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A GEOMETRICAL DESCRIPTION OF VISUAL PERCEPTION

The Leuven Café Erasmus model and the Bristol Café Wall illusion

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Abstract. Intimite relations between the functioning of the natural world in general and of the human intelligence in particular have resulted in some mathematical models which give reasonable formal descriptions of some fundamental objects and processes occuring in nature as observed by human beings. These models constitute the essence of our general scientific knowledge and are illustrations of Feynman's saying that "*Nature speaks to us in the language of mathematics*".

And, in the words of Chern: "*While algebra and analysis provide the foundations of mathematics, geometry is at the core*". Geometry is the field of mathematics whose main source of intuition is human visual perception. So, it seems appropriate that geometry would contribute somewhat to a better understanding of visual perception. Paraphrasing Feynman, what follows may illustrate that "*Nature likes to be looked at with geometer's eyes and brains*".

Basically, a visual observation amounts to the *recording of light-energy* (further on called "luminosity"). In mathematical terms this is well described by *a surface* (further on called "visual-stimulus-surface"). Based on this visual information, our visual system (in the way this has been developed in our ancestors and in ourselves via their and our wider contacts with the observed realities of the surrounding world, and which evolutions indeed also have had and have influence on this recording of light-energy itself) makes us aware of a corresponding image which is our actual registration of this visual observation. And this

image can essentially only be determined by the *geometrical properties* of this surface. My purpose here is to present this natural, and therefore simple, geometrical model in some more detail and to discuss a bit its application to some so-called visual illusions.

1. INTRODUCTION

Following an inspiring lecture on *visual illusions* by d'Ydewalle at Leuven's Laboratory of Experimental Psychology, before heading back to the Department of Mathematics, some members of our research group PADGE made a pitstop at the nearby Café Erasmus. Possibly in addition inspired by the local Stella, there I drew *the natural geometrical model for early vision* (that will be recalled in 3). At that time ($\sim$ 1990) this was just a brief aside from the theme of the visual perception of various kinds of symmetry on which we had mathematical discussions then with some Leuven psychologists and engineers, in particular with Wagemans.

Through frequent recent discussions on the psychology of seeing with the Antwerpen Academie voor Schone Kunsten painter Servellon and with Wagemans and his research team, I got the impression that it might be of interest to some scholars in a variety of scientific disciplines and to some visual artists to write this model up in order that they might give it some thought. Some time ago I visited a few days a very hospitable Bristol University to discuss visual perception with Gregory, whereby also the Café Wall illusion (that will be recalled in 2) came up. In particular, this note gives *the Leuven Erasmus Café model's view on the Bristol Café Wall illusion (4).* As a matter of fact, this model likewise explains all other visual illusions that I am aware of and also other phenomena in human vision, on which will be reported in subsequent papers.

2. THE BRISTOL CAFÉ WALL ILLUSION

From Gregory - Heard's article [1], I quote the following : "*It was noted some time ago ($\sim$ 1973) by a then member of our laboratory, Steve Simpson, that the mortar* *lines of the chessboard-like design of tiles of a café wall in St. Michael's Hill, near our laboratory in Bristol, appear not parallel as they are, but converge markedly in alternate-direction wedges.*"

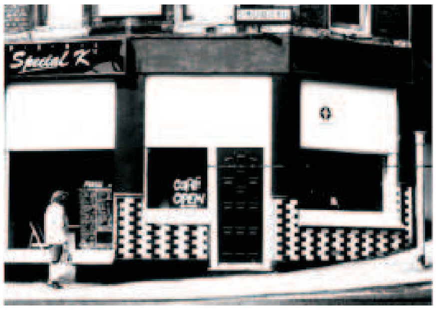

Figure 1. The café is open!

Basically, the question is: "*Why are the horizontal lines in the following figure visually perceived the way they are?*".

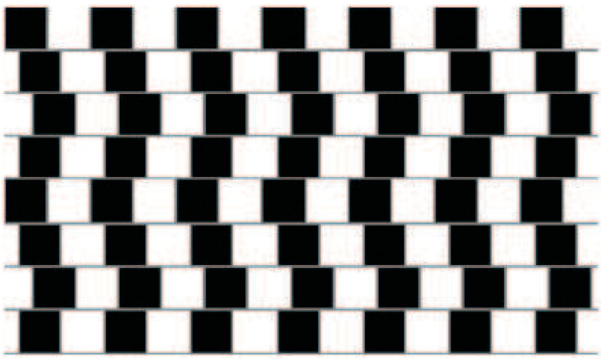

Figure 2. The Bristol Café Wall illusion, also known as the Münsterberg illusion, in one of its more dramatic forms

From Gregory's book "Eye and Brain" [2], I quote the following which is the beginning of his explanation of what is going on here. *"It turns out that there are two processes. The first is small scale: where there is brightness contrast across the neutral mortar line, half the dark and light tiles move towards each other - forming small scale wedges where there is local asymmetry. The eye integrates these little wedges into the long wedges that are seen"*. And quoting further: *"Well-known distortion* *illusions, like the ones of Hering, Poggendorf, Orbison and Zöllner, are very different. They obey completely different laws,.... Physicists and physiologists, psychologists and philosophers, have tried to explain these distortion illusions for over a hundred years. Current explanations are controversial; but I believe we can develop an explanation for these distortions, which throws light on the nature of object perception".*

Also I do believe we can develop an explanation for visual illusions which throws some light on the nature of visual perception. It is given by the following *model for early vision.* The last paragraph of the beautiful "Eye and Brain", which book I fully admit at this stage to have absorbed only very partially indeed, after going through it twice, but which I am sure to experience useful and entertaining also in future readings, goes as follows. "*The physical sciences take immense trouble to avoid errors. Here we seek out and study errors for understanding how we see and to suggest something of how the brain works. The weird and wonderful errors of illusions are not trivial. They are truly phenomenal phenomena, central to art and a major reason for the experimental methods of science*". In my opinion, as a geometer who is not hindred by having seriously studied the unnaturally extended literature on the science of vision, the wonderful visual illusions are neither trivial nor weird, but are quite understandable if one cares a bit to take into account what it means to make a visual observation. To try to avoid errors when not doing mathematics has never been one of my strong points. But, in any case, I do hope to disturb some readers with a natural geometrization of the essence of the registration of visual information which a.o. takes away the wonder, the weirdness and the mystery of the so-called visual illusions. And, consequently, I do hope that some people will give this some further thought, in particular, in the contexts of computer vision, neuroscience and visual arts.

3. THE LEUVEN ERASMUS CAFÉ MODEL

Consider a *static planar image* I that in the mathematical reality is given by a "luminance"-function F, (i.e. when x and y are Cartesian co-ordinates in the image-plane P, then $F(x, y)$ is the luminance at the points (x, y) in P), or, equivalently, by a surface $N : z = F(x, y)$, the graph of the function F in the 3-dimensional Euclidean space E, (with Cartesian co-ordinates x, y and z). Such an image is actually observed in the physical reality as a *"luminance"-function* L, or, equivalently, as a *surface* $M : z = L(x, y)$ in E, which are respectively related to F and N by smoothing (say, by some kind of diffusion). As stated so well by Koenderink and van Doorn [3] in their comments on *the nature of observation*: our only actual knowledge about the "real" image I in P is its observation, and this is most basically formally described by a *"visual-stimulus-surface"* M in Euclidean 3-space E (at least in a *qualitative* way, which is suitable enough for the present purpose; concerning dealing with this matter in a more subtle *quantitative* way, for which many experimental data on visual illusions such as the ones resulting in "the laws of café wall distortion" may have great importance, some comments are given in [4, 5]). In Figures 3 and 4 this is exemplified for (1): a *line-segment* S of *length* ℓ, and, for (2): an *"arrow"* A with S as shaft and thus of length $a = \ell$.

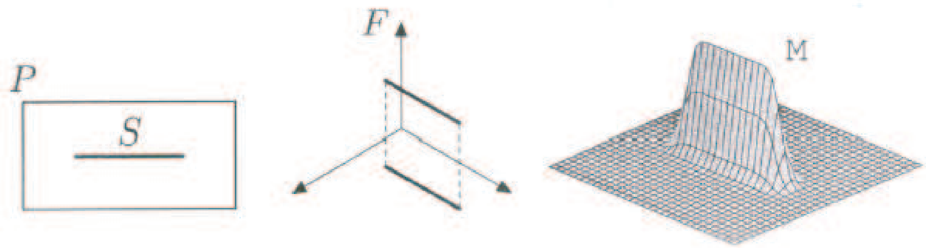

Figure 3. A line-segment S of length ℓ

Thus, in general again, when looking at "real" images I in a plane P, all that we physically actually observe is what is formally well described by their visual-stimulus-surfaces M in Euclidean 3-space E. And then it is only natural to claim that, *in early vision, our visual system essentially registrates the main shape-characteristics of these surfaces* M *in* E. For reasons discussed in [4, 5], as such the distinguished surface shape-characteristic given by the average of the sum of the squares of the

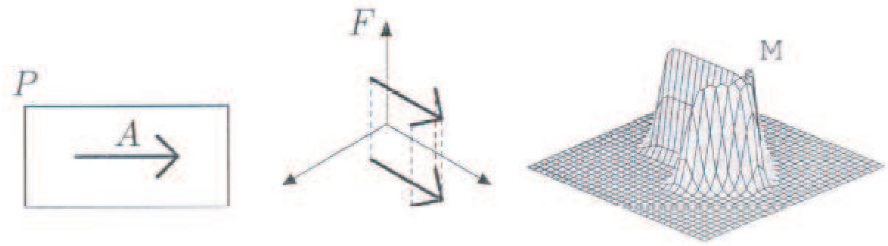

Figure 4. An *"arrow"* A with S as shaft

principle curvatures (or still of the square of the length of the second fundamental form h of M in $E : C = \frac{1}{2}(k_1^2 + k_2^2) = \frac{1}{2}\|h\|^2)$, the so-called *Casorati curvature* of the surfaces M in E, turns out to be like predestined to be used in this respect [6, 7, 8]. This curvature may well be the most simple scalar-valued surface shape-characteristic which measures, in accordance with rudimentary intuition, the degree to which at each of its points a surface in Euclidean space deviates from being part of a plane. At each point (x, y) in P the visual-stimulus-surface M in E corresponding to an image I has a Casorati curvature $C(x, y)$. So, every planar image I has a visual-stimulus-surface $M : z = L(x, y)$ in Euclidean 3-space E of which the shape is significantly represented by its associated Casorati surface $z = C(x, y)$ in E. Figures 5 and 6 show the Casorati surfaces for the Examples (1) and (2), as well as the lines formed by their *main relative extrema* which determine in P a *line-segment* $\tilde{S}$ and an *arrow* $\tilde{A}$ of respective lengths $\tilde{\ell}$ and $\tilde{a}$ whereby *essentially* $\tilde{a} < \tilde{\ell}$.

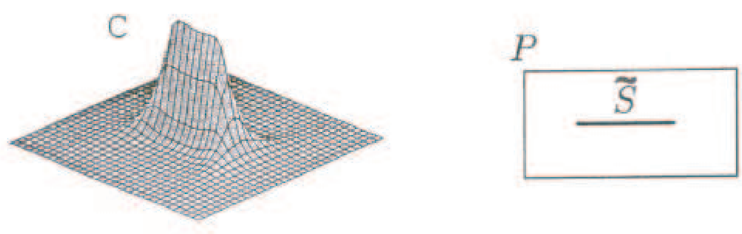

Figure 5.

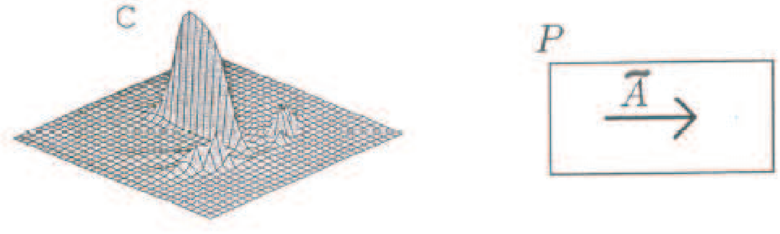

Figure 6.

The announced geometrical *model for early vision* then is the following. Our visual system "registers" visual data, of which we think mathematically, by way of examples, for instance, as a line-segment S or an arrow A, and which we actually observe as visual-stimulus-surfaces M in E, as images formed by the values and in particular by the extrema of the Casorati curvatures of M, which, in mathematical terms, in case of the examples, are a line-segment $\tilde{S}$ and an arrow $\tilde{A}$. In general and in short: *in early vision humans register images $\tilde{I}$ which are determined by the curvatures of the observations M made when looking at an image I.*

In verbal summary: *when looking at an image I in P, the observation that our visual system actually makes is a corresponding visual-stimulus-surface M in E and consequently, by early vision, we naturally register an image $\tilde{I}$ which is determined by the most characteristic features of M in E.* For an arrow, by way of example, the comparison of $A(-)$ and $\tilde{A}(\cdots)$ thus qualitatively is as follows:

Figure 7. Müller-Lyer illusion

which, by the way, properly explains *the Müller-Lyer illusion.*

For some literature in this respect, in particular concerning eye and brain-activities related to early vision and visual perception in general, see e.g. [2, 4, 9, 10, 11, 12, 13, 14, 15].

4. SOME PROMISED EARLY VISIONS

To conclude, and as two further illustrations of the above model of early vision, I would like to mention the following, both related to the quotations made in 2.

(i) Concerning illusions like the ones of Hering, Poggendorf, Orbison, etc., for instance, for *a Zöllner illusion* in its most rudimentary form, the images $\tilde{I}(\cdots)$ and $I(-)$ relate as follows:

Figure 8. Zöllner illusion

(ii) "In the small", the Bristol Café Wall illusion is closely related to the *"brightness illusion"* whereby tiles of the same mathematical sizes are perceived to be larger or smaller when their luminosity is respectively higher or lower than the luminosity of their background. It is a good exercise of the mental visualisation of curvature to think of this "black-and-white"-effect, which for instance is so popular in the world of fashion, in the light of the above model. For the actual situation of the kind of wall some may be facing in reality in this respect, a Münsterberg-tiling and a picture of its associated Casorati curvatures look as follows:

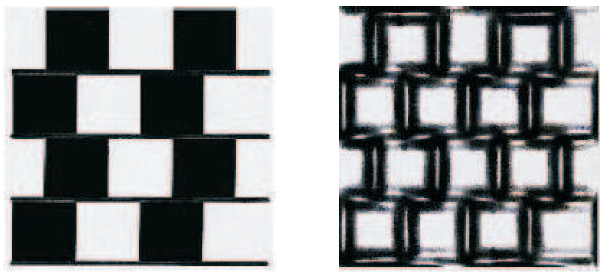

Figure 9.

So, for Bristol Café Wall tilings T, just as for Müller-Lyer arrows A, the visual registrations $\tilde{T}$ and $\tilde{A}$ given by the above model are in accordance with the so-called visual illusions we are said to erroranously have when looking at T or A. *"Wonder en is gheen wonder"* as read Simon Stevin's motto: although far from being quantitatively precise at this moment, the above model geometrically describes what is our visual contact with the physical world around us.

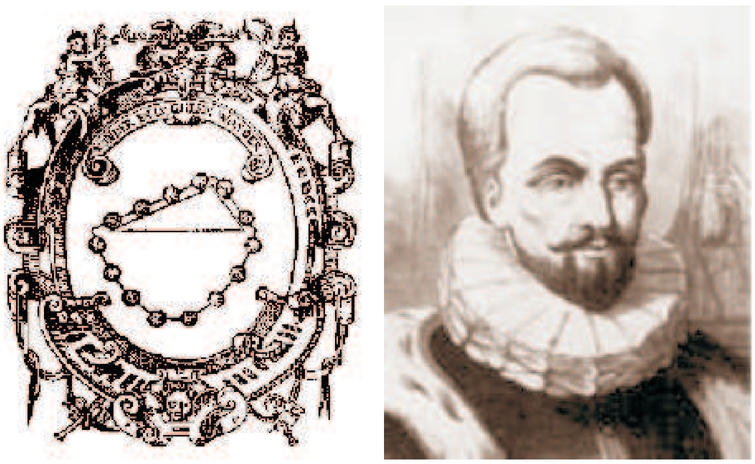

Figure 10. Simon Stevin

I heartily thank Professor Jan Koenderink for his great hospitality at the Helmholtz Institute at the Universiteit Utrecht during the visits I could make there from time to time: I learned so much from talking with his coworkers and visitors there, but most in particular from talking with him on many aspects of true geometry and its applications; (and sometimes, in between, he makes pictures illustrating facts of geometry or of vision, like the above Figure 9).

The experimental data gathered over the years for the above illusions and also for other illusions like for instance the ones of Mach, Hermann, Craik-0'Brain, etc., may be very valuable indeed to elaborate an accurate quantitative version of the above qualitative model.

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PS. Many thanks are due to Bea Peeters and Kristof Lenjou for the editing of this text!

Figure 11. The Erasmus Café at Leuven, with further down, the K. U. Leuven buildings of Philosophy and Psychology (Courtesy Sofie)