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THE DEGREE OF PROGRESS TO GOAL DURING SELF–ORGANIZATION PROCESS

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Abstract. Adeagbo-Sheikh [1] gave the detail about the notions of the 'distance function $(g(t))'$ and the 'working functions $(y_i^*(t), i = 1, 2, ..., n)'$. The working functions are the functions which describe the behaviours of the subsystems during self-organization process. The procedure for obtaining these working functions for a system of three particles which is self-organizing to keep the particles in a straight line was discussed in Olatinwo and Adeagbo-Sheikh[4].

In this paper, we employ some concepts of the probability theory, elementary properties of curve as well as the distance function to determine the degree of progress to goal at any stage during self-organization process. We consider the probability of reaching the goal at any stage of the self-organization process as its degree of process to goal during this process.

The results obtained are in agreement with the axiomatic properties of probability.

1. INTRODUCTION

In his model for self-organizing systems, Adeagbo-Sheikh [1] gave the detail about the views of some notable thinkers such as Ashby [2], Beer [5] and Von Foerster [8] by employing the notions of a 'distance function $(g(t))'$ and that of a 'controlled-disturbance function $(h(g(t)))'$, where t is the time variable. The working functions for a system of three particles which is self-organizing to keep the particles in a straight line were determined in Olatinwo and Adeagbo-Sheikh [4] by reducing the distance-from-goal expression to the distance function.

The objective of this paper is to determine the level or degree of progress to goal at any stage during self-organization process. The results obtained are in agreement with the axiomatic properties of probability. We employ some concept of the probability theory as well as the elementary idea of the curve theory in our study. The study becomes relevant for its applications in diverse areas, espcially in learning/adaptive control and pattern recognition systems. Theories of learning are available in literature and invariably use statistical techniques. See Fu and Mendel [6].

We will take our self-organizing system to be in the sense of Ramon-Margalef (see Beer [5]) in the next section.

2. MAIN RESULTS

In this section, we shall discuss about the determination of the level of progress to goal in self-organization process. However, we recall in this section the following definition:

Definition 2A. *The distance function, $(g(t))$, is the distance from the goal at any time satisfying the following properties:*

(i) $g(t) > 0, t_0 \leq t < t_n < \infty$

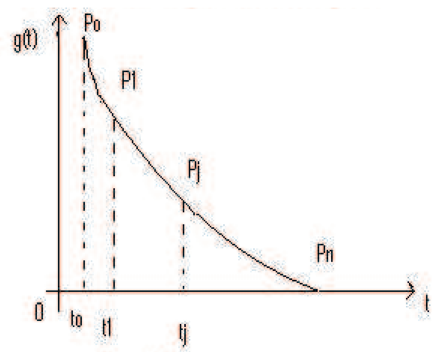
(ii) $g,(t) < 0, t_0 < t < t_n < \infty$

(iii) $g(t_n) = 0, t_0 < t_n < \infty$

(iv) $|g,(t)| < \infty, t_0 < t < t_n < \infty$

See Adeagbo-Sheikh [1] for this definition.

The graph of the function, $g(t)$, is shown in the figure 2.1 below:



This graph is also contained in Olationwo and Adeagbo-Sheikh [1]. The property (ii) of $g(t)$ above shows that $g(t)$ is a monotone decreasing function. The progress of the system to self-organization stage from time t_0 is obvious from the graph. The system begins to self-organize towards some desired state of affairs at time t_0 and the self-organization process gets to completion at time t_n (i.e. $g(t_n) = 0$), property (iii).

Let the successive points $P_0, P_1, ..., P_j, ..., P_n$ on the curve be the successive stages reached during self-organization process at times $t_0, t_1, ..., t_j, ..., t_n$ respectively. Let $\ell(t)$ denote the length of the curve at any time, t. Thus, the length of the curve (see Bruce and Giblin [3]) is given by

$$\ell(t) = \int_{t_0}^{t} ||g'(u)||du \tag{2.1}$$

We assume that the gragh defined by $g(t)$ is regular.

From eqn(2.1), $\ell(t_0) = 0$

$$\ell(t_n) = \int_{t_0}^{t_n} ||g'(u)||du > 0, \qquad \text{since} \qquad t_n > 0$$

Definition 2B. *Let X_k be the event that a self-organizing system attains a stage $P_k$at time t_k during self-organization process. Then, the probability of this event is given by*

$$Prob\{X_k\} = \frac{\ell(t_k)}{\ell(t_n)} \tag{2.2}$$

where $k = 0, 1, 2, \ldots, n$.

The theorem below gives the probability that the self-organizing system attains a stage P_k at time t_k for each $k \in N$ during self - organization process:

Theorem 2A. *Suppose that $[t_0, t_k]$ and $[t_0, t_n]$ are two given time intervals such that $[t_0, t_k] \subseteq [t_0, t_n]$. Let X_k be the event that the self-organizing system attains a stage P_k at time t_k during self-organization process. Then,*

$$Prob\{X_k\} = \frac{\int_{t_0}^{t_k} ||g'(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du} \tag{2.3}$$

where $k \leq n,\ k, n \in N$.

Proof. X_k is the event that the self-organizing system attains a stage P_k at time t_k , $k = 0, 1, \ldots, n$.

We have from equation (2.1) that

$$\ell(t_k) = \int_{t_0}^{t_k} ||g'(u)||du.$$

Substituting for $\ell(t_k)$ and $\ell(t_n)$ in equation (2.2), then

$$Prob\{X_k\} = \frac{\int_{t_0}^{t_k} ||g'(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du}$$

This completes the proof of the theorem.

Corollary 2.A. *Let the hypotheses of theorem (2A) hold. Then,*

$$Prob\{X_k\} = \frac{\sum_{j=1}^{k} \int_{t_{j-1}}^{t_j} ||g'(u)||du}{\sum_{j=1}^{n} \int_{t_{j-1}}^{t_j} ||g'(u)||du} \tag{2.4}$$

where $n, k \in N,\ k \leq n$.

Proof. By considering finite union of intervals (see Kai Lai Chung [7]) which can be split up into disjoint ones,

i.e. $[t_0, t_k] = \cup_{j=1}^{k} [t_{j-1}, t_j]$ and $[t_0, t_n] = \cup_{j=1}^{n} [t_{j-1}, t_j]$

then equation (2.3) becomes

$$\begin{aligned}
Prob\{X_k\} &= \frac{\int_{t_0}^{t_k} ||g'(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du} \\
&= \frac{\int_{t_0}^{t_1} ||g'(u)||du + \int_{t_1}^{t_2} ||g'(u)||du + ... + \int_{t_{k-1}}^{t_k} ||g'(u)||du}{\int_{t_0}^{t_1} ||g'(u)||du + \int_{t_1}^{t_2} ||g'(u)||du + ... + \int_{t_{n-1}}^{t_n} ||g'(u)||du} \\
&= \frac{\sum_{j=1}^{k} \int_{t_{j-1}}^{t_j} ||g'(u)||du}{\sum_{j=1}^{n} \int_{t_{j-1}}^{t_j} ||g'(u)||du}
\end{aligned}$$

This completes the proof of the corollary.

According to Adeagbo-Sheikh [1], $S = \{S_1, S_2, \ldots, S_m\}$ is the set of the subsystems or elements of self-organizing systems. The set $A = \{A_1, A_2, \ldots, A_m\}$ is the corresponding set of activities, and $y(t) = (y_1(t), y_2(t), \ldots, y_m(t))$ is the vector whose components measure the level or aggregate effects of respective activities from time $t_0 \geq 0$ to time t.

In this paper, we are interested in finding the level of contribution or efficiency of each subsystem S_j from time t_0 to time t_n during self-organization process. We then use it to find the probability for the overall level of the self-organization process. This idea is summarized in the results below.

Theorem 2B. *Let X_k be the event that the subsystems S_k have aggregate effects $y_k(t)$, $k = 1, 2, \ldots, m$ in the time interval $[t_0, t_n]$ during self-organization process. If A_k, $k = 1, 2, \ldots, m$ are the corresponding activities over the same time interval, then*

$$Prob\{X_k\} = \frac{\sum_{j=1}^{n} \int_{t_{j-1}}^{t_j} ||y_k'(u)||du}{\sum_{j=1}^{n} \int_{t_{j-1}}^{t_j} ||g'(u)||du} \tag{2.5}$$

where $k = 1, 2, ..., m$.

Proof. Using equations (2.1) and (2.2), we have that

$$Prob\{X_k\} = \frac{\int_{t_0}^{t_n} ||y_k'(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du}$$

Applying corollary (2A) gives

$$Prob\{X_k\} = \frac{\sum_{j=1}^{n} \int_{t_{j-1}}^{t_j} ||y_k'(u)||du}{\sum_{j=1}^{n} \int_{t_{j-1}}^{t_j} ||g'(u)||du}$$

This complete the proof.

Theorem 2C. *Suppose that the subsystems S_k, $k = 1, 2, \ldots, m$ are independent with corresponding activities A_k, $k = 1, 2, \ldots, m$ and let $y_k(t)$, $k = 1, 2, \ldots, m$ be the corresponding aggregate effects in the time interval $[t_0, t_n]$ during self-organization. Let X_k be the event that the subsystems S_k have the aggregate effects $y_k(t)$ over the time interval $[t_0, t_n]$ during self-organization process. Then,*

$$Prob\{\cap_{k=1}^{m} X_k\} = \frac{\Pi_{k=1}^{m} \sum_{j=1}^{n} \int_{t_{j-1}}^{t_j} ||y_k'(u)||du}{\left(\sum_{j=1}^{n} \int_{t_{j-1}}^{t_j} ||g'(u)||du\right)^m} \tag{2.6}$$

Proof. By theorem (2B), we have that

$$Prob\{X_1\} = \frac{\int_{t_0}^{t_n} ||y_1'(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du},$$

$$Prob\{X_2\} = \frac{\int_{t_0}^{t_n} ||y_2'(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du}, \quad \ldots, \quad Prob\{X_m\} = \frac{\int_{t_0}^{t_n} ||y_m'(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du}$$

Since the subsystems are independent, then $X_1, X_2, ..., X_m$ are independent events. Hence

$$Prob\{\cap_{k=1}^{m} X_k\} = Prob\{X_1\} \cdot Prob\{X_2\} \cdots Prob\{X_m\} = Prob\{X_1 \cap X_2 \cap ... \cap X_m\},$$

so that, we have

$$\begin{aligned} Prob\{\cap_{k=1}^{m} X_k\} &= \left(\frac{\int_{t_0}^{t_n} ||y_1'(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du}\right)\left(\frac{\int_{t_0}^{t_n} ||y_2'(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du}\right) \cdots \left(\frac{\int_{t_0}^{t_n} ||y_m'(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du}\right) \\ &= \frac{\Pi_{k=1}^{m} \left(\int_{t_0}^{t_n} ||y_k'(u)||du\right)}{\left(\int_{t_0}^{t_n} ||g'(u)||du\right)^m} \end{aligned}$$

By corollary (2A), we have that

$$Prob\{\cap_{k=1}^{m} X_k\} = \frac{\Pi_{k=1}^{m} \left(\sum_{j=1}^{n} \int_{t_{j-1}}^{t_j} ||y_k'(u)||du\right)}{\left(\sum_{j=1}^{n} \int_{t_{j-1}}^{t_j} ||g'(u)||du\right)^m}$$

This completes the proof of the theorem.

It is important to characterize two stages during the self-organization process. The stages are P_0 and P_n which are obvious from the figure (2.1).

This characterization will be done by the aid of the indicator or characteristic function as follows:

In theorem (2A), when $k = 0$ in eqn (2.3), then X_0 is the event that the self-organizing system attains the stage P_0 at time t_0. Hence,

$$Prob\{X_0\} = \frac{\int_{t_0}^{t_0} ||g'(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du} = 0,$$

i.e. this stage corresponds to the start of the self-organization process. Similarly, when $k = n$ in eqn (2.3), then X_n is the event that the self-organizing system attains the stage P_n at time t_n.

Hence,

$$Prob\{X_n\} = \frac{\int_{t_0}^{t_n} ||g'(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du} = 1$$

i.e. this stage corresponds to the end of the self-organization process (or the self-organization stage is reached at time, t_n). In terms of the indicator function, the two probabilities are expressed in the form:

$$Prob\{A\} = I_A(w) = \begin{cases} 1, & \text{if} \quad \omega \in A \\ 0, & \text{if} \quad \omega \notin A \end{cases}$$

where A denotes the set of members that reach the target at time t_n and A' is the set of those at the point P_0 at time t_0, $\omega \in A$.

Theorem 2D. *Suppose that the subsystems S_k, $k = 1, 2, \ldots, m$ have corresponding activities A_k, $k = 1, 2, \ldots, m$ and let $y_k^*(t)$, $k = 1, 2, \ldots, m$, be the working functions in the interval $[t_0, t_n]$ during self-organization stage. Suppose further that X_k, $k = 1, 2, \ldots, m$ is event that the subsystem S_k, $k = 1, 2, \ldots, m$ when the self-organization stage is reached during the interval $[t_0, t_n]$ and that X is the event that the system $S = \{S_k\}_{k=1}^m$ attains the self-organization stage over the same time interval. Then*

$$Prob\{X\} = 1. \tag{2.8}$$

Proof. By theorem (2B), we have that

$$Prob\{X_1\} = \frac{\int_{t_0}^{t_n} ||{y_1}^{*\prime}(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du},$$

$$Prob\{X_2\} = \frac{\int_{t_0}^{t_n} ||{y_2}^{*\prime}(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du}, \; \ldots \; , \; Prob\{X_m\} = \frac{\int_{t_0}^{t_n} ||{y_m}^{*\prime}(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du}$$

The subsystems $S_1, S_2, \ldots, S_m$ are not independent when the self-organization stage is reached. Rather, the subsystems bias one another and this situation becomes mutually exclusive. Hence,

$$X = \cup_{k=1}^{m} X_k = X_1 \cup X_2 \cup \ldots \cup X_m, \qquad X_1 \cap X_2 \cap \ldots \cap X_m = \phi,$$

$$Prob\{X\} = Prob\{\cup_{k=1}^{m} X_k\} = Prob\{X_1\} + Prob\{X_2\} + ... + Prob\{X_m\}$$

$$\begin{aligned} Prob\{X_1\} &= \left(\frac{\int_{t_0}^{t_n} ||{y_1}^{*\prime}(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du}\right) + \left(\frac{\int_{t_0}^{t_n} ||{y_2}^{*\prime}(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du}\right) + ...\left(\frac{\int_{t_0}^{t_n} ||{y_m}^{*\prime}(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du}\right) \\ &= \frac{\int_{t_0}^{t_n} ||{y_1}^{*\prime}(u)||du + \int_{t_0}^{t_n} ||{y_2}^{*\prime}(u)||du + ... + \int_{t_0}^{t_n} ||{y_m}^{*\prime}(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du} \\ &= \frac{\int_{t_0}^{t_n} ||g'(u)||du}{\int_{t_0}^{t_n} ||g'(u)||du} = 1, \end{aligned}$$

since

$$\int_{t_0}^{t_n} ||g'(u)||du = \int_{t_0}^{t_n} ||{y_1}^{*\prime}(u)||du + ... + \int_{t_0}^{t_n} ||{y_m}^{*\prime}(u)||du,$$

that is, the working functions are determined by the distance function.

3. CONCLUSION

The degree or level of progress to goal at any time and stage of the self-organization process was determined. The results obtained were in agreement with the axiomatic properties of probability.

It was observed from eqn(2.6) of theorem (2C) that the system could not attain self-organization stage when the subsystems were independent, whereas from eqn (2.8) of theorem (2D) it was clear that the subsystems biased one another for self-organization stage to be attained. So the subsystems must be mutually exclusive if the system must reach self-organization stage.

It is our hope that the results obtained in this paper will be generalized in a future paper.

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