Kragujevac J. Math. 30 (2007) 243–252.

INTUITIONISTIC FUZZY STRONGLY IRRESOLUTE PRECONTINUOUS MAPPINGS IN COKER'S SPACES

Biljana Krsteska [1] and Salah Abbas[2]

[1] *"St. Cyril and Methodius University, Faculty of Natural Sciences and Mathematics, Skopje, Macedonia*
(e-mail: biljanak@iunona.pmf.ukim.edu.mk)

[2] *Department of Mathematics, Faculty of Sciences, South Valley University, 82524 Sohag, Egypt*
(e-mail: sabbas73@yahoo.com)

(Received November 06, 2006)

Abstract. Intuitionistic fuzzy strongly irresolute precontinuous mappings between intuitionistic fuzzy topological spaces are introduced. Some of their properties are studied.

1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh in his classic paper [10]. Using the concept of fuzzy sets Chang [2] introduced the fuzzy topological spaces. Since Atanassov [1] introduced the notion of intuitionistic fuzzy sets, Coker [3] defined the intuitionistic fuzzy topological spaces. This approach provided a wide field for investigation in the area of fuzzy topology and its applications. One of directions is related to the properties of intuitionistic fuzzy sets introduced by Coker [4] and Joen

[5]. Also, Joen [6] introduced the concepts of intuitionistic fuzzy α-continuity and intuitionistic fuzzy precontinuity.

Continuing the work in [8,9], as an extension of concept presented in [6,7], we will introduce intuitionistic fuzzy strongly irresolute precontinuous mappings between intuitionistic fuzzy topological spaces in Coker's sense. Some of their properties are studied. We will establish their properties and relationships with other classes of early defined forms of intuitionistic fuzzy continuous mappings.

2. PRELIMINARIES

We introduce some basic notions and results that are used in the sequel.

Definition 2.1. [1] *Let X be a nonempty fixed set and I the closed interval $[0,1]$. An intuitionistic fuzzy set (IFS) A is an object of the following form*

$$A = \{< x, \mu_A(x), \nu_A(x) > \mid x \in X\}$$

where the mappings $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (namely) $\mu_A(x)$) and the degree of nonmembership (namely $\nu_A(x)$) for each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form

$$A = \{< x, \mu_A(x), 1 - \mu_A(x) > \mid x \in X\}.$$

Definition 2.2. [1] *Let A and B are IFSs of the form*
$A = \{< x, \mu_A(x), \nu_A(x) > \mid x \in X\}$ *and* $B = \{< x, \mu_B(x), \nu_B(x) > \mid x \in X\}$. *Then*

(i) $A \subseteq B$ *if and only if* $\mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$;
(ii) $\overline{A} = \{< x, \nu_A(x), \mu_A(x) > \mid x \in X\}$;
(iii) $A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) > \mid x \in X\}$;
(iv) $A \cup B = \{< x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) > \mid x \in X\}$.

We will use the notation $A =< x, \mu_A, \nu_A >$ instead of
$A = \{< x, \mu_A(x), \nu_A(x) > |x \in X\}$. A constant fuzzy set α taking value $\alpha \in [0,1]$

will be denote by $\underline{\alpha}$. The IFSs $0_\sim$ and $1_\sim$ are defined by $0_\sim = \{< x, \underline{0}, \underline{1} > \mid x \in X\}$ and $1_\sim = \{< x, \underline{1}, \underline{0} > \mid x \in X\}$.

Let $\alpha, \beta \in [0,1]$ such that $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP) $p_{(\alpha,\beta)}$ is intuitionistic fuzzy set defined by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha,\beta) & x = p \\ (0,1) & \text{otherwise} \end{cases}$$

Let f be a mapping from an ordinary set X into an ordinary set Y. If $B = \{< y, \mu_B(y), \nu_B(y) > \mid y \in Y\}$ is an IFS in Y, then the inverse image of B under f is an IFS defined by

$$f^{-1}(B) = \{< (x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) > \mid x \in X\}$$

The image of IFS $A = \{< y, \mu_A(y), \nu_A(y) > \mid y \in Y\}$ under f is an IFS defined by $f(A) = \{< y, f(\mu_A)(y), f(\nu_A)(y) > \mid y \in Y\}$

where

$$f(\mu_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & f^{-1}(y) \neq 0, \\ 0 & \text{otherwise} \end{cases}$$

and

$$f(\nu_A)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \nu_A(x) & f^{-1}(y) \neq 0, \\ 1 & \text{otherwise} \end{cases}$$

for each $y \in Y$

Definition 2.3. [3] *An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:*

(T_1) $0_\sim, 1_\sim \in \tau$;

(T_2) $G \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;

(T_3) $\cup G_i \in \tau$ for any arbitrary family $\{G_i \mid i \in J\} \subseteq \tau$.

In this paper by (X, τ) or simply by X we will denote the Coker's intuitionistic fuzzy topological space (IFTS). Each IFS which belongs to τ is called an intuitionistic fuzzy open set (IFOS) in X. The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.4. [3] *Let A be an IFS in IFTS X. Then*

$\text{int}A = \cup\{G \,|\, G \textit{ is an } IFOS \textit{ in } X \textit{ and } G \subseteq A\}$ *is called an intuitionistic fuzzy interior of A;*

$\text{cl}A = \cap\{G \,|\, G \textit{ is an } IFCS \textit{ in } X \textit{ and } G \supseteq A\}$ *is called an intuitionistic fuzzy closure of* A.

Definition 2.5. [5] *An IFS A in an IFTS X is called an intuitionistic fuzzy preopen set (IFPOS) if* $A \subseteq \text{int}(\text{cl}A)$.

The complement $\overline{A}$ of an IFPOS A in IFTS X is called an intuitionistic fuzzy preclosed set (IFPCS) in X.

Definition 2.6. [8] *Let A be an IFS in IFTS* X. *Then*

$p\,\text{int}A = \cup\{G \mid G \textit{ is an } IFPOS \textit{ in } X \textit{ and } G \subseteq A\}$ *is called an intuitionistic fuzzy preinterior of A;*

$p\,\text{cl}A = \cap\{G \mid G \textit{ is an } IFPCS \textit{ in } X \textit{ and } G \supseteq A\}$ *is called an intuitionistic fuzzy preclosure of* A.

Definition 2.7. [8] *An IFS A in an IFTS X is called an intuitionistic fuzzy strongly preopen set (IFSPOS) if* $A \subseteq \text{int}(pclA)$.

The complement $\overline{A}$ of an IFSPOS A in IFTS X is called an intuitionistic fuzzy strongly preclosed set (IFSPCS) in X.

Definition 2.8. [8] *Let A be an IFS in IFTS* X. *Then*

$\text{spint}A = \cup\{G \mid G \textit{ is an } IFSPOS \textit{ in } X \textit{ and } G \subseteq A\}$ *is called an intuitionistic fuzzy strongly preinterior of A;*

$\text{spcl}A = \cap\{G \mid G \textit{ is an } IFSPCS \textit{ in } X \textit{ and } G \supseteq A\}$ *is called an intuitionistic fuzzy strongly preclosure of* A.

Theorem 2.1. [3, 8] *Let A be an IFS of IFTS X. Then*

(i) $\mathrm{cl}\overline{A} = \overline{\mathrm{int}A}$;	(ii) $\mathrm{int}\overline{A} = \overline{\mathrm{cl}A}$;
(iii) $\mathrm{pcl}\overline{A} = \overline{\mathrm{p\,int}A}$;	(iv) $\mathrm{p\,int}\overline{A} = \overline{\mathrm{pcl}A}$
(v) $\mathrm{spcl}\overline{A} = \overline{\mathrm{sp\,int}A}$;	(vi) $\mathrm{sp\,int}\overline{A} = \overline{\mathrm{spcl}A}$

Definition 2.9. [5, 8, 9] *Let $f : X \mapsto Y$ be a mapping from an IFTS X into an IFTS Y. The mapping f is called*

(i) intuitionistic fuzzy continuous if $f^{-1}(B)$ is an IFOS in X, for each IFOS B in Y.

(ii) intuitionistic fuzzy strongly precontinuous if $f^{-1}(B)$ is an IFSPOS in X, for each IFOS B in Y;

(iii) intuitionistic fuzzy strongly preopen (preclosed) mapping if $f(A)$ is an IFSPOS (IFSPCS) in Y, for each IFOS (IFCS) A in X.

Definition 2.10. [5] *Let $p_{(\alpha,\beta)}$ be an IFP in IFTS X. An IFS A in X is called an intuitionistic fuzzy neighborhood (IFN) of $p_{(\alpha,\beta)}$ if there exists an IFOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.*

3. INTUITIONISTIC FUZZY STRONGLY IRRESOLUTE PRECONTINUOUS MAPPINGS

Definition 3.1. *A mapping $f : X \mapsto Y$ from an IFTS X into an IFTS Y is called intuitionistic fuzzy strongly irresolute precontinuous if $f^{-1}(B)$ is an IFSPOS in X for each IFSPOS B in Y.*

Remark 3.1. If f is intuitionistic fuzzy strongly irresolute precontinuous, then f is intuitionistic fuzzy strongly precontinuous. The following example shows that the converse may not be true.

Example 3.1. Let $X = \{A, B, C\}$ and A, B and C are intuitionistic fuzzy sets defined by

$$A = \left\langle x, \left(\frac{a}{0,5}, \frac{b}{0,3}, \frac{c}{0,6}\right), \left(\frac{a}{0,5}, \frac{b}{0,7}, \frac{c}{0,4}\right)\right\rangle$$

$$B = \left\langle x, \left(\frac{a}{0,2}, \frac{b}{0,4}, \frac{c}{0,3}\right), \left(\frac{a}{0,7}, \frac{b}{0,6}, \frac{c}{0,7}\right)\right\rangle$$

$$C = \left\langle x, \left(\frac{a}{0,5}, \frac{b}{0,4}, \frac{c}{0,6}\right), \left(\frac{a}{0,5}, \frac{b}{0,6}, \frac{c}{0,4}\right)\right\rangle$$

Let $\tau_1 = \{0_\sim, A, B, A \cap B, A \cup B, 1_\sim\}$, $\tau_2 = \{0_\sim, C, 1_\sim\}$ and $f = \mathrm{id} : (X, \tau_1) \to (X, \tau_2)$. Then f is intuitionistic fuzzy strongly precontinuous, but f is not intuitionistic fuzzy strongly irresolute precontinuous.

Theorem 3.1. *Let $f : X \mapsto Y$ be a mapping from an IFTS X into an IFTS Y. Then the following statements are equivalent:*

(i) f is an intuitionistic fuzzy strongly irresolute precontinuous mapping;

(ii) $f^{-1}(B)$ is an IFSPCS in X for each IFSPCS B in Y;

(iii) $spclf^{-1}(B) \subseteq f^{-1}(spclB)$ for each IFS B of Y;

(iv) $f^{-1}(spintB) \subseteq spintf^{-1}(B)$for each IFS B of Y.

Proof. (i) $\Rightarrow$ (ii) It can be proved by using the complement and the Definition 3.1.

(ii) $\Rightarrow$ (iii) Let B be any IFS in Y. From $B \subseteq spclB$ follows that $f^{-1}(B) \subseteq f^{-1}(spclB)$. Since $\mathrm{spcl}B$ is an IFSPCS in Y, according to the assumption we have that $f^{-1}(spclB)$ is an IFSPC in X. Therefore $\mathrm{spcl}f^{-1}(B) \subseteq f^{-1}(spclB)$.

(iii) $\Rightarrow$ (iv) It can be proved by using the complement.

(iv) $\Rightarrow$ (i) Let B be any IFSPOS in Y. Then $\mathrm{spint}B = B$. According to the assumption we have $f^{-1}(B) = f^{-1}(\mathrm{spint}B) \subseteq \mathrm{spint}f^{-1}(B)$, so $f^{-1}(B)$ is an IFSPOS in X. Hence f is an intuitionistic fuzzy strongly irresolute precontinuous mapping. $\square$

Theorem 3.2. *Let $f : X \mapsto Y$ be a mapping from an IFTS X into an IFTS Y. Then the following statements are equivalent:*

(i) f is an intuitionistic fuzzy strongly irresolute precontinuous mapping.

(ii) $cl(\mathrm{pint} f^{-1}(B)) \subseteq f^{-1}(spclB)$, for each IFCS B in Y;

(iii) $f^{-1}(\mathrm{spint}B) \subseteq \mathrm{int}(pclf^{-1}(B))$, for each IFSO B in Y;

(iv) $f(cl(\mathrm{pint}A)) \subseteq spclf(A)$, for each IFS A in X.

Proof. (i) $\Rightarrow$ (ii) Let B be any IFCS in Y. According to the assumption we have that $f^{-1}(spclB)$ is IFSPCS in X. Hence
$f^{-1}(spclB) \supseteq cl(\mathrm{pint} f^{-1}(spclB)) \supseteq cl(\mathrm{pint} f^{-1}(B))$.

(ii) $\Rightarrow$ (iii) It can be proved by using the complement.

(iii) $\Rightarrow$ (iv) Let A be any IFS in X. We put $B = f(A)$. Then $A \subseteq f^{-1}(B)$. According to the assumption we have

$$\overline{\mathrm{int}(pcl\overline{A}))} \subseteq \overline{\mathrm{int}(pclf^{-1}(\overline{B}))} \subseteq \overline{f^{-1}(\mathrm{spint}\overline{B}))}.$$

Thus

$$cl(\mathrm{pint}A) \subseteq cl(\mathrm{pint} f^{-1}(B)) \subseteq f^{-1}(spclB).$$

Hence

$$f(cl(\mathrm{pint}A)) \subseteq ff^{-1}(spclB) \subseteq spclB = spclf(A).$$

(iv) $\Rightarrow$ (i) Let B be any IFSPCS in Y. According to the assumption we obtain

$$f(cl(\mathrm{pint} f^{-1}(B)) \subseteq spclff^{-1}(B) \subseteq spclB = B.$$

Then

$$cl(\mathrm{pint} f^{-1}(B)) \subseteq f^{-1}f(cl(\mathrm{pint} f^{-1}(B)) \subseteq f^{-1}(B).$$

Thus $f^{-1}(B)$ is IFSPCS in X, so f is an intuitionistic fuzzy strongly irresolute precontinuous mapping. $\square$

Theorem 3.3. *Let $f : X \mapsto Y$ be an intuitionistic fuzzy strongly irresolute precontinuous mapping from an IFTS X into an IFTS Y. Then*
$f^{-1}(B) \subseteq \mathrm{spint} f^{-1}(\mathrm{int}(pclB))$, for each IFSPOS B in Y.

Proof. Let f be any intuitionistic fuzzy strongly irresolute precontinuous mapping, and B any IFSPOS in Y. Then $f^{-1}(B) \subseteq f^{-1}(\mathrm{int}(pclB))$. Since $f^{-1}(\mathrm{int}(pclB))$ is an IFSPOS in X it follows that $f^{-1}(B) \subseteq sp\mathrm{int} f^{-1}(\mathrm{int}(pclB))$. □

Theorem 3.4. *A mapping $f : X \mapsto Y$ from an IFTS X into an IFTS Y is intuitionistic fuzzy strongly irresolute precontinuous if and only if for each IFP $p_{(\alpha,\beta)}$ in X and IFSPOS B in Y such that $f(p_{(\alpha,\beta)})$ there exists an IFSPOS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.*

Proof. Let f be any intuitionistic fuzzy strongly irresolute precontinuous mapping, $p_{(\alpha,\beta)}$ an IFP in X and B any IFSPOS in Y such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(B) = sp\mathrm{int} f^{-1}(B)$. We put $A = sp\mathrm{int} f^{-1}(B)$. Then A is an IFSPOS in X which containing IFP $p_{(\alpha,\beta)}$ and $f(A) = f(sp\mathrm{int} f^{-1}(B)) \subseteq f f^{-1}(B) \subseteq B$.

Conversely, let B be any IFSPOS in Y and $p_{(\alpha,\beta)}$ IFP in X such that $p_{(\alpha,\beta)} \in f^{-1}(B)$. According to the assumption there exists IFSPOS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$. Therefore $p_{(\alpha,\beta)} \in A \subseteq f^{-1}(B)$ and $p_{(\alpha,\beta)} \in A = sp\mathrm{int}A \subseteq sp\mathrm{int} f^{-1}(B)$. Since $p_{(\alpha,\beta)}$ is an arbitrary IFP and $f^{-1}(B)$ is union of all IFP containing in $f^{-1}(B)$, we obtain that $f^{-1}(B) = sp\mathrm{int} f^{-1}(B)$, so f is an intuitionistic fuzzy strongly irresolute precontinuous mapping. □

Corrolary 3.5. *A mapping $f : X \mapsto Y$ from an IFTS X into an IFTS Y is intuitionistic fuzzy strongly irresolute precontinuous if and only if for each IFP $p_{(\alpha,\beta)}$ in X and IFSPOS B in Y such that $f(p_{(\alpha,\beta)}) \in B$ there exists an IFSPOS A in X such that $p_{(\alpha,\beta)} \in A$ and $A \subseteq f^{-1}(B)$.*

Theorem 3.6. *A mapping $f : X \mapsto Y$ from an IFTS X into an IFTS Y is intuitionistic fuzzy strongly preirresolute continuous if and only if for each IFP $p_{(\alpha,\beta)}$ in X and IFSPOS B in Y such that $f(p_{(\alpha,\beta)}) \in B$ $pclf^{-1}(B)$ is IFN of IFP $p_{(\alpha,\beta)}$ in X.*

Proof. Let f be any intuitionistic fuzzy strongly preirresolute continuous mapping, $p_{(\alpha,\beta)}$ an IFP in X and B any IFSPOS in Y such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(B) \subseteq \mathrm{int}(pclf^{-1}(B)) \subseteq pclf^{-1}(B)$, so $pclf^{-1}(B)$ is IFN of IFP $p_{(\alpha,\beta)}$ in X.

Conversely, let B be any IFSPOS in Y and $p_{(\alpha,\beta)}$ IFP in X such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(B)$. According to the assumption $pcl f^{-1}(B)$ is IFN of IFP $p_{(\alpha,\beta)}$ in X. Thus $p_{(\alpha,\beta)} \in \mathrm{int}(pcl f^{-1}(B))$, so $f^{-1}(B) \subseteq \mathrm{int}(pcl f^{-1}(B))$. Therefore f is intuitionistic fuzzy strongly irresolute precontinuous. $\square$

Theorem 3.7. *Let $f : X \mapsto Y$ and $g : Y \mapsto Z$ are mappings, where X, Y and Z are IFTS.*

(1) If f and g are intuitionistic fuzzy strongly irresolute precontinuous mappings, then gf is an intuitionistic fuzzy strongly irresolute precontinuous mapping.

(2) If f is an intuitionistic fuzzy strongly irresolute precontinuous mapping and g an is intuitionistic fuzzy strongly precontinuous mapping, then gf is an intuitionistic fuzzy strongly precontinuous mapping.

(3) If f is an intuitionistic fuzzy strongly irresolute precontinuous mapping and g is an intuitionistic fuzzy continuous mapping, then gf is an intuitionistic fuzzy strongly precontinuous mapping.

(4) If gf is an intuitionistic fuzzy strongly irresolute precontinuous mapping and g is an injective intuitionistic fuzzy strongly preopen (preclosed) mapping, then f is an intuitionistic fuzzy strongly precontinuous mapping.

(5) If gf is an intuitionistic fuzzy strongly preopen (preclosed) mapping and g is an injective intuitionistic fuzzy strongly irresolute precontinuous mapping, then f is an intuitionistic fuzzy strongly preopen (preclosed) mapping.

Proof. It follows from the relations $(gf)^{-1}(C) = f^{-1}(g^{-1}(C))$, for each IFS C in Z; $f^{-1}(B) = (gf)^{-1}g(B)$, for each IFS B in Y and g is injective; and $f(A) = g^{-1}(gf)(A)$, for each IFS A in X and g is injective. $\square$

Theorem 3.8. *Let X, X_1 and X_2 are IFTSs and $p_i : X_1 \times X_2 \to X_i$ $(i = 1, 2)$ are the projections of $X_1 \times X_2$ onto X_i. If $f : X \to X_1 \times X_2$ is intuitionistic fuzzy strongly irresolute precontinuous, then $p_i f$ are intuitionistic fuzzy strongly precontinuous mappings.*

Proof. It follows from the fact that p_i $(i = 1, 2)$ are intuitionistic fuzzy continuous mappings. $\square$

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