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Osc^kM ADMITTING f-STRUCTURE

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Abstract. The theory of Osc^kM was introduced by R. Miron and Gh. Atanasiu in [4], [5]. R. Miron in [6], [7] gave the comprehend theory of higher order geometry and its application. In [1] and [2] the special adapted basis of Miron's Osc^kM was constructed. Using the above results here different structures of Osc^kM will be examined.[1]

1. SPECIAL ADAPTED BASIS IN $T(Osc^kM)$ AND $T^*(Osc^kM)$

Here Osc^kM will be defined as a C^∞ manifold in which the transformations of form (1.1) are allowed. It is formed as a tangent space of higher order of the base manifold M.

Let $E = Osc^kM$ be a $(k+1)n$ dimensional C^∞ manifold. In some local chart (U, φ) some point $u \in E$ has coordinates

$$(x^a, y^{1a}, y^{2a}, \dots, y^{ka}) = (y^{0a}, y^{1a}, y^{2a}, \dots, y^{ka}) = (y^{Aa}),$$

where $x^a = y^{0a}$ and

$$a, b, c, d, e, \dots = 1, 2, \dots, n, \quad A, B, C, D, \dots = 0, 1, 2, \dots, k.$$

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The following abbreviations:

$$\partial_{Aa} = \frac{\partial}{\partial y^{Aa}}, \quad A = 1, 2, \ldots, k, \quad \partial_a = \partial_{0a} = \frac{\partial}{\partial x^a} = \frac{\partial}{\partial y^{0a}}$$

will be used.

If in some other chart (U', φ') the point $u \in E$ has coordinates $(x^{a'}, y^{1a'}, y^{2a'}, \ldots, y^{ka'})$, then in $U \cap U'$ the allowable coordinate transformations are given by:

$$\begin{aligned} x^{a'} &= x^{a'}(x^1, x^2, \ldots, x^n), \\ y^{1a'} &= (\partial_a x^{a'}) y^{1a} = (\partial_{0a} y^{0a'}) y^{1a}, \\ y^{2a'} &= (\partial_{0a} y^{1a'}) y^{1a} + (\partial_{1a} y^{1a'}) y^{2a}, \ldots, \\ y^{ka'} &= (\partial_{0a} y^{(k-1)a}) y^{1a} + (\partial_{1a} y^{(k-1)a}) y^{2a} + \cdots + (\partial_{(k-1)a} y^{(k-1)a}) y^{ka}. \end{aligned} \tag{1.1}$$

The natural basis $\bar{B}$ of $T(E)$ is

$$\bar{B} = \{\partial_{0a}, \partial_{1a}, \ldots, \partial_{ka}\}. \tag{1.2}$$

The natural basis $\bar{B}^*$ of $T^*(E)$ is

$$\bar{B}^* = \{dy^{0a}, dy^{1a}, \ldots, dy^{ka}\}. \tag{1.3}$$

We shall use the notations

$$[dy^{(a)}] = \begin{bmatrix} dy^{0a} \\ dy^{1a} \\ \vdots \\ dy^{ka} \end{bmatrix}, \quad [\delta y^{(a)}] = \begin{bmatrix} \delta y^{0a} \\ \delta y^{1a} \\ \vdots \\ \delta y^{ka} \end{bmatrix}, \quad [\partial_{(a)}] = [\partial_{0a} \partial_{1a} \ldots \partial_{ka}],$$

$$[\delta_{(a)}] = [\delta_{0a} \delta_{1a} \ldots \delta_{ka}], \quad {}^{(0)}B_a^{a'} = \partial_{0a} y^{0a'},$$

$$[B_{(a)}^{(a')}] = \begin{bmatrix} \partial_{0a} y^{0a'} & 0 & \cdots & 0 \\ \partial_{0a} y^{1a'} & \partial_{1a} y^{1a'} & \cdots & 0 \\ \vdots & & & \\ \partial_{0a} y^{ka'} & \partial_{1a} y^{ka'} & \cdots & \partial_{ka} y^{ka'} \end{bmatrix}.$$

Definition 1.1. *The special adapted basis B^* of $T^*(E)$ is defined by*

$$[\delta y^{(a)}] = [M_{(b)}^{(a)}][dy^{(b)}], \tag{1.4}$$

where

$$[M_{(b)}^{(a)}] = \begin{bmatrix} \binom{0}{0}\delta_b^a & 0 & 0 & 0 \\ \binom{1}{0}M_{0b}^{1a} & \binom{1}{1}\delta_b^a & 0 & 0 \\ \binom{2}{0}M_{0b}^{2a} & \binom{2}{1}M_{0b}^{1a} & \binom{2}{2}\delta_b^a & 0 \\ \vdots & & & \\ \binom{k}{0}M_{0b}^{ka} & \binom{k}{1}M_{0b}^{(k-1)a} & \vdots & \binom{k}{k}\delta_b^a \end{bmatrix}. \tag{1.5}$$

Theorem 1.1. *The necessary and sufficient conditions that $\delta y^{Aa} (A = 0, 1, \dots, k)$ are transformed as d-tensors are:*

$$[M_{(b)}^{(a)}]^{(0)}B_a^{a'} = [M_{(b')}^{(a')}][B_{(a)}^{(b')}]. \tag{1.6}$$

Definition 1.2. *The special adapted basis B of $T(E)$ is given by*

$$[\delta_{(a)}] = [\partial_{(b)}][N_{(a)}^{(b)}], \tag{1.7}$$

where

$$[N_{(a)}^{(b)}] = \begin{bmatrix} \binom{0}{0}\delta_a^b & 0 & 0 & \dots & 0 \\ -\binom{1}{0}N_{0a}^{1b} & \binom{1}{1}\delta_a^b & 0 & \dots & 0 \\ -\binom{2}{0}N_{0a}^{2b} & -\binom{2}{1}N_{0a}^{1b} & \binom{2}{2}\delta_a^b & \dots & 0 \\ \vdots & & & & \vdots \\ -\binom{k}{0}N_{0a}^{kb} & -\binom{k}{1}N_{0a}^{(k-1)b} & \dots & \dots & \binom{k}{k}\delta_a^b \end{bmatrix}. \tag{1.8}$$

Theorem 1.2. *The necessary and sufficient conditions for $\delta_{Aa} (A = 0, 1, \dots, k)$ given by (1.7) to be d-tensors is the following matrix equation:*

$$[B_{(b)}^{(c')}][N_{(a)}^{(b)}] = [N_{(a')}^{(c')}]^{(0)}B_a^{a'}. \tag{1.9}$$

Theorem 1.3. *The special adapted basis B^* is dual to special adapted basis B if and only if*

$$[M_{(c)}^{(b)}][N_{(a)}^{(c)}] = \delta_a^b I. \tag{1.10}$$

The proof of Theorems 1.1-1.3 can be found in [2].

2. THE J STRUCTURE

Definition 2.1. *The k-tangent structure J is a $\mathcal{F}(E)$-linear mapping*

$$J : \chi(E) \to \chi(E)$$

defined by

$$\begin{gathered} J\partial_{0i} = \partial_{1i}, \quad J\partial_{1i} = 2\partial_{2i}, \quad \dots, \\ J\partial_{\alpha i} = (\alpha+1)\partial_{(\alpha+1)i}, \quad \dots, \quad J\partial_{(k-1)i} = k\partial_{ki}, \quad J\partial_{ki} = 0. \end{gathered} \tag{2.1}$$

The k-structure J determined by Definition 2.1 is the same as J used in [6], [7], but there it is represented in different basis of the tangent space.

For the k-tangent structure J the relation

$$J^{k+1} = 0 \tag{2.2}$$

is valid. In the natural bases $\bar{B}$ and $\bar{B}^*$ of $T(E)$ and $T^*(E)$ it can be written in the form

$$\begin{aligned} J &= [\partial_{0a}\partial_{1a}\dots\partial_{ka}] \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 0 & & 0 & 0 \\ 0 & 2 & 0 & & 0 & 0 \\ 0 & 0 & 3 & & 0 & 0 \\ \vdots & & \vdots & & k & 0 \end{bmatrix} \otimes \begin{bmatrix} dy^{0a} \\ dy^{1a} \\ dy^{2a} \\ \vdots \\ dy^{ka} \end{bmatrix} \\ &= \partial_{1a} \otimes dy^{0a} + 2\partial_{2a} \otimes dy^{1a} + 3\partial_{3a} \otimes dy^{2a} + \dots + k\partial_{ka} \otimes dy^{(k-1)a} \end{aligned} \tag{2.3}$$

Theorem 2.1. *The k-tangent structure J defined by Definition (2.1) the elements of basis $B = \{\delta_{0a}, \delta_{1a}, \dots, \delta_{ka}\}$ determined by (1.7) transform in the following way*

$$\begin{gathered} J\delta_{0a} = \delta_{1a}, \quad J\delta_{1a} = 2\delta_{2a}, \quad J\delta_{Aa} = (A+1)\delta_{(A+1)a}, \dots \\ J\delta_{(k-1)a} = k\partial_{ka}, \quad J\delta_{ka} = 0. \end{gathered} \tag{2.4}$$

Theorem 2.2. *The k-tangent structure J given by (2.1) satisfies the relations*

$$dy^{0b}J = 0, dy^{1b}J = dy^{0b}, dy^{2b}J = 2dy^{1b}, \dots, dy^{kb}J = kdy^{(k-1)b}. \tag{2.5}$$

Theorem 2.3. *For the k-tangent structure J given by (2.1) we have*

$$\delta y^{0b}J = 0, \delta y^{1b}J = \delta y^{0b}, \delta y^{2b}J = 2\delta y^{1b}, \ldots, \delta y^{kb}J = k\delta y^{(k-1)b} \tag{2.6}$$

where $\{\delta y^{0b}, \delta y^{1b}, \ldots, \delta y^{kb}\}$ is the special adapted basis B^* of $T(E)$ determined by (1.4).

Theorem 2.4. *The structure J in the adapted basis $B = \{\delta_{0a}, \delta_{1a}, \ldots, \delta_{ka}\}$ and $B^* = \{\delta y^{0a}, \delta y^{1a}, \ldots, \delta y^{ka}\}$ is given by*

$$J = \delta_{1a} \otimes \delta y^{0a} + 2\delta_{2a} \otimes \delta y^{1a} + 3\delta_{3a} \otimes \delta y^{2a} + \ldots + k\delta_{ka} \otimes \delta y^{(k-1)a}. \tag{2.7}$$

The proof of Theorems 2.1-2.4 can be found in [2].

3. $f(2t+1,-1)$-STRUCTURE IN $Osc^k M$

In the special adapted basis $B = \{\delta_{0a}, \delta_{1a}, \ldots, \delta_{ka}\}$ of $T(E)$, the vectors $\{\delta_{0a}\}$ span the n-dimensional space $T_H(E)$, and the vectors $\{\delta_{1a}, \delta_{2a}, \ldots, \delta_{ka}\}$ the $k{\cdot}n$-dimensional $T_V(E)$ and

$$T(E) = T_H(E) \oplus T_V(E).$$

With respect to the metric tensor G:

$$G = g_{0a\,0b}\delta y^{0a} \otimes \delta y^{0b} + g_{Aa\,Bb}\delta y^{Aa} \otimes \delta y^{Bb},\ A = 1, 2, \ldots, k$$

$T_H(E)$ is orthogonal to $T_V(E)$.

Definition 3.1. *Let $E = Osc^k M$ be a $m = (k+1)n$-dimensional differentiable manifold of class C^∞, and let there be given a tensor field $f \neq 0$ of the type (1,1) and of class C^∞ such that*

$$f^{2t+1} - f = 0,\ \ f^{2i+1} - f \neq 0 \ \text{ for }\ 1 \leq i < t, \tag{3.1}$$

where t is a fixed integer greater than 1. Let rank $f = r$ be constant. We call such a structure an $f(2t+1,-1)$-structure or an f-structure of the rank r and of degree $2t+1$.

Theorem 3.1. *For a tensor field $f, f \neq 0$ satisfying (2.1), the operators*

$$\mathbf{m} = I - f^{2t}, \ \ \mathbf{l} = f^{2t} \tag{3.2}$$

are the complementary projection operators where I denotes the identity operator applied to the tangent space at a point of the manifold.

Proof. We have

$$\mathbf{l} + \mathbf{m} = I, \ \mathbf{l}^2 = \mathbf{l}, \ \mathbf{m}^2 = \mathbf{m}, \ \mathbf{ml} = \mathbf{lm} = 0$$

by virtue of (3.1), which proves the theorem.

Let L and M be the complementary distributions corresponding to the operators $\mathbf{l}$ and $\mathbf{m}$, respectively. If rank $f = r$ is constant and $\dim L = r$, then $\dim M = m - r$.

Proposition 3.1. *Let an f-structure of the rank r and degree $2t+1$ be given on E, then $f^{2t}\mathbf{l} = \mathbf{l}$ and $f^{2t}\mathbf{m} = 0$, i.e. f^t acts on L as an almost product structure operator and on M as a null operator.*

We shall assume that E is a $Osc^k M$ space of dimension $m = (k+1)n$, and that rank $f = r = k \cdot n$. Then $\dim L = k \cdot n, \dim M = n$ and $M = T_H(E), L = T_V(E)$.

If we denote by h the projection morphism of $T(E)$ to $T_H(E)$, we can construct the mapping α which is defined in [10] by

$$\alpha(X,Y) = \frac{1}{2}[\overline{h}(\mathbf{l}X, \mathbf{l}Y)] + \overline{h}(\mathbf{m}X, \mathbf{m}Y)], \ \ \forall X, Y \in T(E),$$

where $\overline{h} = Gh$, is a pseudo-Riemannian structure on $T(E)$, such that $\alpha(X,Y) = 0, \forall X \in M, Y \in L$.

If we put $g(X,Y) = \frac{1}{2t}[\alpha(X,Y) + \alpha(fX, fY) + \cdots + \alpha(f^{2t-1}X, f^{2t-1}Y)]$, it is easy to see that $g(X,Y) = 0, \forall X \in M, Y \in L$.

Also, using (3.2) we get $g(fX, fY) = \frac{1}{2t}[\alpha(fX, fY) + \alpha(f^2X, f^2Y) + \cdots + \alpha(X,Y)] = g(X,Y)$. Thus f is an isometry with respect to g.

We assume that f_L^i (the restriction from f^i on L, $(i < 2t)$) is not identity operator of L. Then f_L is a linear transformation of L with the minimal polynomial $x^{2t} - 1 = 0$. (We know that $f^{2t} = 1$ on L.) The polynomial $(x^t - 1)(x^t + 1) = 0$ has simple roots:

$$e^{2\frac{\pi i}{t}}, e^{4\frac{\pi i}{t}}, \ldots, e^{2t\frac{\pi i}{t}}, e^{\frac{\pi i}{t}}, e^{3\frac{\pi i}{t}}, \ldots, e^{(2t-1)\frac{\pi i}{t}}.$$

The eigenvectors which correspond to these eigenvalues are $e_2, e_4, \ldots, e_{2t}$, $e_1, e_3, \ldots, e_{2t-1}$, respectively. Let us denote by L_1 the vector space generated by the vectors $e_2, e_4, \ldots, e_{2t}$ and by L_2 the vector space generated by the vectors $e_1, e_3, \ldots, e_{2t-1}$. Then

$$f^t = 1 \text{ on } L_1, \; f^t = -1 \text{ on } L_2.$$

For $X \in L_1$ and $Y \in L_2$, we have

$$g(X,Y) = g(fX, fY) = g(f^tX, f^tY) = g(X, -Y) = -g(X,Y).$$

Hence, L_1 and L_2 are orthogonal with respect to the metric g.

We assume that $f^j - 1 \neq 0$ on $L_1, j < t$ and $f^j + 1 \neq 0$ on $L_2, j < t$. Then, f is a linear transformation of L_2 with the minimal polynomial $x^t + 1 = 0$, with the eigenvalue $\sqrt[t]{-1}$, to which correspond the eigenvectors $e'_1, e'_2, \ldots, e'_t$ and $L_2 = L_2^1 \oplus L_2^2 \oplus \ldots \oplus L_2^t$ where L_2^s is the subspace of L_2 generated by the vector $e'_s, s = 1, \ldots, t$.

It is also an f linear transformation on L_1 with the minimal polynomial $x^t - 1 = 0$, with the eigenvalue $\sqrt[t]{1}$, to which the eigenvectors $e'_{t+1}, e'_{t+2}, \ldots, e'_{2t}$ correspond. Now $L_1 = L_1^{t+1} \oplus L_1^{t+2} \oplus \ldots \oplus L_1^{2t}$, where L_1^{t+s} is the subspace of L_1 generated by the vector $e'_{t+s}, s = 1, \ldots, t$.

L_1^{t+p} and $L_1^{t+r}, (p, r < t)$, are orthogonal with respect to g if $t = 2^k$, $k \in N$, which is then shown by induction, see [10]. In the sequel $t = 2^k, k \in N$.

In [3] the following theorem is proved: If $f^t = \begin{bmatrix} 0 & E_p \\ -E_p & 0 \end{bmatrix}$, then $t \leq p$ and p is divisible by $t, (p = s \cdot t)$.

An analogous situation is on the space $L_2(\dim L_2 = 2p, p = s \cdot 2^{k-1})$.

If we assume that E is a Osc^kM space of dimension $m = (K+1)n$, and that rank $f = r = k \cdot n = 2 \cdot p \cdot k$, then $\dim L = k \cdot n \quad \dim M = n$ and $M = T_H(E)$, $L = T_V(E)$.

Let $u_1, \ldots, u_{2p}$ be an orthogonal basis of L_2 and $u_{2p+1}, u_{2p+2}, \ldots, u_{r-2p}$ be an orthogonal basis of L_1, both with respect to g, then $u_1, \ldots, u_{2p}, u_{2p+1}, \ldots, u_{r-2p}$ is an orthogonal basis of L such that

$$L_2 : \begin{cases} f(u_i) = u_{i+\frac{2p}{2^k}}, & f(u_{j+2p-\frac{2p}{2^k}}) = -u_j, \\ i = 1, 2, \dots, 2p - 2p/2^k, & j = 1, 2, \dots, 2p/2^k. \end{cases}$$

$$L_1 : \begin{cases} f(u_{2p+i}) = u_{2p+i+\frac{2p}{2^{k-1}}}, & f(u_{4p+j-\frac{2p}{2^{k-1}}}) = -u_{2p+j} \\ i = 1, 2, \dots, 2p - 2p/2^{k-1}, & j = 1, 2, \dots, 2p/2^{k-1} \\ f(u_{4p+i}) = u_{4p+i+\frac{2p}{2^{k-2}}}, & f(u_{6p+j-\frac{2p}{2^{k-2}}}) = -u_{4p+j} \\ i = 1, 2, \dots, 2p - 2p/2^{k-2}, & j = 1, 2, \dots, 2p/2^{k-2} \\ \vdots & \\ f(u_{2(p-1)k+i}) = u_{2(p-1)k+i+p}, & f(u_{2pk-j}) = -u_{2(p-1)k+j} \\ i = 1, 2, \dots, p, & j = 1, 2, \dots, p \\ f(u_{2kp+i}) = u_{2pk+i}, & f(u_{2(k+1)p-i}) = -u_{2p(k+1)-i} \\ i = 1, 2, \dots, p. & \end{cases}$$

Next, we choose in $M = T_H(E)$ an orthogonal basis $u_{r+1}, \dots, u_{(k+1)n}$ with respect to g, $\dim Osc^k M = m = n + r = (k+1)n$. Then, with respect to the orthogonal frame $u_1, \dots, u_m$, the tensors g and f have components as in [9]:

$$f = \begin{bmatrix} 0 & E_{2p-\frac{2p}{2^k}} & & & & & & & & \\ -E_{\frac{2p}{2^k}} & 0 & & & & & & & & \\ & & \ddots & & & & & & & \\ & & & 0 & E_{2p-\frac{p}{2}} & & & & & \\ & & & -E_{\frac{p}{2}} & 0 & & & & & \\ & & & & & 0 & E_p & & & \\ & & & & & -E_p & 0 & & & \\ & & & & & & & E_p & 0 & \\ & & & & & & & 0 & -E_p & \\ & & & & & & & & & 0_{m-r} \end{bmatrix} \qquad (3.3)$$

$$g = \begin{bmatrix} E_{2p} & & & \\ & \ddots & & \\ & & E_{2p} & \\ & & & E_{m-r} \end{bmatrix}$$

We call such a frame an adapted frame of $f(2 \cdot 2^k + 1, -1)$ structure.

Let $\overline{u}_1, \dots, \overline{u}_m$ be another adapted frame with respect to which the metric tensor g and the tensor f have the same components as (3.3). We put $\overline{u}_i = \gamma_i^j u_j$, then we find that γ has the form

$$\gamma = \begin{bmatrix} S_{(\frac{2p}{2^k})} & & & & & & & \\ & S_{(\frac{2p}{2^{k-1}})} & & & & & & \\ & & \ddots & & & & & \\ & & & S_{(\frac{p}{2})} & & & & \\ & & & & A_p & B_p & & \\ & & & & -B_p & A_p & & \\ & & & & & & 0_{2p} & \\ & & & & & & & 0_{m-r} \end{bmatrix} \tag{3.4}$$

where $S_{(\frac{2p}{i})}, i = 2, 4, \ldots, 2^k$ is a matrix of format $2p = n$ and has the form

$$S_{(\frac{2p}{i})} = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 & \ldots & A_i \\ -A_i & A_1 & A_2 & A_3 & \ldots & A_{i-1} \\ -A_{i-1} & -A_i & A_1 & A_2 & \ldots & A_{i-2} \\ \vdots & & & & & \\ -A_3 & -A_4 & -A_5 & & & A_2 \\ -A_2 & -A_3 & -A_4 & & \ldots & A_1 \end{bmatrix} \tag{3.5}$$

where each matrix $A_l, l = 1, \ldots, i$ has a format $(\frac{2p}{i}) \times (\frac{2p}{i})$, i.e. $s \times s$.

Let $\overline{S}_{(\frac{2p}{i})}$ be the tangent group defined by $S_{(\frac{2p}{i})}$. Then we can say that the group of the tangent bundle of the manifold can be reduced to

$$\overline{S}_{(\frac{2p}{2^k})} \times \overline{S}_{(\frac{2p}{2^{k-1}})} \times \ldots \times \overline{S}_{(\frac{2p}{4})} \times U_p \times 0_{2p} \times 0_{m-r}.$$

Theorem 3.2. *A necessary and sufficient condition for a space E of dimension $(k+1)n$ to admit a tensor field $f \neq 0$ of type (1,1) and of rank $k \cdot n$, such that $f^{2 \cdot 2^k+1} - f = 0$, is that*

i) $r = k \cdot 2p = k \cdot n$,

ii) $2p = s \cdot 2^k = s \cdot t$, $s \in N$, $t = 2^k$,

iii) *the group of the tangent bundle of the manifold be reduced to the group*

$$\overline{S}_{(\frac{2p}{2^k})} \times \overline{S}_{(\frac{2p}{2^{k-1}})} \times \ldots \times \overline{S}_{(\frac{2p}{4})} \times U_p \times O_{2p} \times O_{m-r}.$$

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