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FUZZY PAIRWISE ALMOST STRONG PRECONTINUITY

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Abstract. The concept of fuzzy pairwise almost strongly precontinuous mappings has been introduced and studied. Their properties and relationships with other classes of early defined weaker forms of fuzzy pairwise continuous mappings has been investigated.

1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh in his classic paper [**8**]. Chang [**2**] first introduced the fuzzy topological spaces by using the fuzzy sets. Kandil [**3**] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces. Recently, Kumar [**6,7**] defined the (τ_i, τ_j) – fuzzy semiopen (semiclosed) sets, (τ_i, τ_j) – fuzzy preopen (preclosed) sets and (τ_i, τ_j) – fuzzy strongly semiopen (semiclosed) sets. Continuing his work, Kumar [**6,7**] defined the fuzzy pairwise semicontinuous mappings, fuzzy pairwise precontinuous, fuzzy pairwise strongly semicontinuous mappings. The author [**4**] defined the concept of fuzzy pairwise strongly precontinuous mappings.

In this paper, we will define the concept of fuzzy pairwise almost strongly precontinuous mappings. We will establish their properties and relationships with other classes of early defined weaker forms of fuzzy pairwise continuous mappings.

2. PRELIMINARIES

A triple (X, τ_1, τ_2) consisting of a nonempty set X with two fuzzy topologies τ_1 and τ_2 on X is called a fuzzy bitopological spaces, shortly fbts. Throughout this paper, the indices i and j take values in $\{1, 2\}$ and $i \neq j$. For a fuzzy set A of a fbts (X, τ_1, τ_2) , τ_1- int A and τ_j- clA means, respectively, the interior and closure of A with respect to the fuzzy topologies τ_i and τ_j.

Definition 2.1 [4, 6, 7] *Let A be a fuzzy set of a fbts X . Then A is called*

(1) a (τ_i, τ_j) – fuzzy semiopen set if and only if there exists τ_i – fuzzy open set U such that $U \leq A \leq \tau_j - clU$;

(2) a (τ_i, τ_j) – fuzzy preopen set if and only if $A \leq \tau_i - int(\tau_j - clA)$;

(3) a (τ_i, τ_j) – fuzzy strongly semiopen set if and only if $A \leq \tau_i - int(\tau_j - cl(\tau_i - int A))$;

(4) a (τ_i, τ_j) – fuzzy semipreopen set if and only if $A \leq \tau_i - cl(\tau_j - int(\tau_i - clA))$;

(5) a (τ_i, τ_j) – fuzzy strongly preopen set if and only if $A \leq \tau_i - int((\tau_j, \tau_i) - pclA)$;

(6) a (τ_i, τ_j) – fuzzy regular open set if and only if $A = \tau_i - int(\tau_j - clA)$.

The family of all (τ_i, τ_j) – fuzzy semiopen sets, (τ_i, τ_j) – fuzzy preopen sets, (τ_i, τ_j) – fuzzy strongly semiopen sets, (τ_i, τ_j) – fuzzy semipreopen sets, (τ_i, τ_j) – fuzzy strongly preopen sets and (τ_i, τ_j) – fuzzy regular open sets of a fbts (X, τ_1, τ_2) will be denote by (τ_i, τ_j) – FSO, (τ_i, τ_j) – FPO, (τ_i, τ_j) – FSSO, (τ_i, τ_j) – FSEPO, (τ_i, τ_j) – FSPO and (τ_i, τ_j) – FRO respectively.

Definition 2.2 [5, 7, 8] *Let A be a fuzzy set of a fbts X. Then A is called*

(1) a (τ_i, τ_j) – fuzzy semiclosed set if and only if A^c is a (τ_i, τ_j) – fuzzy semiopen set;

(2) a (τ_i, τ_j) – fuzzy preclosed set if and only if A^c is a (τ_i, τ_j) – fuzzy preopen set;

(3) a (τ_i, τ_j) – fuzzy strongly semiclosed set if A^c is a (τ_i, τ_j) – fuzzy strongly semiopen set;

(4) a (τ_i, τ_j) – fuzzy semipreclosed set if A^c is a (τ_i, τ_j) – fuzzy semipreopen set;

(5) a (τ_i, τ_j) – fuzzy strongly preclosed set if and only if A^c is a (τ_i, τ_j) – fuzzy strongly preopen set;

(6) a (τ_i, τ_j) – fuzzy regular closed set if and only if A^c is a (τ_i, τ_j) – fuzzy regular open set.

Similary, the family of all (τ_i, τ_j) – fuzzy semiclosed sets, (τ_i, τ_j) – fuzzy preclosed sets, (τ_i, τ_j) – fuzzy strongly semiclosed sets, (τ_i, τ_j) – fuzzy semipreclosed sets, (τ_i, τ_j) – fuzzy strongly preclosed sets and (τ_i, τ_j) – fuzzy regular closed sets will be denote by (τ_i, τ_j) – FSC, (τ_i, τ_j) – FPC, (τ_i, τ_j) – FSSC, (τ_i, τ_j) – FSEPC, (τ_i, τ_j) – FSPC and (τ_i, τ_j) – FRC, respectively.

Definition 2.3 [4, 6, 7] *A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \eta_1, \eta_2)$ from a fbts X into a fbts Y is called*

(1) a fuzzy pairwise semicontinuous if $f^{-1}(B)$ is a (τ_i, τ_j) – fuzzy semiopen set of X for each η_i – fuzzy open set B of Y;

(2) a fuzzy pairwise precontinuous if $f^{-1}(B)$ is a (τ_i, τ_j) – fuzzy preopen set of X for each η_i – fuzzy open set B of Y;

(3) a fuzzy pairwise strongly semicontinuous if $f^{-1}(B)$ is a (τ_i, τ_j) – fuzzy strongly semiopen set of X, for each η_i – fuzzy open set B of Y;

(4) a pairwise semiprecontnuous if $f^{-1}(B)$ is a (τ_i, τ_j) fuzzy semipreopen set of X, for each η_i – fuzzy open set B of Y ;

(5) a fuzzy pairwise strongly precontinuous if $f^{-1}(B)$ is a (τ_i, τ_j) fuzzy strongly preopen set of X, for each fuzzy η_i- open set B of Y;

(6) a fuzzy pairwise strong precontinuous irresolution if $f^{-1}(B)$ is a (τ_i, τ_j) – fuzzy strongly preopen set of X, for each (τ_i, τ_j) – fuzzy strongly preopen set B of Y.

Definition 2.4 [1] *A fuzzy set A of a fbts X is called fuzzy (τ_i, τ_j) - $\delta-$ open if and only if there exists fuzzy (τ_i, τ_j) – regular open sets $A_k, k \in I$ such that $A = \vee_{k \in I} A_k$.*

Definition 2.5 [1] *A fuzzy set A of a fbts X is called fuzzy (τ_i, τ_j) - $\delta-$ closed if and only if A^c is a fuzzy (τ_i, τ_j) - $\delta-$ open set.*

Definition 2.6 *Let A be a fuzzy set of a fbts (X, τ_1, τ_2). Then,*

$$(\tau_i, \tau_j) - int_\delta A = \Big\{ B \mid B \leq A, B \in (\tau_i, \tau_j) - FRO \Big\}$$

is called the fuzzy $(\tau_i, \tau_j) - \delta$ – interior of A.

$$(\tau_i, \tau_j) - cl_\delta A = \Big\{ B \mid B \geq A, B \in (\tau_i, \tau_j) - FRC \Big\}$$

is called the fuzzy $(\tau_i, \tau_j) - \delta$ – closure of A.

3. FUZZY PAIRWISE ALMOST STRONG PRECONTINUITY

Definition 3.1 *A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ from a fbts X into a fbts Y is called fuzzy pairwise almost strongly precontinuous if $f^{-1}(B)$ is a (τ_i, τ_j) – fuzzy strongly preopen set of X for each (η_i, η_j) – fuzzy regular open set B of X.*

Remark 3.1. Let $f : X \to Y$ be a mapping from a fbts X into a fbts Y. If f is fuzzy pairwise strongly precontinuous, then f is a fuzzy pairwise almost strongly precontinuous mapping. The following example shows that the converse statement may not be true.

Example 3.1. Let $X = \{a, b, c\}$ and A, B, C and be fuzzy sets of X defined as follows:

$$A(a) = 0,5 \qquad A(b) = 0,3 \qquad A(c) = 0,6;$$
$$B(a) = 0,3 \qquad B(b) = 0,4 \qquad B(c) = 0,3;$$
$$C(a) = 0,5 \qquad C(b) = 0,5 \qquad C(c) = 0,6.$$

If we put $\tau_1 = \tau_2 = \{0, B, A \vee B, 1\}$, $\eta_1 = \eta_2 = \{0, A, B, A \wedge B, A \vee B, 1\}$ and $f = id : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ we conclude that f is fuzzy pairwise almost strongly precontinuous but f is not fuzzy pairwise strong precontinuous mapping.

Theorem 3.1 *Let $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a mapping from a fbts X into a fbts Y. Then the following statements are equivalent:*

(i) f *is a fuzzy pairwise almost strongly precontinuous mapping;*

(ii) $f^{-1}(B)$ *is a* (τ_i, τ_j) *– fuzzy strongly preclosed set of* X *for each* (η_i, η_j) *– fuzzy regular closed set* B *of* Y*;*

(iii) $(\tau_i, \tau_j) - spcl\, f^{-1}(\eta_i - cl(\eta_j - int\, B)) \leq f^{-1}(B)$ *for each* η_i *– fuzzy closed set* B *of* Y*;*

(iv) $f^{-1}(B) \leq (\tau_i, \tau_j) - spint\, f^{-1}(\eta_i - int\,(\eta_j - clB))$*, for each* η_i *– fuzzy open set* B *of* Y*;*

(v) $f^{-1}(B) \leq (\tau_i, \tau_j) - spint\, f^{-1}(\eta_i - int\,(\eta_j - cl(\eta_i - int\, B)))$ *for each* (τ_i, τ_j) *fuzzy strongly semiopen set* B *of* Y*;*

(vi) $(\tau_i, \tau_j) - spcl\, f^{-1}(\eta_i - cl(\eta_j - int\,(\eta_i - clB))) \leq f^{-1}(B)$*, for each* (τ_i, τ_j) *– fuzzy strongly semiclosed set* B *of* Y*;*

(vii) $(\tau_i, \tau_j) - spcl\, f^{-1}(\eta_i - cl(\eta_j - int\, B)) \leq f^{-1}(B)$*, for each* (τ_i, τ_j) *– fuzzy preclosed set* B *of* Y*;*

(viii) $f^{-1}(B) \leq (\tau_i, \tau_j) - spint\, f^{-1}(\eta_i - int\,(\eta_j - clB))$*, for each* (τ_i, τ_j) *– fuzzy preopen set* B *of* Y*;*

(ix) $(\tau_i, \tau_j) - spcl\, f^{-1}(\eta_i - cl(\eta_j - int\,(\eta_i - clB))) \leq f^{-1}(\eta_i - clB)$*, for each fuzzy set* B *of* Y*;*

(x) $f^{-1}(\eta_i - int\, B) \leq (\tau_i, \tau_j) - spint\, f^{-1}(\eta_i - int\,(\eta_j - cl(\eta_i - int\, B)))$*, for each fuzzy set* B *of* Y*;*

Proof. $(i) \Rightarrow (ii)$ Let B be any (η_i, η_j) – fuzzy regular closed set of fbts Y. Then B^c is a (η_i, η_j) – fuzzy open set of fbts Y. By assumption and the Definition 3.1 we obtain that $f^{-1}(B^c)$ is a (τ_i, τ_j) – fuzzy strongly preopen set of fbts X. From $f^{-1}(B^c) = (f^{-1}(B))^c$ it follows that $f^{-1}(B)$ is a (τ_i, τ_j) – fuzzy strongly preclosed set of fbts X.

$(ii) \Rightarrow (iii)$ Let B be any η_i – fuzzy closed set of fbts Y. Then $\eta_i - cl(\eta_j - \mathrm{int}\, B) \leq B$, so $f^{-1}(\eta_i - cl(\eta_j - \mathrm{int}\, B)) \leq f^{-1}(B)$. Since $\eta_i - cl(\eta_j - \mathrm{int}\, B)$ is a (η_i, η_j)-fuzzy regular closed set, by assumptions it follows that $f^{-1}(B)$ is a (τ_i, τ_j) – strongly preclosed set. Hence (τ_i, τ_j) – spcl $f^{-1}(\eta_i - cl(\eta_j - \mathrm{int}\, B)) \leq f^{-1}(B)$.

$(iii) \Rightarrow (iv)$ It can be proved by using the complement.

$(iv) \Rightarrow (v)$ Let B be any (η_i, η_j) – fuzzy strongly semiopen set of fbts Y. Then $B \leq \eta_i - \operatorname{int}(\eta_j - cl(\eta_i - \operatorname{int} B))$, so $f^{-1}B \leq f^{-1}(\eta_i - \operatorname{int}(\eta_j - cl(\eta_j - \operatorname{int} B))$. Since $\eta_i - \operatorname{int}(\eta_j - cl(\eta_i - \operatorname{int} B))$ is any η_i – fuzzy open set, by the assumption it follows that

$$\begin{aligned} &f^{-1}(\eta_i - \operatorname{int}(\eta_j - cl(\eta_i - \operatorname{int} B))) \\ &\qquad \leq (\tau_i, \tau_j) - \operatorname{spint} f^{-1}(\eta_i - \operatorname{int}(\eta_j - cl(\eta_i - \operatorname{int}(\eta_j - cl(\eta_i - \operatorname{int} B))))) \\ &\qquad = (\tau_i, \tau_j) - \operatorname{spint} f^{-1}(\eta_i - \operatorname{int}(\eta_j - cl(\eta_i - \operatorname{int} B))). \end{aligned}$$

$(v) \Rightarrow (vi)$ It can be proved by using the complement.

$(vi) \Rightarrow (vii)$ Let B be any (η_i, η_j) – fuzzy preclosed set of fbts Y. Then $B \geq \eta_i - cl(\eta_j - \operatorname{int} B)$, so $f^{-1}(B) \geq f^{-1}(\eta_i - cl(\eta_j - \operatorname{int} B))$. Since $\eta_i - cl(\eta_j - \operatorname{int} B)$ is a (η_i, η_j) – fuzzy strongly semiclosed set, by the assumptions it folows that

$$\begin{aligned} f^{-1}(B) &\geq f^{-1}(cl(\operatorname{int} B)) \\ &\geq (\tau_i, \tau_j) - \operatorname{spcl} f^{-1}(\eta_i - cl(\eta_j - \operatorname{int}(\eta_i - cl(\eta_j - cl(\eta_i - \operatorname{int} B))))) \\ &\geq (\tau_i, \tau_j) - \operatorname{spcl} f^{-1}(\eta_i - cl(\eta_i - cl(\eta_j - \operatorname{int} B))). \end{aligned}$$

$(vii) \Rightarrow (viii)$ It can be proved by using the complement.

$(viii) \Rightarrow (ix)$ Let B be any fuzzy set of fbts Y. Then $\eta_i - \operatorname{int} B^c$ is a (τ_i, τ_j) – fuzzy preopen set, so

$$f^{-1}(\eta_i - \operatorname{int} B^c) \leq f^{-1}(\eta_i - \operatorname{int}(\eta_j - cl(\eta_i - \operatorname{int} B^c))) \leq f^{-1}(\eta_i - clB).$$

$(ix) \Rightarrow (x)$ It can be proved by using the complement.

$(x) \Rightarrow (i)$ Let B be η_i- regular open set of fbts Y. Then

$$\begin{aligned} f^{-1}(B) &= f^{-1}(\eta_i - \operatorname{int} B) \leq \\ &\leq spint\, f^{-1}(\eta_i - \operatorname{int}(\eta_j - cl(\eta_i - \operatorname{int} B))) \\ &= (\tau_i, \tau_j) - -\operatorname{spint} f^{-1}(B), \end{aligned}$$

so $f^{-1}(B)$ is a (τ_i, τ_j) – fuzzy strongly preopen set of X. Hence f is fuzzy pairwise almost strongly precontinuous. □

Theorem 3.2 *Let $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a mapping from a fbts X into a fbts Y. Then the following statements are equivalent:*

(i) f *is a fuzzy pairwise almost strongly precontinuous mapping.*

(ii) (τ_i, τ_j) $-spcl\, f^{-1}(B) \leq f^{-1}(\eta_i - clB)$*, for each* (η_1, η_2) *– fuzzy semiopen set* B *of* Y*.*

(iii) $f^{-1}(\eta_i - int\, B) \leq (\tau_i, \tau_j) - spint\, f^{-1}(B)$*, for each* (η_i, η_j) *– fuzzy semiclosed set* B *of* Y*.*

(iv) $f^{-1}(\eta_i - int\, B) \leq (\tau_i, \tau_j) - spint\, f^{-1}(B)$*, for each* (η_i, η_j) *– fuzzy semipreclosed set* B *of* Y*.*

(v) (τ_i, τ_j) *–* $spcl\, f^{-1}(B) \leq f^{-1}(\eta_i - clB)$*, for each* (η_i, η_j) *– fuzzy semipreopen set* B *of* Y*.*

Proof. $(i) \Rightarrow (ii)$ Let B be any (η_i, η_j) – fuzzy semiopen set of fbts Y. Then $B \leq \eta_i - cl(\eta_j - \operatorname{int} B)$, so $f^{-1}(B) \leq f^{-1}(\eta_i - cl(\eta_j - \operatorname{int} B))$. Since $\eta_i - cl(\eta_j - \operatorname{int} B)$ is a (η_i, η_j) – fuzzy regular closed set, by the assumptions it follows that $f^{-1}(B)$ is a (τ_i, τ_j) – strongly preclosed set. Hence $(\tau_i, \tau_j) - \operatorname{spcl} f^{-1}(\eta_i - cl(\eta_j - \operatorname{int} B)) \leq f^{-1}(\eta_i - clB)$.

$(ii) \Rightarrow (iii)$ It can be proved by using the complement.

$(iii) \Rightarrow (iv)$ Let B be any (η_i, η_j) – fuzzy semipreclosed open set of fbts Y. From $B \geq \eta_i - \operatorname{int}(\eta_j - cl(\eta_i - \operatorname{int} B))$ it follows that $\eta_i - \operatorname{int} B \geq \eta_i - \operatorname{int}(\eta_j - cl(\eta_i - \operatorname{int} B))$, so $\eta_i - \operatorname{int} B$ is a (η_i, η_j) – fuzzy semiclosed set. According to the assumption we get

$$f^{-1}(\eta_i - \operatorname{int} B) \leq (\tau_i, \tau_j) - \operatorname{spint} f^{-1}(\eta_i - \operatorname{int} B) \leq (\tau_i, \tau_j) - \operatorname{spint} f^{-1}(B).$$

$(iv) \Rightarrow (v)$ It can be proved by using the complement.

$(v) \Rightarrow (i)$ Let B be any (η_i, η_j) – fuzzy regular closed set of fbts Y. Then B is a (η_i, η_j) –fuzzy semipreopen set. From the assumption we get $f^{-1}(\eta_i - clB) \geq \operatorname{spcl} f^{-1}(B)$, so $f^{-1}(B)$ is a fuzzy strongly preclosed set. Hence f is a fuzzy pairwise almost strongly precontinuous. □

Corrolary 3.3 *Let* $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ *be a fuzzy almost strongly precontinuous mapping from a fbts* X *into a fbts* Y*. Then the following statements holds:*

(i) (τ_i, τ_j) *-*$spcl\, f^{-1}(B) \leq f^{-1}(\eta_i - clB)$*, for each* η_i*–fuzzy open set* B *of* Y*.*

(ii) $f^{-1}(\eta_i - int\, B) \leq (\tau_i, \tau_j)-$ *spint* $f^{-1}(B)$, *for each* η_i- *fuzzy closed set* B *of* Y.

The following theorem gives some local characterizations of the fuzzy almost strongly precontinuous mappings.

Theorem 3.4 *Let* $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ *be a mapping from a fbts* X *into an fbts* Y. *Then the following statements are equivalent:*

(i) f *is a fuzzy pairwise almost strongly precontinuous mapping.*

(ii) for each fuzzy singleton x_α *of* X *and* η_i- *fuzzy open set* B *contain* $f(x_\alpha)$, *there exists fuzzy strongly preopen set* A *of* X *containing* x_α *such that* $f(A) \leq \eta_i - int\,(\eta_j - clB)$.

(iii) for each fuzzy singleton x_α *of* X *and* (η_i, η_j) *– fuzzy regular open set* B *containing* $f(x_\alpha)$ *there exists* (τ_i, τ_j) *– fuzzy strongly preopen set* A *of* X *containing* x_α *such that* $f(A) \leq B$.

Proof. *(i)* $\Rightarrow$ *(iii)* Let f be η_i – fuzzy pairwise almost strongly precontinuous, x_α be a fuzzy singleton of X and let B be an η_i- fuzzy open set of Y such that $f(x_\alpha) \leq B$. Then

$$x_\alpha \leq f^{-1}(B) \leq (\tau_i, \tau_j) - \text{spint}\, f^{-1}(\eta_i - \text{int}\,(\eta_j - clB)).$$

Let

$$A = (\tau_i, \tau_j) - \text{spint}\, f^{-1}(\eta_i - \text{int}\,(\eta_j - clB)).$$

Then A is a (τ_i, τ_j) – fuzzy strongly preopen set and

$$\begin{aligned} f(A) &= f((\tau_i, \tau_j) - \text{spint}\, f^{-1}(\eta_i - \text{int}\,(\eta_j - clB)) \\ &\leq ff^{-1}(\eta_i - \text{int}\,(\eta_j - clB)) \leq \eta_i - \text{int}\,(\eta_j - clB)). \end{aligned}$$

(ii) $\Rightarrow$ *(iii)* Let x_α be a fuzzy singleton of X and let B be a (η_i, η_j) – fuzzy regular open set of Y containing $f(x_\alpha)$. Then B is a η_i – fuzzy open set. According to the assumption there exists a (τ_i, τ_j) – fuzzy strongly preopen set A of X containing x_α such that $f(A) \leq \eta_i - \text{int}\,(\eta_j - clB) = B$.

(iii) $\Rightarrow$ *(i)* Let B be a (η_i, η_j) – fuzzy regularly open set of Y and let x_α be a fuzzy singleton of X such that $x_\alpha \leq f^{-1}(B)$. According to the assumption there exists a (τ_i, τ_j) -fuzzy strongly preopen set A of X such that $x_\alpha \leq A$ and $f(A) \leq B$. Hence

$$x_\alpha \leq A \leq f^{-1}f(A) \leq f^{-1}(B)$$

and

$$x_\alpha \leq A = (\tau_i, \tau_j) - \text{spint}\, A \leq (\tau_i, \tau_j) - \text{spint}\, f^{-1}(B).$$

Since x_α is arbitrary and $f^{-1}(B)$ is the union of all fuzzy singletons of $f^{-1}(B)$, $f^{-1}(B) \leq (\tau_i, \tau_j) - \text{spint}\, f^{-1}(B)$. Thus f is fuzzy pairwise almost strongly precontinuous mapping. $\square$

Theorem 3.5 *Let $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a mapping from a fbts X into a fbts Y. Then the following statements are equivalent:*

(i) f is a fuzzy pairwise almost strongly precontinuous mapping.

(ii) $f^{-1}(B)$ is a (τ_i, τ_j) – fuzzy strongly preopen set of X, for each (η_i, η_j) – fuzzy δ – open set B of Y.

(iii) $f^{-1}(B)$ is a (τ_i, τ_j) – fuzzy strongly preclosed set of X, for each (η_i, η_j) – fuzzy δ closed set B of Y.

(iv) $f((\tau_i, \tau_j) - spcl\, A) \leq (\eta_i, \eta_j) - cl_\delta f(A)$, for each fuzzy set A of X.

(v) (τ_i, τ_j) – $spcl\, f^{-1}(B) \leq f^{-1}((\eta_i, \eta_j) - cl_\delta B)$, for each fuzzy set B of Y.

(vi) $f^{-1}((\eta_i, \eta_j) - int_\delta B) \leq (\tau_i, \tau_j) - spint\, f^{-1}(B)$, for each fuzzy set B of Y.

Proof. $(i) \Rightarrow (ii)$ Let B be any (η_i, η_j) – fuzzy δ – open set of Y. Then $B = \vee_{\alpha \in I} B_\alpha$, where B_α is a (η_i, η_j) – fuzzy regular open set of Y, for each $\alpha \in I$. From

$$f^{-1}(B) = f^{-1}(\vee_{\alpha \in I} B_\alpha) = \vee_{\alpha \in I} f^{-1}(B_\alpha)$$

it follows that $f^{-1}(B)$ is a (τ_i, τ_j) – fuzzy strongly preopen set as a union of (τ_i, τ_j) – fuzzy strongly preopen sets.

$(ii) \Rightarrow (iii)$ Can be proved by using the complement.

$(iii) \Rightarrow (iv)$ Let A be any fuzzy set of X. Then $(\eta_i, \eta_j) - cl_\delta f(A)$ is a (η_i, η_j) – fuzzy δ– closed set of Y. According to the assumption $f^{-1}((\eta_i, \eta_j) - cl_\delta f(A))$ is a (τ_i, τ_j) – fuzzy strongly preclosed set of X. Hence

$$\begin{aligned}
(\tau_i, \tau_j) - \text{spcl}\, A &\leq (\tau_i, \tau_j) - \text{spcl}\, f^{-1}(f(A)) \\
&\leq (\tau_i, \tau_j) - \text{spcl}\, f^{-1}((\eta_i, \eta_j) - cl_\delta f(A)) \\
&= f^{-1}((\eta_i, \eta_j) - cl_\delta f(A)).
\end{aligned}$$

so

$$f((\tau_i,\tau_j)-\operatorname{spcl} A)\leq(\eta_i,\eta_j)-cl_\delta f(A).$$

$(iv)\Rightarrow(v)$ Let B be any fuzzy set of Y. From the assumption it follows that

$$(\tau_i,\tau_j)-\operatorname{spcl} f^{-1}(B)\leq f^{-1}f(\tau_i,\tau_j)-\operatorname{spcl} f^{-1}(B)\leq f^{-1}(\eta_i,\eta_j)-cl_\delta B.$$

$(v)\Rightarrow(vi)$ Can be proved by using the complement.

$(vi)\Rightarrow(i)$ Let B be any (η_i,η_j) – fuzzy regular open set of Y. Then $B=(\eta_i,\eta_j)-\operatorname{int}_\delta B$. According to the assumption

$$\begin{aligned} f^{-1}(B) &= f^{-1}((\eta_i,\eta_j)-\operatorname{int}_\delta B)\\ &\leq (\tau_i,\tau_j)-\operatorname{spint} f^{-1}(B)\\ &\leq f^{-1}(B). \end{aligned}$$

Hence $f^{-1}(B)=(\tau_i,\tau_j)-\operatorname{spint} f^{-1}(B)$, so $f^{-1}(B)$ is a (τ_i,τ_j) – fuzzy strongly preopen set. Thus f is a fuzzy pairwise almost strongly precontinuous mapping. □

Theorem 3.6 *Let $f:(X,\tau_1,\tau_2)\to(Y,\eta_1,\eta_2)$ be a bijective mapping from a fbts X into an fbts Y. The mapping f is a fuzzy pairwise almost strongly precontinuous if and only if*

$$(\eta_i,\eta_j)-int_\delta f(A)\leq f((\tau_i,\tau_j)-spint\, A),$$

for each fuzzy set A of X.

Proof. We suppose that f is fuzzy pairwise almost strongly precontinuous. Then $f^{-1}((\eta_i,\eta_j)-\operatorname{int}_\delta f(A))$ is a (τ_i,τ_j) – fuzzy strongly preopen set of X, for any fuzzy set A of X. Since f is injective, from Theorem 3.5 it follows that

$$\begin{aligned} f^{-1}((\eta_i,\eta_j)-\operatorname{int}_\delta f(A)) &= (\tau_i,\tau_j)-\operatorname{spint} f^{-1}((\eta_i,\eta_j)-\operatorname{int}_\delta f(A))\\ &\leq (\tau_i,\tau_j)-\operatorname{spint} f^{-1}f(A)\\ &= (\tau_i,\tau_j)-\operatorname{spint} A. \end{aligned}$$

Again, since f is surjective, we obtain

$$\begin{aligned} (\eta_i,\eta_j)-\operatorname{int}_\delta f(A) &= (\eta_i,\eta_j)-ff^{-1}((\eta_i,\eta_j)-\operatorname{int}_\delta f(A))\\ &\leq f((\tau_i,\tau_j)-\operatorname{spint} A). \end{aligned}$$

Conversely, let B be any (η_i, η_j) – fuzzy δ – open set of Y . Then (η_i, η_j) – $\operatorname{int}_\delta B = B$. According to the assumption,

$$\begin{aligned} f((\tau_i, \tau_j) - \operatorname{spint} f^{-1}(B)) &\geq (\eta_i, \eta_j) - \operatorname{int}_\delta f f^{-1}(B) \\ &= (\eta_i, \eta_j) - \operatorname{int}_\delta B \\ &= B. \end{aligned}$$

Thus implies that

$$f^{-1} f((\tau_i, \tau_j) - \operatorname{spint} f^{-1}(B)) \geq f^{-1}(B).$$

Since f is injective we obtain

$$(\tau_i, \tau_j) - \operatorname{spint} f^{-1}(B) = f^{-1} f((\tau_i, \tau_j) - \operatorname{spint} f^{-1}(B)) \geq f^{-1}(B).$$

Hence

$$(\tau_i, \tau_j) - \operatorname{spint} f^{-1}(B) = f^{-1}(B),$$

so $f^{-1}(B)$ is a (τ_i, τ_j) – fuzzy strongly preopen set.

Thus f is a fuzzy pairwise almost strongly precontinuous mapping. □

Definition 3.2 *A mapping $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ from a fbts X into a fbts Y is called*

(1) fuzzy pairwise almost precontinuity if $f^{-1}(B) \in (\tau_i, \tau_j)$ – FPO for each $B \in (\eta_i, \eta_j)$ – FRO

(2) fuzzy pairwise almost strong semicontinuity if $f^{-1}(B) \in (\tau_i, \tau_j)$ – FSSO for each $B \in (\eta_i, \eta_j)$ – FRO

(3) fuzzy pairwise almost semicontinuity if $f^{-1}(B) \in (\tau_i, \tau_j)$ – FSO for each $B \in (\eta_i, \eta_j)$ – FRO.

Theorem 3.7 *Let (X_1, τ_2, τ_2), $(X_2, \omega_1, \omega_2)$, (Y_1, η_1, η_2) and $(Y_2, \sigma_1, \sigma_2)$ be fbts's such that X_1 is a product related to X_2. Then the product*

$$f_1 \times f_2 : (X_1 \times X_2, \theta_1, \theta_2) \to Y_1 \times (Y_1 \times Y_2, \lambda_1, \lambda_2)$$

where θ_k (resp. λ_k is the fuzzy product topology generated by σ_k and ω_k (resp. η_k and σ_k) (for $k = 1, 2$) of fuzzy almost strong precontinuous mappings $f_1 : (X_1, \tau_2, \tau_2) \to (X_2, \omega_1, \omega_2)$ and $f_2 : (Y_1, \eta_1, \eta_2) \to (Y_2, \sigma_1, \sigma_2)$ is a fuzzy pairwise almost precontinuous mapping.

Proof. Let $B = \vee\Big(U_\alpha \times V_\beta\Big)$, where U_α and V_β are η_i- fuzzy open sets of Y_1 and σ_i – fuzzy open sets of Y_2. From

$$\begin{aligned}
(f_1 \times f_2)^{-1}(B) &= \vee\Big[f_1 \times f_2\Big]^{-1}(U_\alpha \times V_\beta)\Big] \\
&= \vee(f_1^{-1}(U_\alpha) \times f_2^{-1}(V_\beta)) \\
&\le \vee\Big((\tau_i,\tau_j) - \text{spint}\, f_1^{-1}(\eta_i - \text{int}\,(\eta_j - clU_\alpha)) \\
&\qquad \times(\omega_i,\omega_j) - \text{spint}\, f_2^{-1}(\eta_i - \text{spint}\,(\eta_j - clV_\beta)\Big) \\
&\le \vee\Big((\tau_i,\tau_j) - p\ \text{spint}\, f_1^{-1}(\eta_i - \text{int}\,(\eta_j - clU_\alpha)) \\
&\qquad \times(\omega_i,\omega_j) - p\text{int}\, f_2^{-1}(\eta_i - \text{int}\,(\eta_j - clV_\beta)\Big) \\
&\le \vee\Big((\theta_i,\theta_j) - p\ \text{int}\,(f_1^{-1}(\eta_i - \text{int}\,(\eta_j - clU_\alpha))) \\
&\qquad \times f_2^{-1}(\eta_i - \text{int}\,(\eta_j - cl(V_\beta)))\Big) \\
&\le (\theta_i,\theta_j) - p\text{int}\,(\vee\Big(f_1^{-1}(\eta_i - \text{int}\,(\eta_j - clU_\alpha)) \\
&\qquad \times f_2^{-1}(\eta_i - \text{int}\,(\eta_j - clV_\beta))\Big) \\
&\le (\theta_i,\theta_j) - p\text{int}\,(f_1 \times f_2)^{-1}\Big(\vee\,(\eta_i - \text{int}\,(\eta_j - clU_\alpha) \\
&\qquad \times \eta_i - \text{int}\,(\eta_i - clV_\beta))\Big) \\
&\le (\theta_i,\theta_j) - p\text{int}\,(f_1 \times f_2)^{-1}\Big(\vee\,(\lambda_i - \text{int}\,(\lambda_j - cl(U_\alpha \times V_\beta)))\Big) \\
&\le (\theta_i,\theta_j) - p\text{int}\,(f_1 \times f_2)^{-1}\Big((\lambda_i - \text{int}\,(\lambda_j - cl(\vee(U_\alpha \times V_\beta)))\Big) \\
&= (\theta_i,\theta_j) - p\text{int}\,(f_1 \times f_2)^{-1}\Big(\lambda_i - \text{int}\,(\lambda_j - clB)\Big),
\end{aligned}$$

it follows that $f_1 \times f_2$ is fuzzy pairwise almost precontinuous mapping. □

Theorem 3.8 *Let X, X_1, X_2 be fbts's and $p_k : X_1 \times X_2 \to X_i (k = 1, 2)$ are the projections of $X_1 \times X_2$ onto X_i. If $f : X \to X_1 \times X_2$ is a fuzzy parwise almost strongly precontinuous, then $p_i f$ are also fuzzy pairwise almost strongly precontinuous mappings.*

Proof. Follows from the fact that the projection mappings are fuzzy continuous $(p_k : (X_1 \times X_2) \to X_k,\ k = 1, 2)$ □

Theorem 3.9 *Let $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a mapping from a fbts X into a fbts Y. If the graph $g : (X, \tau_1, \tau_2) \to (X \times Y, \theta_1, \theta_2)$ of f is fuzzy pairwise almost strongly precontinuous, then is fuzzy pairwise almost strongly precontinuous.*

Proof. By Lemma 2.2 of [1] $f^{-1}(B) = 1 \wedge f^{-1}(B) = g^{-1}(1 \times B)$, for each (η_i, η_j) – fuzzy regular open set B of Y. Since g is fuzzy pairwise almost precontinuous and $1 \times B$ is a $(\theta_i, \theta_j\ )$ – fuzzy regular open set of $X \times Y, f^{-1}(B)$ is a (τ_i, τ_j) – fuzzy strongly preopen set of X , so f is fuzzy pairwise almost strongly precontinuous. □

Theorem 3.10 *Let $f : (X, \tau_1, \tau_2) \to (Y, \eta_1, \eta_2)$ be a fuzzy pairwise strongly preopen and fuzzy pairwise strongly precontinuous irresolution from a fbts X into a fbts Y and let*

$$g : (Y, \eta_1, \eta_2) \to (Z, \sigma_1, \sigma_2)$$

be a mapping from fbts Y into fbts Z. The mapping gf is a fuzzy pairwise almost strongly precontinuous if and only if g is a fuzzy pairwise almost strongly precontinuous.

Proof. Let gf be a fuzzy pairwise almost strongly precontinuous. Then

$$g^{-1}(C) = f(gf)^{-1}(C)$$

is a (η_i, η_j) – fuzzy strongly preopen set of Y , for each (σ_1, σ_2) – fuzzy regular open set C of Z. Hence g is a fuzzy almost strongly precontinuous mapping.

Conversely, let g be a fuzzy pairwise almost strongly precontinuous mapping and let C be a (σ_1, σ_2) – fuzzy regular open set of Z. From $(gf)^{-1}(C) = f^{-1}(g^{-1}(C))$ it follows that gf is a fuzzy pairwise almost strongly precontinuous mapping. □

References

[1] K. K. Azad, *On fuzzy precontinuity, fuzzy almost continuity and fuzzy weakly continuity*, J. Math. Anal. Appl. **82** (1981), 14–32.

[2] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. **24** (1968), 182–190.

[3] B. Krsteska, *Fuzzy strongly preopen sets and fuzzy strong precontinuity*, Mat. Vesnik, **50** (1998), 111–123.

[4] B. Krsteska, *Fuzzy pairwise strong precontinuity*, submited.

[5] A. Kandil, *Biproximaties and fuzzy bitopological spaces*, Simon Steven **63** (1989), 45–66.

[6] S. Sampath Kumar, *Semiopen sets, semicontinuity and semiopen mappings in fuzzy bitopological spaces*, Fuzzy Sets and Systems **64** (1994), 421–426.

[7] S. Sampath Kumar, *On fuzzy pairwise α – continuity and fuzzy pairwise precontinuity*, Fuzzy Sets and Systems **62** (1994), 231–238.

[8] L. A. Zadeh, *Fuzzy sets*, Information and Control **8** (1965), 338–353.