p$, $m_2 = qp$ and $n = (q + 1)p + 1$, where $q \in \mathbb{N}$. Using (1.2) we obtain $r = p(|\lambda_3| - q\lambda_2)$. Let $|\lambda_3| - q\lambda_2 = t$ where $t = 1, 2, \ldots, q$. Let $\lambda_2 = k$ where k is a positive integer. Then, (i) $\lambda_3 = -(qk + t)$; (ii) $\tau - \theta = -((q - 1)k + t)$; (iii) $\delta = (q + 1)k + t$; (iv) $r = pt$ and (v) $\theta = pt - qk^2 - kt$. Using (ii), (iv) and (v) we can easily see that (1.1) is reduced to

$$(p + 1)t^2 - \Big((q + 1)p + 1\Big)t + q(q + 1)k^2 + 2qkt = 0. \tag{1.3}$$

Second, let $m_2 = p$, $m_3 = qp$ and $n = (q+1)p+1$, where $q \in \mathbb{N}$. Using (1.2) we obtain $r = p(q|\lambda_3| - \lambda_2)$. Let $q|\lambda_3| - \lambda_2 = t$, where $t = 1, 2, \ldots, q$. Let $\lambda_3 = -k$, where k is a positive integer. Then, (i) $\lambda_2 = qk - t$; (ii) $\tau - \theta = (q - 1)k - t$; (iii) $\delta = (q + 1)k - t$;

(iv) $r = pt$ and (v) $\theta = pt - qk^2 + kt$. Using (ii), (iv) and (v) we can easily see that (1.1) is reduced to

$$(p+1)t^2 - \Big((q+1)p+1\Big)t + q(q+1)k^2 - 2qkt = 0. \tag{1.4}$$

Using (1.3) and (1.4) we can obtain for $t = 1, 2, \dots, q$ the corresponding classes of strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, respectively.

2. Main Results

Remark 2.1. Since $m_2(\overline{G}) = m_3(G)$ and $m_3(\overline{G}) = m_2(G)$ we note that if $m_2(G) = qm_3(G)$, then $m_3(\overline{G}) = qm_2(\overline{G})$.

Remark 2.2. In Theorems 2.1, 2.2, ..., 2.11 the complements of strongly regular graphs appear in pairs in (k^0) and $(\overline{k}^0)$ classes, where k denotes the corresponding number of a class.

Remark 2.3. $\overline{\alpha K_\beta}$ is a strongly regular graph of order $n = \alpha\beta$ and degree $r = (\alpha-1)\beta$ with $\tau = (\alpha - 2)\beta$ and $\theta = (\alpha - 1)\beta$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -\beta$ with $m_2 = \alpha(\beta - 1)$ and $m_3 = \alpha - 1$.

In order to demonstrate a method which is applied to describe the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, we shall establish the parameters of strongly regular graphs with $m_2 = 2m_3$ and $m_3 = 2m_2$, as follows.

Proposition 2.1. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 2m_3$. Then G belongs to the class $(\overline{2}^0)$ or (3^0) represented in Theorem* 2.1.

Proof. Let $m_3 = p$, $m_2 = 2p$ and $n = 3p + 1$ where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(|\lambda_3| - 2\lambda_2)$. Let $|\lambda_3| - 2\lambda_2 = t$ where $t = 1, 2$. Let $\lambda_2 = k$ where k is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_3 = -(2k + t)$; (ii) $\tau - \theta = -(k + t)$; (iii) $\delta = 3k + t$; (iv) $r = pt$ and (v) $\theta = pt - 2k^2 - kt$. In this case we can easily see that (1.3) is reduced to

$$(p+1)t^2 - (3p+1)t + 6k^2 + 4kt = 0. \tag{2.1}$$

Case 1. ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(2k+1)$, $\tau - \theta = -(k+1)$, $\delta = 3k+1$, $r = p$ and $\theta = p - 2k^2 - k$. Using (2.1) we find that $p = k(3k + 2)$. So, we obtain that G is a strongly regular graph of order $n = (3k + 1)^2$ and degree $r = k(3k + 2)$ with $\tau = k^2 - 1$ and $\theta = k(k + 1)$.

Case 2. ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(2k + 2)$, $\tau - \theta = -(k + 2)$, $\delta = 3k + 2$, $r = 2p$ and $\theta = 2p - 2k^2 - 2k$. Using (2.1) we find that $p = (k + 1)(3k + 1)$. Replacing k with $k - 1$ we arrive at $p = k(3k-2)$. So, we obtain that G is a strongly regular graph of order $n = (3k-1)^2$ and degree $r = 2k(3k - 2)$ with $\tau = (k - 1)(4k + 1)$ and $\theta = 2k(2k - 1)$. □

Proposition 2.2. *Let G be a connected strongly regular graph of order n and degree r with $m_3 = 2m_2$. Then G belongs to the class (2^0) or $(\overline{3}^0)$ represented in Theorem 2.1.*

Proof. Let $m_2 = p$, $m_3 = 2p$ and $n = 3p + 1$ where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(2|\lambda_3| - \lambda_2)$. Let $2|\lambda_3| - \lambda_2 = t$ where $t = 1, 2$. Let $\lambda_3 = -k$ where k is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_2 = 2k - t$; (ii) $\tau - \theta = k - t$; (iii) $\delta = 3k - t$; (iv) $r = pt$ and (v) $\theta = pt - 2k^2 + kt$. In this case we can easily see that (1.4) is reduced to

$$(p + 1)t^2 - (3p + 1)t + 6k^2 - 4kt = 0. \tag{2.2}$$

Case 1. ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 2k - 1$ and $\lambda_3 = -k$, $\tau - \theta = k - 1$, $\delta = 3k - 1$, $r = p$ and $\theta = p - 2k^2 + k$. Using (2.2) we find that $p = k(3k - 2)$. So we obtain that G is a strongly regular graph of order $n = (3k - 1)^2$ and degree $r = k(3k - 2)$ with $\tau = k^2 - 1$ and $\theta = k(k - 1)$.

Case 2. ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 2k - 2$ and $\lambda_3 = -k$, $\tau - \theta = k - 2$, $\delta = 3k - 2$, $r = 2p$ and $\theta = 2p - 2k^2 + 2k$. Using (2.2) we find that $p = (k - 1)(3k - 1)$. Replacing k with $k + 1$ we arrive at $p = k(3k + 2)$. So we obtain that G is a strongly regular graph of order $n = (3k + 1)^2$ and degree $r = 2k(3k + 2)$ with $\tau = (k + 1)(4k - 1)$ and $\theta = 2k(2k + 1)$. □

Theorem 2.1. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 2m_3$ or $m_3 = 2m_2$. Then, G is one of the following strongly regular graphs.*

- (1^0) *G is the complete bipartite graph of order $n = 4$ and degree $r = 2$ with $\tau = 0$ and $\theta = 2$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -2$ with $m_2 = 2$ and $m_3 = 1$.*
- (2^0) *G is a strongly regular graph of order $n = (3k - 1)^2$ and degree $r = k(3k - 2)$ with $\tau = k^2 - 1$ and $\theta = k(k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 2k - 1$ and $\lambda_3 = -k$ with $m_2 = k(3k - 2)$ and $m_3 = 2k(3k - 2)$.*
- $(\overline{2}^0)$ *G is a strongly regular graph of order $n = (3k - 1)^2$ and degree $r = 2k(3k - 2)$ with $\tau = (k - 1)(4k + 1)$ and $\theta = 2k(2k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -2k$ with $m_2 = 2k(3k - 2)$ and $m_3 = k(3k - 2)$.*
- (3^0) *G is a strongly regular graph of order $n = (3k + 1)^2$ and degree $r = k(3k + 2)$ with $\tau = k^2 - 1$ and $\theta = k(k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(2k + 1)$ with $m_2 = 2k(3k + 2)$ and $m_3 = k(3k + 2)$.*
- $(\overline{3}^0)$ *G is a strongly regular graph of order $n = (3k + 1)^2$ and degree $r = 2k(3k + 2)$ with $\tau = (k + 1)(4k - 1)$ and $\theta = 2k(2k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 2k$ and $\lambda_3 = -(k + 1)$ with $m_2 = k(3k + 2)$ and $m_3 = 2k(3k + 2)$.*

Proof. First, according to Remark 2.3 we have $\alpha(\beta - 1) = 2(\alpha - 1)$, from which we find that $\alpha = 2$, $\beta = 2$. In view of this we obtain the strongly regular graph represented in Theorem 2.1 (1^0). Next, according to Proposition 2.1 it turns out that G belongs to the class $(\overline{2}^0)$ or (3^0) if $m_2 = 2m_3$. According to Proposition 2.2 it turns out that G belongs to the class (2^0) or $(\overline{3}^0)$ if $m_3 = 2m_2$. □

Theorem 2.2 ([4]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 3m_3$ or $m_3 = 3m_2$. Then, G is one of the following strongly regular graphs.*

- (1^0) *G is the strongly regular graph $\overline{3K_3}$ of order $n = 9$ and degree $r = 6$ with $\tau = 3$ and $\theta = 6$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -3$ with $m_2 = 6$ and $m_3 = 2$.*
- (2^0) *G is a strongly regular graph of order $n = (4k-1)^2$ and degree $r = 2k(2k-1)$ with $\tau = k^2 + k - 1$ and $\theta = k(k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 3k - 1$ and $\lambda_3 = -k$ with $m_2 = 2k(2k-1)$ and $m_3 = 6k(2k-1)$.*
- ($\overline{2}^0$) *G is a strongly regular graph of order $n = (4k-1)^2$ and degree $r = 6k(2k-1)$ with $\tau = 9k^2 - 5k - 1$ and $\theta = 3k(3k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -3k$ with $m_2 = 6k(2k-1)$ and $m_3 = 2k(2k-1)$.*
- (3^0) *G is a strongly regular graph of order $n = (4k+1)^2$ and degree $r = 2k(2k+1)$ with $\tau = k^2 - k - 1$ and $\theta = k(k+1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(3k+1)$ with $m_2 = 6k(2k+1)$ and $m_3 = 2k(2k+1)$.*
- ($\overline{3}^0$) *G is a strongly regular graph of order $n = (4k+1)^2$ and degree $r = 6k(2k+1)$ with $\tau = 9k^2 + 5k - 1$ and $\theta = 3k(3k+1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 3k$ and $\lambda_3 = -(k+1)$ with $m_2 = 2k(2k+1)$ and $m_3 = 6k(2k+1)$.*

Theorem 2.3 ([4]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 4m_3$ or $m_3 = 4m_2$. Then, G is one of the following strongly regular graphs.*

- (1^0) *G is the complete bipartite graph $K_{3,3}$ of order $n = 6$ and degree $r = 3$ with $\tau = 0$ and $\theta = 3$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -3$ with $m_2 = 4$ and $m_3 = 1$.*
- (2^0) *G is the strongly regular graph $\overline{4K_4}$ of order $n = 16$ and degree $r = 12$ with $\tau = 8$ and $\theta = 12$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -4$ with $m_2 = 12$ and $m_3 = 3$.*
- (3^0) *G is a strongly regular graph of order $n = (5k-1)^2$ and degree $r = k(5k-2)$ with $\tau = k^2 + 2k - 1$ and $\theta = k(k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 4k - 1$ and $\lambda_3 = -k$ with $m_2 = k(5k-2)$ and $m_3 = 4k(5k-2)$.*
- ($\overline{3}^0$) *G is a strongly regular graph of order $n = (5k-1)^2$ and degree $r = 4k(5k-2)$ with $\tau = 16k^2 - 7k - 1$ and $\theta = 4k(4k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -4k$ with $m_2 = 4k(5k-2)$ and $m_3 = k(5k-2)$.*
- (4^0) *G is a strongly regular graph of order $n = (5k+1)^2$ and degree $r = k(5k+2)$ with $\tau = k^2 - 2k - 1$ and $\theta = k(k+1)$, where $k \geq 3$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(4k+1)$ with $m_2 = 4k(5k+2)$ and $m_3 = k(5k+2)$.*
- ($\overline{4}^0$) *G is a strongly regular graph of order $n = (5k+1)^2$ and degree $r = 4k(5k+2)$ with $\tau = 16k^2 + 7k - 1$ and $\theta = 4k(4k+1)$, where $k \geq 3$. Its eigenvalues are $\lambda_2 = 4k$ and $\lambda_3 = -(k+1)$ with $m_2 = k(5k+2)$ and $m_3 = 4k(5k+2)$.*
- (5^0) *G is a strongly regular graph of order $n = 6(5k-1)^2$ and degree $r = 2(30k^2 - 12k + 1)$ with $\tau = 24k^2 - 15k + 1$ and $\theta = 6k(4k-1)$, where $k \in \mathbb{N}$. Its*

eigenvalues are $\lambda_2 = 3k-1$ *and* $\lambda_3 = -(12k-2)$ *with* $m_2 = 4(30k^2-12k+1)$ *and* $m_3 = 30k^2-12k+1$.

($\overline{5}^0$) *G is a strongly regular graph of order* $n = 6(5k-1)^2$ *and degree* $r = 3(30k^2-12k+1)$ *with* $\tau = 18k(3k-1)$ *and* $\theta = 3(3k-1)(6k-1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 12k-3$ *and* $\lambda_3 = -3k$ *with* $m_2 = 30k^2-12k+1$ *and* $m_3 = 4(30k^2-12k+1)$.

(6^0) *G is a strongly regular graph of order* $n = 6(5k+1)^2$ *and degree* $r = 2(30k^2+12k+1)$ *with* $\tau = 24k^2+15k+1$ *and* $\theta = 6k(4k+1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 12k+2$ *and* $\lambda_3 = -(3k+1)$ *with* $m_2 = 30k^2+12k+1$ *and* $m_3 = 4(30k^2+12k+1)$.

($\overline{6}^0$) *G is a strongly regular graph of order* $n = 6(5k+1)^2$ *and degree* $r = 3(30k^2+12k+1)$ *with* $\tau = 18k(3k+1)$ *and* $\theta = 3(3k+1)(6k+1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 3k$ *and* $\lambda_3 = -(12k+3)$ *with* $m_2 = 4(30k^2+12k+1)$ *and* $m_3 = 30k^2+12k+1$.

Theorem 2.4 ([5])**.** *Let G be a connected strongly regular graph of order n and degree r with* $m_2 = 5m_3$ *or* $m_3 = 5m_2$. *Then, G is one of the following strongly regular graphs.*

(1^0) *G is the strongly regular graph* $\overline{5K_5}$ *of order* $n = 25$ *and degree* $r = 20$ *with* $\tau = 15$ *and* $\theta = 20$. *Its eigenvalues are* $\lambda_2 = 0$ *and* $\lambda_3 = -5$ *with* $m_2 = 20$ *and* $m_3 = 4$.

(2^0) *G is a strongly regular graph of order* $n = (6k-1)^2$ *and degree* $r = 2k(3k-1)$ *with* $\tau = k^2+3k-1$ *and* $\theta = k(k-1)$, *where* $k \geq 2$. *Its eigenvalues are* $\lambda_2 = 5k-1$ *and* $\lambda_3 = -k$ *with* $m_2 = 2k(3k-1)$ *and* $m_3 = 10k(3k-1)$.

($\overline{2}^0$) *G is a strongly regular graph of order* $n = (6k-1)^2$ *and degree* $r = 10k(3k-1)$ *with* $\tau = 25k^2-9k-1$ *and* $\theta = 5k(5k-1)$, *where* $k \geq 2$. *Its eigenvalues are* $\lambda_2 = k-1$ *and* $\lambda_3 = -5k$ *with* $m_2 = 10k(3k-1)$ *and* $m_3 = 2k(3k-1)$.

(3^0) *G is a strongly regular graph of order* $n = (6k+1)^2$ *and degree* $r = 2k(3k+1)$ *with* $\tau = k^2-3k-1$ *and* $\theta = k(k+1)$, *where* $k \geq 4$. *Its eigenvalues are* $\lambda_2 = k$ *and* $\lambda_3 = -(5k+1)$ *with* $m_2 = 10k(3k+1)$ *and* $m_3 = 2k(3k+1)$.

($\overline{3}^0$) *G is a strongly regular graph of order* $n = (6k+1)^2$ *and degree* $r = 10k(3k+1)$ *with* $\tau = 25k^2+9k-1$ *and* $\theta = 5k(5k+1)$, *where* $k \geq 4$. *Its eigenvalues are* $\lambda_2 = 5k$ *and* $\lambda_3 = -(k+1)$ *with* $m_2 = 2k(3k+1)$ *and* $m_3 = 10k(3k+1)$.

Theorem 2.5 ([5])**.** *Let G be a connected strongly regular graph of order n and degree r with* $m_2 = 6m_3$ *or* $m_3 = 6m_2$. *Then, G is one of the following strongly regular graphs.*

(1^0) *G is the complete bipartite graph* $K_{4,4}$ *of order* $n = 8$ *and degree* $r = 4$ *with* $\tau = 0$ *and* $\theta = 4$. *Its eigenvalues are* $\lambda_2 = 0$ *and* $\lambda_3 = -4$ *with* $m_2 = 6$ *and* $m_3 = 1$.

(2^0) *G is the strongly regular graph* $\overline{3K_5}$ *of order* $n = 15$ *and degree* $r = 10$ *with* $\tau = 5$ *and* $\theta = 10$. *Its eigenvalues are* $\lambda_2 = 0$ *and* $\lambda_3 = -5$ *with* $m_2 = 12$ *and* $m_3 = 2$.

(3^0) *G is the strongly regular graph $\overline{6K_6}$ of order $n = 36$ and degree $r = 30$ with $\tau = 24$ and $\theta = 30$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -6$ with $m_2 = 30$ and $m_3 = 5$.*

(4^0) *G is a strongly regular graph of order $n = (7k-1)^2$ and degree $r = k(7k-2)$ with $\tau = k^2 + 4k - 1$ and $\theta = k(k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 6k - 1$ and $\lambda_3 = -k$ with $m_2 = k(7k-2)$ and $m_3 = 6k(7k-2)$.*

($\overline{4}^0$) *G is a strongly regular graph of order $n = (7k-1)^2$ and degree $r = 6k(7k-2)$ with $\tau = 36k^2 - 11k - 1$ and $\theta = 6k(6k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -6k$ with $m_2 = 6k(7k-2)$ and $m_3 = k(7k-2)$.*

(5^0) *G is a strongly regular graph of order $n = (7k+1)^2$ and degree $r = k(7k+2)$ with $\tau = k^2 - 4k - 1$ and $\theta = k(k+1)$, where $k \geq 5$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(6k+1)$ with $m_2 = 6k(7k+2)$ and $m_3 = k(7k+2)$.*

($\overline{5}^0$) *G is a strongly regular graph of order $n = (7k+1)^2$ and degree $r = 6k(7k+2)$ with $\tau = 36k^2 + 11k - 1$ and $\theta = 6k(6k+1)$, where $k \geq 5$. Its eigenvalues are $\lambda_2 = 6k$ and $\lambda_3 = -(k+1)$ with $m_2 = k(7k+2)$ and $m_3 = 6k(7k+2)$.*

(6^0) *G is a strongly regular graph of order $n = 2(7k-2)^2$ and degree $r = 3(14k^2 - 8k + 1)$ with $\tau = 18k^2 - 16k + 2$ and $\theta = 6k(3k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 2k - 1$ and $\lambda_3 = -(12k-3)$ with $m_2 = 6(14k^2 - 8k + 1)$ and $m_3 = 14k^2 - 8k + 1$.*

($\overline{6}^0$) *G is a strongly regular graph of order $n = 2(7k-2)^2$ and degree $r = 4(14k^2 - 8k + 1)$ with $\tau = 2k(16k-7)$ and $\theta = 4(2k-1)(4k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 12k - 4$ and $\lambda_3 = -2k$ with $m_2 = 14k^2 - 8k + 1$ and $m_3 = 6(14k^2 - 8k + 1)$.*

(7^0) *G is a strongly regular graph of order $n = 2(7k+2)^2$ and degree $r = 3(14k^2 + 8k + 1)$ with $\tau = 18k^2 + 16k + 2$ and $\theta = 6k(3k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 12k + 3$ and $\lambda_3 = -(2k+1)$ with $m_2 = 14k^2 + 8k + 1$ and $m_3 = 6(14k^2 + 8k + 1)$.*

($\overline{7}^0$) *G is a strongly regular graph of order $n = 2(7k+2)^2$ and degree $r = 4(14k^2 + 8k + 1)$ with $\tau = 2k(16k+7)$ and $\theta = 4(2k+1)(4k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 2k$ and $\lambda_3 = -(12k+4)$ with $m_2 = 6(14k^2 + 8k + 1)$ and $m_3 = 14k^2 + 8k + 1$.*

(8^0) *G is a strongly regular graph of order $n = 15(7k-1)^2$ and degree $r = 2(105k^2 - 30k + 2)$ with $\tau = 60k^2 - 35k + 3$ and $\theta = 10k(6k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k - 1$ and $\lambda_3 = -(30k-4)$ with $m_2 = 6(105k^2 - 30k + 2)$ and $m_3 = 105k^2 - 30k + 2$.*

($\overline{8}^0$) *G is a strongly regular graph of order $n = 15(7k-1)^2$ and degree $r = 5(105k^2 - 30k + 2)$ with $\tau = 5(5k-1)(15k-1)$ and $\theta = 5(5k-1)(15k-2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 30k - 5$ and $\lambda_3 = -5k$ with $m_2 = 105k^2 - 30k + 2$ and $m_3 = 6(105k^2 - 30k + 2)$.*

(9^0) *G is a strongly regular graph of order $n = 15(7k+1)^2$ and degree $r = 2(105k^2 + 30k + 2)$ with $\tau = 60k^2 + 35k + 3$ and $\theta = 10k(6k+1)$, where $k \in \mathbb{N}$. Its* *eigenvalues are* $\lambda_2 = 30k + 4$ *and* $\lambda_3 = -(5k + 1)$ *with* $m_2 = 105k^2 + 30k + 2$ *and* $m_3 = 6(105k^2 + 30k + 2)$.

($\overline{9}^0$) *G is a strongly regular graph of order* $n = 15(7k+1)^2$ *and degree* $r = 5(105k^2 + 30k + 2)$ *with* $\tau = 5(5k + 1)(15k + 1)$ *and* $\theta = 5(5k + 1)(15 + 2)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 5k$ *and* $\lambda_3 = -(30k + 5)$ *with* $m_2 = 6(105k^2 + 30k + 2)$ *and* $m_3 = 105k^2 + 30k + 2$.

Theorem 2.6 ([5]). *Let G be a connected strongly regular graph of order n and degree r with* $m_2 = 7m_3$ *or* $m_3 = 7m_2$. *Then, G is one of the following strongly regular graphs.*

(1^0) *G is the strongly regular graph* $\overline{7K_7}$ *of order* $n = 49$ *and degree* $r = 42$ *with* $\tau = 35$ *and* $\theta = 42$. *Its eigenvalues are* $\lambda_2 = 0$ *and* $\lambda_3 = -7$ *with* $m_2 = 42$ *and* $m_3 = 6$.

(2^0) *G is a strongly regular graph of order* $n = (8k - 1)^2$ *and degree* $r = 2k(4k - 1)$ *with* $\tau = k^2 + 5k - 1$ *and* $\theta = k(k - 1)$, *where* $k \geq 2$. *Its eigenvalues are* $\lambda_2 = 7k - 1$ *and* $\lambda_3 = -k$ *with* $m_2 = 2k(4k - 1)$ *and* $m_3 = 14k(4k - 1)$.

($\overline{2}^0$) *G is a strongly regular graph of order* $n = (8k - 1)^2$ *and degree* $r = 14k(4k - 1)$ *with* $\tau = 49k^2 - 13k - 1$ *and* $\theta = 7k(7k - 1)$, *where* $k \geq 2$. *Its eigenvalues are* $\lambda_2 = k - 1$ *and* $\lambda_3 = -7k$ *with* $m_2 = 14k(4k - 1)$ *and* $m_3 = 2k(4k - 1)$.

(3^0) *G is a strongly regular graph of order* $n = (8k + 1)^2$ *and degree* $r = 2k(4k + 1)$ *with* $\tau = k^2 - 5k - 1$ *and* $\theta = k(k + 1)$, *where* $k \geq 6$. *Its eigenvalues are* $\lambda_2 = k$ *and* $\lambda_3 = -(7k + 1)$ *with* $m_2 = 14k(4k + 1)$ *and* $m_3 = 2k(4k + 1)$.

($\overline{3}^0$) *G is a strongly regular graph of order* $n = (8k + 1)^2$ *and degree* $r = 14k(4k + 1)$ *with* $\tau = 49k^2 + 13k - 1$ *and* $\theta = 7k(7k + 1)$, *where* $k \geq 6$. *Its eigenvalues are* $\lambda_2 = 7k$ *and* $\lambda_3 = -(k + 1)$ *with* $m_2 = 2k(4k + 1)$ *and* $m_3 = 14k(4k + 1)$.

(4^0) *G is a strongly regular graph of order* $n = 105(8k - 3)^2$ *and degree* $r = 3(840k^2 - 630k + 118)$ *with* $\tau = 945k^2 - 765k + 153$ *and* $\theta = 15(3k - 1)(21k - 8)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 15k - 6$ *and* $\lambda_3 = -(105k - 39)$ *with* $m_2 = 7(840k^2 - 630k + 118)$ *and* $m_3 = 840k^2 - 630k + 118$.

($\overline{4}^0$) *G is a strongly regular graph of order* $n = 105(8k - 3)^2$ *and degree* $r = 5(840k^2 - 630k + 118)$ *with* $\tau = 5(525k^2 - 387k + 71)$ *and* $\theta = 15(5k - 2)(35k - 13)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 105k - 40$ *and* $\lambda_3 = -(15k - 5)$ *with* $m_2 = 840k^2 - 630k + 118$ *and* $m_3 = 7(840k^2 - 630k + 118)$.

(5^0) *G is a strongly regular graph of order* $n = 105(8k + 3)^2$ *and degree* $r = 3(840k^2 + 630k + 118)$ *with* $\tau = 945k^2 + 765k + 153$ *and* $\theta = 15(3k + 1)(21k + 8)$, *where* $k \geq 0$. *Its eigenvalues are* $\lambda_2 = 105k + 39$ *and* $\lambda_3 = -(15k + 6)$ *with* $m_2 = 840k^2 + 630k + 118$ *and* $m_3 = 7(840k^2 + 630k + 118)$.

($\overline{5}^0$) *G is a strongly regular graph of order* $n = 105(8k + 3)^2$ *and degree* $r = 5(840k^2 + 630k + 118)$ *with* $\tau = 5(525k^2 + 387k + 71)$ *and* $\theta = 15(5k + 2)(35k + 13)$, *where* $k \geq 0$. *Its eigenvalues are* $\lambda_2 = 15k + 5$ *and* $\lambda_3 = -(105k + 40)$ *with* $m_2 = 7(840k^2 + 630k + 118)$ *and* $m_3 = 840k^2 + 630k + 118$.

Theorem 2.7 ([5]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 8m_3$ or $m_3 = 8m_2$. Then, G is one of the following strongly regular graphs.*

(1^0) *G is the complete bipartite graph $K_{5,5}$ of order $n = 10$ and degree $r = 5$ with $\tau = 0$ and $\theta = 5$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -5$ with $m_2 = 8$ and $m_3 = 1$.*

(2^0) *G is the strongly regular graph $\overline{4K_7}$ of order $n = 28$ and degree $r = 21$ with $\tau = 14$ and $\theta = 21$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -7$ with $m_2 = 24$ and $m_3 = 3$.*

(3^0) *G is the strongly regular graph $\overline{8K_8}$ of order $n = 64$ and degree $r = 56$ with $\tau = 48$ and $\theta = 56$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -8$ with $m_2 = 56$ and $m_3 = 7$.*

(4^0) *G is a strongly regular graph of order $n = (9k-1)^2$ and degree $r = k(9k-2)$ with $\tau = k^2 + 6k - 1$ and $\theta = k(k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 8k - 1$ and $\lambda_3 = -k$ with $m_2 = k(9k-2)$ and $m_3 = 8k(9k-2)$.*

($\overline{4}^0$) *G is a strongly regular graph of order $n = (9k-1)^2$ and degree $r = 8k(9k-2)$ with $\tau = 64k^2 - 15k - 1$ and $\theta = 8k(8k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -8k$ with $m_2 = 8k(9k-2)$ and $m_3 = k(9k-2)$.*

(5^0) *G is a strongly regular graph of order $n = (9k+1)^2$ and degree $r = k(9k+2)$ with $\tau = k^2 - 6k - 1$ and $\theta = k(k+1)$, where $k \geq 7$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(8k+1)$ with $m_2 = 8k(9k+2)$ and $m_3 = k(9k+2)$.*

($\overline{5}^0$) *G is a strongly regular graph of order $n = (9k+1)^2$ and degree $r = 8k(9k+2)$ with $\tau = 64k^2 + 15k - 1$ and $\theta = 8k(8k+1)$, where $k \geq 7$. Its eigenvalues are $\lambda_2 = 8k$ and $\lambda_3 = -(k+1)$ with $m_2 = k(9k+2)$ and $m_3 = 8k(9k+2)$.*

(6^0) *G is a strongly regular graph of order $n = 4(9k-4)^2$ and degree $r = 3(2k-1)(18k-7)$ with $\tau = 18k(2k-1)$ and $\theta = (3k-2)(12k-5)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 24k - 11$ and $\lambda_3 = -(3k-1)$ with $m_2 = (2k-1)(18k-7)$ and $m_3 = 8(2k-1)(18k-7)$.*

($\overline{6}^0$) *G is a strongly regular graph of order $n = 4(9k-4)^2$ and degree $r = 6(2k-1)(18k-7)$ with $\tau = 3(48k^2 - 45k + 10)$ and $\theta = 2(3k-1)(24k-11)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k - 2$ and $\lambda_3 = -(24k-10)$ with $m_2 = 8(2k-1)(18k-7)$ and $m_3 = (2k-1)(18k-7)$.*

(7^0) *G is a strongly regular graph of order $n = 4(9k+4)^2$ and degree $r = 3(2k+1)(18k+7)$ with $\tau = 18k(2k+1)$ and $\theta = (3k+2)(12k+5)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 3k+1$ and $\lambda_3 = -(24k+11)$ with $m_2 = 8(2k+1)(18k+7)$ and $m_3 = (2k+1)(18k+7)$.*

($\overline{7}^0$) *G is a strongly regular graph of order $n = 4(9k+4)^2$ and degree $r = 6(2k+1)(18k+7)$ with $\tau = 3(48k^2+45k+10)$ and $\theta = 2(3k+1)(24k+11)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 24k+10$ and $\lambda_3 = -(3k+2)$ with $m_2 = (2k+1)(18k+7)$ and $m_3 = 8(2k+1)(18k+7)$.*

(8^0) *G is a strongly regular graph of order* $n = 10(9k-1)^2$ *and degree* $r = 4(90k^2 - 20k + 1)$ *with* $\tau = 160k^2 - 55k + 3$ *and* $\theta = 20k(8k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 5k - 1$ *and* $\lambda_3 = -(40k-4)$ *with* $m_2 = 8(90k^2 - 20k + 1)$ *and* $m_3 = 90k^2 - 20k + 1$*.*

($\overline{8}^0$) *G is a strongly regular graph of order* $n = 10(9k-1)^2$ *and degree* $r = 5(90k^2 - 20k + 1)$ *with* $\tau = 10k(25k-4)$ *and* $\theta = 5(5k-1)(10k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 40k - 5$ *and* $\lambda_3 = -5k$ *with* $m_2 = 90k^2 - 20k + 1$ *and* $m_3 = 8(90k^2 - 20k + 1)$*.*

(9^0) *G is a strongly regular graph of order* $n = 10(9k+1)^2$ *and degree* $r = 4(90k^2 + 20k + 1)$ *with* $\tau = 160k^2 + 55k + 3$ *and* $\theta = 20k(8k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 40k + 4$ *and* $\lambda_3 = -(5k+1)$ *with* $m_2 = 90k^2 + 20k + 1$ *and* $m_3 = 8(90k^2 + 20k + 1)$*.*

($\overline{9}^0$) *G is a strongly regular graph of order* $n = 10(9k+1)^2$ *and degree* $r = 5(90k^2 + 20k + 1)$ *with* $\tau = 10k(25k+4)$ *and* $\theta = 5(5k+1)(10k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 5k$ *and* $\lambda_3 = -(40k+5)$ *with* $m_2 = 8(90k^2 + 20k + 1)$ *and* $m_3 = 90k^2 + 20k + 1$*.*

(10^0) *G is a strongly regular graph of order* $n = 28(9k-1)^2$ *and degree* $r = 2(252k^2 - 56k + 3)$ *with* $\tau = 112k^2 - 63k + 5$ *and* $\theta = 14k(8k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 7k - 1$ *and* $\lambda_3 = -(56k-6)$ *with* $m_2 = 8(252k^2 - 56k + 3)$ *and* $m_3 = 252k^2 - 56k + 3$*.*

($\overline{10}^0$) *G is a strongly regular graph of order* $n = 28(9k-1)^2$ *and degree* $r = 7(252k^2 - 56k + 3)$ *with* $\tau = 14(7k-1)(14k-1)$ *and* $\theta = 7(7k-1)(28k-3)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 56k - 7$ *and* $\lambda_3 = -7k$ *with* $m_2 = 252k^2 - 56k + 3$ *and* $m_3 = 8(252k^2 - 56k + 3)$*.*

(11^0) *G is a strongly regular graph of order* $n = 28(9k+1)^2$ *and degree* $r = 2(252k^2 + 56k + 3)$ *with* $\tau = 112k^2 + 63k + 5$ *and* $\theta = 14k(8k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 56k + 6$ *and* $\lambda_3 = -(7k+1)$ *with* $m_2 = 252k^2 + 56k + 3$ *and* $m_3 = 8(252k^2 + 56k + 3)$*.*

($\overline{11}^0$) *G is a strongly regular graph of order* $n = 28(9k+1)^2$ *and degree* $r = 7(252k^2 + 56k + 3)$ *with* $\tau = 14(7k+1)(14k+1)$ *and* $\theta = 7(7k+1)(28k+3)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 7k$ *and* $\lambda_3 = -(56k+7)$ *with* $m_2 = 8(252k^2 + 56k + 3)$ *and* $m_3 = 252k^2 + 56k + 3$*.*

Theorem 2.8 ([6])**.** *Let G be a connected strongly regular graph of order n and degree r with* $m_2 = 9m_3$ *or* $m_3 = 9m_2$*. Then, G is one of the following strongly regular graphs.*

(1^0) *G is the strongly regular graph* $\overline{3K_7}$ *of order* $n = 21$ *and degree* $r = 14$ *with* $\tau = 7$ *and* $\theta = 14$*. Its eigenvalues are* $\lambda_2 = 0$ *and* $\lambda_3 = -7$ *with* $m_2 = 18$ *and* $m_3 = 2$*.*

(2^0) *G is the strongly regular graph* $\overline{9K_9}$ *of order* $n = 81$ *and degree* $r = 72$ *with* $\tau = 63$ *and* $\theta = 72$*. Its eigenvalues are* $\lambda_2 = 0$ *and* $\lambda_3 = -9$ *with* $m_2 = 72$ *and* $m_3 = 8$*.*

(3^0) *G is a strongly regular graph of order $n = (10k-1)^2$ and degree $r = 2k(5k-1)$ with $\tau = k^2 + 7k - 1$ and $\theta = k(k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 9k - 1$ and $\lambda_3 = -k$ with $m_2 = 2k(5k-1)$ and $m_3 = 18k(5k-1)$.*

($\overline{3}^0$) *G is a strongly regular graph of order $n = (10k-1)^2$ and degree $r = 18k(5k-1)$ with $\tau = 81k^2 - 17k - 1$ and $\theta = 9k(9k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -9k$ with $m_2 = 18k(5k-1)$ and $m_3 = 2k(5k-1)$.*

(4^0) *G is a strongly regular graph of order $n = (10k+1)^2$ and degree $r = 2k(5k+1)$ with $\tau = k^2 - 7k - 1$ and $\theta = k(k+1)$, where $k \geq 8$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(9k+1)$ with $m_2 = 18k(5k+1)$ and $m_3 = 2k(5k+1)$.*

($\overline{4}^0$) *G is a strongly regular graph of order $n = (10k+1)^2$ and degree $r = 18k(5k+1)$ with $\tau = 81k^2 + 17k - 1$ and $\theta = 9k(9k+1)$, where $k \geq 8$. Its eigenvalues are $\lambda_2 = 9k$ and $\lambda_3 = -(k+1)$ with $m_2 = 2k(5k+1)$ and $m_3 = 18k(5k+1)$.*

(5^0) *G is a strongly regular graph of order $n = 9(10k-3)^2$ and degree $r = 2(90k^2 - 54k + 8)$ with $\tau = 36k^2 + 4k - 5$ and $\theta = (2k-1)(18k-5)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 36k - 11$ and $\lambda_3 = -(4k-1)$ with $m_2 = 90k^2 - 54k + 8$ and $m_3 = 9(90k^2 - 54k + 8)$.*

($\overline{5}^0$) *G is a strongly regular graph of order $n = 9(10k-3)^2$ and degree $r = 8(90k^2 - 54k+8)$ with $\tau = 4(4k-1)(36k-13)$ and $\theta = 4(4k-1)(36k-11)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 4k-2$ and $\lambda_3 = -(36k-10)$ with $m_2 = 9(90k^2-54k+8)$ and $m_3 = 90k^2 - 54k + 8$.*

(6^0) *G is a strongly regular graph of order $n = 9(10k-3)^2$ and degree $r = 5(90k^2 - 54k+8)$ with $\tau = 225k^2-155k+25$ and $\theta = (5k-1)(45k-14)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k-2$ and $\lambda_3 = -(45k-13)$ with $m_2 = 9(90k^2-54k+8)$ and $m_3 = 90k^2 - 54k + 8$.*

($\overline{6}^0$) *G is a strongly regular graph of order $n = 9(10k-3)^2$ and degree $r = 5(90k^2 - 54k+8)$ with $\tau = 225k^2 - 115k + 13$ and $\theta = (5k-2)(45k-13)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 45k-14$ and $\lambda_3 = -(5k-1)$ with $m_2 = 90k^2-54k+8$ and $m_3 = 9(90k^2 - 54k + 8)$.*

(7^0) *G is a strongly regular graph of order $n = 9(10k+3)^2$ and degree $r = 2(90k^2 + 54k+8)$ with $\tau = 36k^2 - 4k - 5$ and $\theta = (2k+1)(18k+5)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 4k+1$ and $\lambda_3 = -(36k+11)$ with $m_2 = 9(90k^2+54k+8)$ and $m_3 = 90k^2 + 54k + 8$.*

($\overline{7}^0$) *G is a strongly regular graph of order $n = 9(10k+3)^2$ and degree $r = 8(90k^2 + 54k+8)$ with $\tau = 4(4k+1)(36k+13)$ and $\theta = 4(4k+1)(36k+11)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 36k+10$ and $\lambda_3 = -(4k+2)$ with $m_2 = 90k^2+54k+8$ and $m_3 = 9(90k^2 + 54k + 8)$.*

(8^0) *G is a strongly regular graph of order $n = 9(10k+3)^2$ and degree $r = 5(90k^2 + 54k+8)$ with $\tau = 225k^2+115k+13$ and $\theta = (5k+2)(45k+13)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 5k+1$ and $\lambda_3 = -(45k+14)$ with $m_2 = 9(90k^2+54k+8)$ and $m_3 = 90k^2 + 54k + 8$.*

($\overline{8}^0$) *G is a strongly regular graph of order $n = 9(10k+3)^2$ and degree $r = 5(90k^2 + 54k+8)$ with $\tau = 225k^2 + 155k + 25$ and $\theta = (5k+1)(45k+14)$, where $k \geq 0$.*

Its eigenvalues are $\lambda_2 = 45k+13$ *and* $\lambda_3 = -(5k+2)$ *with* $m_2 = 90k^2+54k+8$ *and* $m_3 = 9(90k^2+54k+8)$.

(9^0) *G is a strongly regular graph of order* $n = 21(10k-1)^2$ *and degree* $r = 3(210k^2-42k+2)$ *with* $\tau = 189k^2-77k+5$ *and* $\theta = 21k(9k-1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 7k-1$ *and* $\lambda_3 = -(63k-6)$ *with* $m_2 = 9(210k^2-42k+2)$ *and* $m_3 = 210k^2-42k+2$.

($\overline{9}^0$) *G is a strongly regular graph of order* $n = 21(10k-1)^2$ *and degree* $r = 7(210k^2-42k+2)$ *with* $\tau = 7(147k^2-27k+1)$ *and* $\theta = 7(7k-1)(21k-2)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 63k-7$ *and* $\lambda_3 = -7k$ *with* $m_2 = 210k^2-42k+2$ *and* $m_3 = 9(210k^2-42k+2)$.

(10^0) *G is a strongly regular graph of order* $n = 21(10k+1)^2$ *and degree* $r = 3(210k^2+42k+2)$ *with* $\tau = 189k^2+77k+5$ *and* $\theta = 21k(9k+1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 63k+6$ *and* $\lambda_3 = -(7k+1)$ *with* $m_2 = 210k^2+42k+2$ *and* $m_3 = 9(210k^2+42k+2)$.

($\overline{10}^0$) *G is a strongly regular graph of order* $n = 21(10k+1)^2$ *and degree* $r = 7(210k^2+42k+2)$ *with* $\tau = 7(147k^2+27k+1)$ *and* $\theta = 7(7k+1)(21k+2)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 7k$ *and* $\lambda_3 = -(63k+7)$ *with* $m_2 = 9(210k^2+42k+2)$ *and* $m_3 = 210k^2+42k+2$.

Theorem 2.9 ([6]). *Let G be a connected strongly regular graph of order n and degree r with* $m_2 = 10m_3$ *or* $m_3 = 10m_2$. *Then, G is one of the following strongly regular graphs.*

(1^0) *G is the complete bipartite graph* $K_{6,6}$ *of order* $n = 12$ *and degree* $r = 6$ *with* $\tau = 0$ *and* $\theta = 6$. *Its eigenvalues are* $\lambda_2 = 0$ *and* $\lambda_3 = -6$ *with* $m_2 = 10$ *and* $m_3 = 1$.

(2^0) *G is the strongly regular graph* $\overline{5K_9}$ *of order* $n = 45$ *and degree* $r = 36$ *with* $\tau = 27$ *and* $\theta = 36$. *Its eigenvalues are* $\lambda_2 = 0$ *and* $\lambda_3 = -9$ *with* $m_2 = 40$ *and* $m_3 = 4$.

(3^0) *G is the strongly regular graph* $\overline{10K_{10}}$ *of order* $n = 100$ *and degree* $r = 90$ *with* $\tau = 80$ *and* $\theta = 90$. *Its eigenvalues are* $\lambda_2 = 0$ *and* $\lambda_3 = -10$ *with* $m_2 = 90$ *and* $m_3 = 9$.

(4^0) *G is a strongly regular graph of order* $n = (11k-1)^2$ *and degree* $r = k(11k-2)$ *with* $\tau = k^2+8k-1$ *and* $\theta = k(k-1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = 10k-1$ *and* $\lambda_3 = -k$ *with* $m_2 = k(11k-2)$ *and* $m_3 = 10k(11k-2)$.

($\overline{4}^0$) *G is a strongly regular graph of order* $n = (11k-1)^2$ *and degree* $r = 10k(11k-2)$ *with* $\tau = 100k^2-19k-1$ *and* $\theta = 10k(10k-1)$, *where* $k \in \mathbb{N}$. *Its eigenvalues are* $\lambda_2 = k-1$ *and* $\lambda_3 = -10k$ *with* $m_2 = 10k(11k-2)$ *and* $m_3 = k(11k-2)$.

(5^0) *G is a strongly regular graph of order* $n = (11k+1)^2$ *and degree* $r = k(11k+2)$ *with* $\tau = k^2-8k-1$ *and* $\theta = k(k+1)$, *where* $k \geq 9$. *Its eigenvalues are* $\lambda_2 = k$ *and* $\lambda_3 = -(10k+1)$ *with* $m_2 = 10k(11k+2)$ *and* $m_3 = k(11k+2)$.

($\overline{5}^0$) *G is a strongly regular graph of order $n = (11k+1)^2$ and degree $r = 10k(11k+2)$ with $\tau = 100k^2 + 19k - 1$ and $\theta = 10k(10k + 1)$, where $k \geq 9$. Its eigenvalues are $\lambda_2 = 10k$ and $\lambda_3 = -(k+1)$ with $m_2 = k(11k+2)$ and $m_3 = 10k(11k+2)$.*

(6^0) *G is a strongly regular graph of order $n = 3(11k-2)^2$ and degree $r = 5(33k^2 - 12k + 1)$ with $\tau = 75k^2 - 42k + 4$ and $\theta = 15k(5k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k-1$ and $\lambda_3 = -(30k-5)$ with $m_2 = 10(33k^2 - 12k+1)$ and $m_3 = 33k^2 - 12k + 1$.*

($\overline{6}^0$) *G is a strongly regular graph of order $n = 3(11k-2)^2$ and degree $r = 6(33k^2 - 12k + 1)$ with $\tau = 27k(4k - 1)$ and $\theta = 6(3k - 1)(6k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 30k - 6$ and $\lambda_3 = -3k$ with $m_2 = 33k^2 - 12k + 1$ and $m_3 = 10(33k^2 - 12k + 1)$.*

(7^0) *G is a strongly regular graph of order $n = 3(11k+2)^2$ and degree $r = 5(33k^2 + 12k + 1)$ with $\tau = 75k^2 + 42k + 4$ and $\theta = 15k(5k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 30k + 5$ and $\lambda_3 = -(3k + 1)$ with $m_2 = 33k^2 + 12k + 1$ and $m_3 = 10(33k^2 + 12k + 1)$.*

($\overline{7}^0$) *G is a strongly regular graph of order $n = 3(11k+2)^2$ and degree $r = 6(33k^2 + 12k + 1)$ with $\tau = 27k(4k + 1)$ and $\theta = 6(3k + 1)(6k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k$ and $\lambda_3 = -(30k + 6)$ with $m_2 = 10(33k^2 + 12k + 1)$ and $m_3 = 33k^2 + 12k + 1$.*

(8^0) *G is a strongly regular graph of order $n = 5(11k-3)^2$ and degree $r = 2(55k^2 - 30k + 4)$ with $\tau = 20k^2 - 33k + 7$ and $\theta = 2k(10k - 3)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 3k-1$ and $\lambda_3 = -(30k-8)$ with $m_2 = 10(55k^2 - 30k+4)$ and $m_3 = 55k^2 - 30k + 4$.*

($\overline{8}^0$) *G is a strongly regular graph of order $n = 5(11k-3)^2$ and degree $r = 9(55k^2 - 30k + 4)$ with $\tau = 27(3k - 1)(5k - 1)$ and $\theta = 9(3k - 1)(15k - 4)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 30k - 9$ and $\lambda_3 = -3k$ with $m_2 = 55k^2 - 30k + 4$ and $m_3 = 10(55k^2 - 30k + 4)$.*

(9^0) *G is a strongly regular graph of order $n = 5(11k+3)^2$ and degree $r = 2(55k^2 + 30k + 4)$ with $\tau = 20k^2 + 33k + 7$ and $\theta = 2k(10k + 3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 30k + 8$ and $\lambda_3 = -(3k + 1)$ with $m_2 = 55k^2 + 30k + 4$ and $m_3 = 10(55k^2 + 30k + 4)$.*

($\overline{9}^0$) *G is a strongly regular graph of order $n = 5(11k+3)^2$ and degree $r = 9(55k^2 + 30k + 4)$ with $\tau = 27(3k+1)(5k+1)$ and $\theta = 9(3k+1)(15k+4)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k$ and $\lambda_3 = -(30k+9)$ with $m_2 = 10(55k^2 + 30k+4)$ and $m_3 = 55k^2 + 30k + 4$.*

(10^0) *G is a strongly regular graph of order $n = 15(11k - 5)^2$ and degree $r = 3(165k^2 - 150k + 34)$ with $\tau = 3(45k^2 - 54k + 15)$ and $\theta = 3(3k - 1)(15k - 7)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 6k - 3$ and $\lambda_3 = -(60k - 27)$ with $m_2 = 10(165k^2 - 150k + 34)$ and $m_3 = 165k^2 - 150k + 34$.*

($\overline{10}^0$) *G is a strongly regular graph of order $n = 15(11k - 5)^2$ and degree $r = 8(165k^2 - 150k+34)$ with $\tau = 2(480k^2 - 429k+95)$ and $\theta = 24(2k-1)(20k-9)$,* *where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 60k - 28$ and $\lambda_3 = -(6k-2)$ with $m_2 = 165k^2 - 150k + 34$ and $m_3 = 10(165k^2 - 150k + 34)$.*

(11^0) *G is a strongly regular graph of order $n = 15(11k+5)^2$ and degree $r = 3(165k^2 + 150k + 34)$ with $\tau = 3(45k^2 + 54k + 15)$ and $\theta = 3(3k+1)(15k+7)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 60k + 27$ and $\lambda_3 = -(6k+3)$ with $m_2 = 165k^2 + 150k + 34$ and $m_3 = 10(165k^2 + 150k + 34)$.*

($\overline{11}^0$) *G is a strongly regular graph of order $n = 15(11k+5)^2$ and degree $r = 8(165k^2+150k+34)$ with $\tau = 2(480k^2+429k+95)$ and $\theta = 24(2k+1)(20k+9)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 6k+2$ and $\lambda_3 = -(60k+28)$ with $m_2 = 10(165k^2 + 150k + 34)$ and $m_3 = 165k^2 + 150k + 34$.*

(12^0) *G is a strongly regular graph of order $n = 70(11k-5)^2$ and degree $r = 4(770k^2-700k+159)$ with $\tau = 2(560k^2-469k+97)$ and $\theta = 28(2k-1)(20k-9)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 140k - 64$ and $\lambda_3 = -(14k-6)$ with $m_2 = 770k^2 - 700k + 159$ and $m_3 = 10(770k^2 - 700k + 159)$.*

($\overline{12}^0$) *G is a strongly regular graph of order $n = 70(11k-5)^2$ and degree $r = 7(770k^2 - 700k + 159)$ with $\tau = 14(245k^2 - 226k + 52)$ and $\theta = 14(7k-3)(35k-16)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 14k - 7$ and $\lambda_3 = -(140k-63)$ with $m_2 = 10(770k^2 - 700k + 159)$ and $m_3 = 770k^2 - 700k + 159$.*

(13^0) *G is a strongly regular graph of order $n = 70(11k+5)^2$ and degree $r = 4(770k^2+700k+159)$ with $\tau = 2(560k^2+469k+97)$ and $\theta = 28(2k+1)(20k+9)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 14k + 6$ and $\lambda_3 = -(140k+64)$ with $m_2 = 10(770k^2 + 700k + 159)$ and $m_3 = 770k^2 + 700k + 159$.*

($\overline{13}^0$) *G is a strongly regular graph of order $n = 70(11k+5)^2$ and degree $r = 7(770k^2 + 700k + 159)$ with $\tau = 14(245k^2 + 226k + 52)$ and $\theta = 14(7k+3)(35k+16)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 140k + 63$ and $\lambda_3 = -(14k+7)$ with $m_2 = 770k^2 + 700k + 159$ and $m_3 = 10(770k^2 + 700k + 159)$.*

Theorem 2.10 ([7])**.** *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 11m_3$ or $m_3 = 11m_2$. Then, G is one of the following strongly regular graphs.*

(1^0) *G is the strongly regular graph $\overline{11K_{11}}$ of order $n = 121$ and degree $r = 110$ with $\tau = 99$ and $\theta = 110$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -11$ with $m_2 = 110$ and $m_3 = 10$.*

(2^0) *G is a strongly regular graph of order $n = (12k-1)^2$ and degree $r = 2k(6k-1)$ with $\tau = k^2 + 9k - 1$ and $\theta = k(k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 11k - 1$ and $\lambda_3 = -k$ with $m_2 = 2k(6k-1)$ and $m_3 = 22k(6k-1)$.*

($\overline{2}^0$) *G is a strongly regular graph of order $n = (12k-1)^2$ and degree $r = 22k(6k-1)$ with $\tau = 121k^2 - 21k - 1$ and $\theta = 11k(11k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -11k$ with $m_2 = 22k(6k-1)$ and $m_3 = 2k(6k-1)$.*

(3^0) *G is a strongly regular graph of order $n = (12k+1)^2$ and degree $r = 2k(6k+1)$ with $\tau = k^2 - 9k - 1$ and $\theta = k(k+1)$, where $k \geq 10$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(11k+1)$ with $m_2 = 22k(6k+1)$ and $m_3 = 2k(6k+1)$.*

($\overline{3}^0$) *G is a strongly regular graph of order $n = (12k+1)^2$ and degree $r = 22k(6k+1)$ with $\tau = 121k^2 + 21k - 1$ and $\theta = 11k(11k+1)$, where $k \geq 10$. Its eigenvalues are $\lambda_2 = 11k$ and $\lambda_3 = -(k+1)$ with $m_2 = 2k(6k+1)$ and $m_3 = 22k(6k+1)$.*

(4^0) *G is a strongly regular graph of order $n = 385(12k-5)^2$ and degree $r = 5(4620k^2 - 3850k + 802)$ with $\tau = 5(1925k^2 - 1645k + 351)$ and $\theta = 35(5k-2)(55k-23)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 35k-15$ and $\lambda_3 = -(385k-160)$ with $m_2 = 11(4620k^2 - 3850k + 802)$ and $m_3 = 4620k^2 - 3850k + 802$.*

($\overline{4}^0$) *G is a strongly regular graph of order $n = 385(12k-5)^2$ and degree $r = 7(4620k^2 - 3850k + 802)$ with $\tau = 7(2695k^2 - 2225k + 459)$ and $\theta = 35(7k-3)(77k-32)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 385k - 161$ and $\lambda_3 = -(35k-14)$ with $m_2 = 4620k^2-3850k+802$ and $m_3 = 11(4620k^2-3850k+802)$.*

(5^0) *G is a strongly regular graph of order $n = 385(12k+5)^2$ and degree $r = 5(4620k^2 + 3850k + 802)$ with $\tau = 5(1925k^2 + 1645k + 351)$ and $\theta = 35(5k+2)(55k+23)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 385k + 160$ and $\lambda_3 = -(35k+15)$ with $m_2 = 4620k^2+3850k+802$ and $m_3 = 11(4620k^2+3850k+802)$.*

($\overline{5}^0$) *G is a strongly regular graph of order $n = 385(12k+5)^2$ and degree $r = 7(4620k^2 + 3850k + 802)$ with $\tau = 7(2695k^2 + 2225k + 459)$ and $\theta = 35(7k+3)(77k+32)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 35k+14$ and $\lambda_3 = -(385k+161)$ with $m_2 = 11(4620k^2 + 3850k + 802)$ and $m_3 = 4620k^2 + 3850k + 802$.*

Theorem 2.11 ([7])**.** *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 12m_3$ or $m_3 = 12m_2$. Then, G is one of the following strongly regular graphs.*

(1^0) *G is the complete bipartite graph $K_{7,7}$ of order $n = 14$ and degree $r = 7$ with $\tau = 0$ and $\theta = 7$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -7$ with $m_2 = 12$ and $m_3 = 1$.*

(2^0) *G is the strongly regular graph $\overline{3K_9}$ of order $n = 27$ and degree $r = 18$ with $\tau = 9$ and $\theta = 18$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -9$ with $m_2 = 24$ and $m_3 = 2$.*

(3^0) *G is the strongly regular graph $\overline{4K_{10}}$ of order $n = 40$ and degree $r = 30$ with $\tau = 20$ and $\theta = 30$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -10$ with $m_2 = 36$ and $m_3 = 3$.*

(4^0) *G is the strongly regular graph $\overline{6K_{11}}$ of order $n = 66$ and degree $r = 55$ with $\tau = 44$ and $\theta = 55$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -11$ with $m_2 = 60$ and $m_3 = 5$.*

(5^0) *G is the strongly regular graph $\overline{12K_{12}}$ of order $n = 144$ and degree $r = 132$ with $\tau = 120$ and $\theta = 132$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -12$ with $m_2 = 132$ and $m_3 = 11$.*

(6^0) *G is a strongly regular graph of order $n = (13k-1)^2$ and degree $r = k(13k-2)$ with $\tau = k^2 + 10k - 1$ and $\theta = k(k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 12k - 1$ and $\lambda_3 = -k$ with $m_2 = k(13k-2)$ and $m_3 = 12k(13k-2)$.*

($\overline{6}^0$) *G is a strongly regular graph of order* $n = (13k-1)^2$ *and degree* $r = 12k(13k-2)$ *with* $\tau = 144k^2 - 23k - 1$ *and* $\theta = 12k(12k-1)$*, where* $k \geq 2$*. Its eigenvalues are* $\lambda_2 = k - 1$ *and* $\lambda_3 = -12k$ *with* $m_2 = 12k(13k-2)$ *and* $m_3 = k(13k-2)$*.*

(7^0) *G is a strongly regular graph of order* $n = (13k+1)^2$ *and degree* $r = k(13k+2)$ *with* $\tau = k^2 - 10k - 1$ *and* $\theta = k(k+1)$*, where* $k \geq 11$*. Its eigenvalues are* $\lambda_2 = k$ *and* $\lambda_3 = -(12k+1)$ *with* $m_2 = 12k(13k+2)$ *and* $m_3 = k(13k+2)$*.*

($\overline{7}^0$) *G is a strongly regular graph of order* $n = (13k+1)^2$ *and degree* $r = 12k(13k+2)$ *with* $\tau = 144k^2 + 23k - 1$ *and* $\theta = 12k(12k+1)$*, where* $k \geq 11$*. Its eigenvalues are* $\lambda_2 = 12k$ *and* $\lambda_3 = -(k+1)$ *with* $m_2 = k(13k+2)$ *and* $m_3 = 12k(13k+2)$*.*

(8^0) *G is a strongly regular graph of order* $n = 3(13k-3)^2$ *and degree* $r = 4(39k^2 - 18k + 2)$ *with* $\tau = 48k^2 - 45k + 7$ *and* $\theta = 12k(4k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 3k-1$ *and* $\lambda_3 = -(36k-8)$ *with* $m_2 = 12(39k^2 - 18k + 2)$ *and* $m_3 = 39k^2 - 18k + 2$*.*

($\overline{8}^0$) *G is a strongly regular graph of order* $n = 3(13k-3)^2$ *and degree* $r = 9(39k^2 - 18k + 2)$ *with* $\tau = 3(81k^2 - 34k + 3)$ *and* $\theta = 9(3k-1)(9k-2)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 36k - 9$ *and* $\lambda_3 = -3k$ *with* $m_2 = 39k^2 - 18k + 2$ *and* $m_3 = 12(39k^2 - 18k + 2)$*.*

(9^0) *G is a strongly regular graph of order* $n = 3(13k+3)^2$ *and degree* $r = 4(39k^2 + 18k + 2)$ *with* $\tau = 48k^2 + 45k + 7$ *and* $\theta = 12k(4k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 36k + 8$ *and* $\lambda_3 = -(3k+1)$ *with* $m_2 = 39k^2 + 18k + 2$ *and* $m_3 = 12(39k^2 + 18k + 2)$*.*

($\overline{9}^0$) *G is a strongly regular graph of order* $n = 3(13k+3)^2$ *and degree* $r = 9(39k^2 + 18k + 2)$ *with* $\tau = 3(81k^2 + 34k + 3)$ *and* $\theta = 9(3k+1)(9k+2)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 3k$ *and* $\lambda_3 = -(36k+9)$ *with* $m_2 = 12(39k^2 + 18k + 2)$ *and* $m_3 = 39k^2 + 18k + 2$*.*

(10^0) *G is a strongly regular graph of order* $n = 10(13k-2)^2$ *and degree* $r = 3(130k^2 - 40k + 3)$ *with* $\tau = 90k^2 - 70k + 8$ *and* $\theta = 15k(6k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 5k-1$ *and* $\lambda_3 = -(60k-9)$ *with* $m_2 = 12(130k^2 - 40k + 3)$ *and* $m_3 = 130k^2 - 40k + 3$*.*

($\overline{10}^0$) *G is a strongly regular graph of order* $n = 10(13k-2)^2$ *and degree* $r = 10(130k^2 - 40k + 3)$ *with* $\tau = 5(200k^2 - 59k + 4)$ *and* $\theta = 10(5k-1)(20k-3)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 60k - 10$ *and* $\lambda_3 = -5k$ *with* $m_2 = 130k^2 - 40k + 3$ *and* $m_3 = 12(130k^2 - 40k + 3)$*.*

(11^0) *G is a strongly regular graph of order* $n = 10(13k+2)^2$ *and degree* $r = 3(130k^2 + 40k + 3)$ *with* $\tau = 90k^2 + 70k + 8$ *and* $\theta = 15k(6k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 60k + 9$ *and* $\lambda_3 = -(5k+1)$ *with* $m_2 = 130k^2 + 40k + 3$ *and* $m_3 = 12(130k^2 + 40k + 3)$*.*

($\overline{11}^0$) *G is a strongly regular graph of order* $n = 10(13k+2)^2$ *and degree* $r = 10(130k^2 + 40k + 3)$ *with* $\tau = 5(200k^2 + 59k + 4)$ *and* $\theta = 10(5k+1)(20k+3)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 5k$ *and* $\lambda_3 = -(60k+10)$ *with* $m_2 = 12(130k^2 + 40k + 3)$ *and* $m_3 = 130k^2 + 40k + 3$*.*

(12^0) *G is a strongly regular graph of order* $n = 14(13k-1)^2$ *and degree* $r = 6(182k^2 - 28k + 1)$ *with* $\tau = 504k^2 - 119k + 5$ *and* $\theta = 42k(12k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 7k-1$ *and* $\lambda_3 = -(84k-6)$ *with* $m_2 = 12(182k^2-28k+1)$ *and* $m_3 = 182k^2 - 28k + 1$.

($\overline{12}^0$) *G is a strongly regular graph of order* $n = 14(13k-1)^2$ *and degree* $r = 7(182k^2 - 28k + 1)$ *with* $\tau = 14k(49k-5)$ *and* $\theta = 7(7k-1)(14k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 84k-7$ *and* $\lambda_3 = -7k$ *with* $m_2 = 182k^2-28k+1$ *and* $m_3 = 12(182k^2 - 28k + 1)$.

(13^0) *G is a strongly regular graph of order* $n = 14(13k+1)^2$ *and degree* $r = 6(182k^2+28k+1)$ *with* $\tau = 504k^2+119k+5$ *and* $\theta = 42k(12k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 84k+6$ *and* $\lambda_3 = -(7k+1)$ *with* $m_2 = 182k^2+28k+1$ *and* $m_3 = 12(182k^2 + 28k + 1)$.

($\overline{13}^0$) *G is a strongly regular graph of order* $n = 14(13k+1)^2$ *and degree* $r = 7(182k^2+28k+1)$ *with* $\tau = 14k(49k+5)$ *and* $\theta = 7(7k+1)(14k+1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 7k$ *and* $\lambda_3 = -(84k+7)$ *with* $m_2 = 12(182k^2+28k+1)$ *and* $m_3 = 182k^2 + 28k + 1$.

(14^0) *G is a strongly regular graph of order* $n = 30(13k-6)^2$ *and degree* $r = 5(390k^2-360k+83)$ *with* $\tau = 10(75k^2-76k+19)$ *and* $\theta = 10(5k-2)(15k-7)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 10k-5$ *and* $\lambda_3 = -(120k-55)$ *with* $m_2 = 12(390k^2 - 360k + 83)$ *and* $m_3 = 390k^2 - 360k + 83$.

($\overline{14}^0$) *G is a strongly regular graph of order* $n = 30(13k-6)^2$ *and degree* $r = 8(390k^2-360k+83)$ *with* $\tau = 2(960k^2-865k+194)$ *and* $\theta = 40(2k-1)(24k-11)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 120k-56$ *and* $\lambda_3 = -(10k-4)$ *with* $m_2 = 390k^2 - 360k + 83$ *and* $m_3 = 12(390k^2 - 360k + 83)$.

(15^0) *G is a strongly regular graph of order* $n = 30(13k+6)^2$ *and degree* $r = 5(390k^2+360k+83)$ *with* $\tau = 10(75k^2+76k+19)$ *and* $\theta = 10(5k+2)(15k+7)$*, where* $k \geq 0$*. Its eigenvalues are* $\lambda_2 = 120k+55$ *and* $\lambda_3 = -(10k+5)$ *with* $m_2 = 390k^2 + 360k + 83$ *and* $m_3 = 12(390k^2 + 360k + 83)$.

($\overline{15}^0$) *G is a strongly regular graph of order* $n = 30(13k+6)^2$ *and degree* $r = 8(390k^2+360k+83)$ *with* $\tau = 2(960k^2+865k+194)$ *and* $\theta = 40(2k+1)(24k+11)$*, where* $k \geq 0$*. Its eigenvalues are* $\lambda_2 = 10k+4$ *and* $\lambda_3 = -(120k+56)$ *with* $m_2 = 12(390k^2 + 360k + 83)$ *and* $m_3 = 390k^2 + 360k + 83$.

(16^0) *G is a strongly regular graph of order* $n = 66(13k-1)^2$ *and degree* $r = 2(858k^2 - 132k + 5)$ *with* $\tau = 264k^2 - 143k + 9$ *and* $\theta = 22k(12k-1)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 11k-1$ *and* $\lambda_3 = -(132k-10)$ *with* $m_2 = 12(858k^2 - 132k + 5)$ *and* $m_3 = 858k^2 - 132k + 5$.

($\overline{16}^0$) *G is a strongly regular graph of order* $n = 66(13k-1)^2$ *and degree* $r = 11(858k^2-132k+5)$ *with* $\tau = 22(11k-1)(33k-2)$ *and* $\theta = 11(11k-1)(66k-5)$*, where* $k \in \mathbb{N}$*. Its eigenvalues are* $\lambda_2 = 132k-11$ *and* $\lambda_3 = -11k$ *with* $m_2 = 858k^2 - 132k + 5$ *and* $m_3 = 12(858k^2 - 132k + 5)$.

(17^0) *G is a strongly regular graph of order $n = 66(13k+1)^2$ and degree $r = 2(858k^2+132k+5)$ with $\tau = 264k^2+143k+9$ and $\theta = 22k(12k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 132k+10$ and $\lambda_3 = -(11k+1)$ with $m_2 = 858k^2+132k+5$ and $m_3 = 12(858k^2+132k+5)$.*

($\overline{17}^0$) *G is a strongly regular graph of order $n = 66(13k+1)^2$ and degree $r = 11(858k^2+132k+5)$ with $\tau = 22(11k+1)(33k+2)$ and $\theta = 11(11k+1)(66k+5)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 11k$ and $\lambda_3 = -(132k+11)$ with $m_2 = 12(858k^2+132k+5)$ and $m_3 = 858k^2+132k+5$.*

3. Concluding Remarks

Using equations (1.3) and (1.4), and applying the same procedure as in articles [4–7], we can establish the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for any fixed value $q \in \mathbb{N}$, by using only one parameter k. All results presented in this work has been verified using a computer program `srgpar.exe`, which was written by the author in the programming language `Borland C++ Builder 5.5`. using only one parameter k.

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