Kragujevac J. Math. 33 (2010) 5–15.

A GEOMETRICAL DESCRIPTION OF VISUAL SENSATION

An anisotropic retouche of Koenderink and van Doorn's view on the nature of observation in accordance with the work of Milutin Borisavljević on scientific aesthetics

Bart Ons[1] and Paul Verstraelen[2]

[1] *Katholieke Universiteit Leuven, Laboratory of Experimental Psychology*
(e-mail: Bart.Ons@psy.kuleuven.be)
[2]*Katholieke Universiteit Leuven, Section of Geometry*
(e-mail: Leopold.Verstraelen@wis.kuleuven.be)

(Received February 17, 2010)

Milutin Borisavljević, Andrea van Doorn and Jan Koenderink (courtesy Sergio Servellon)

Abstract. The following are some *oversimplified verbal definitions regarding sensation and perception*, which however might be not entirely useless, even if only by keeping them somewhere in the back of our heads while scientifically studying perception and sensation within the field of psychology. A sensation is the observation made when observing something. In particular, a visual sensation is the observation of light energy when looking at something. A perception is the appreciation made of a sensation. In particular, a visual perception is what we see when looking at something: in ”*early vision*” this refers to what could be called instinctive perceptions and in ”*cognitive vision*” this refers to possibly various ways of thinking about or interpreting the naked perceptions made in early vision.

In [1] [2] [3] [4] essentially *perception is defined via the Casorati curvature of sensation*, or, more precisely, ”early” human perceptions are defined as the most rudimentary, the most intuitive surface curvatures of human sensations, whereby the latter are defined to be human observations as described in detail by Koendernik and van Doorn basically for all most natural human observations, and, in particular, very concretely for visual human observations, in [5] [6] [7]. The main purpose of the present article is to bring a kind of refinement to Koenderink and van Doorn's description of human visual observation in order to take into account the factual anisotropy which hereby occurs, most dramatically illustrated by the different evaluations of geometrically equal lengths in the horizontal and vertical directions, respectively, cfr. [8] [9], which ”*horizontal-vertical visual effect*” for instance can be observed pretty distinctly in the following 1858 figure of W. Wundt (see Figure 1).

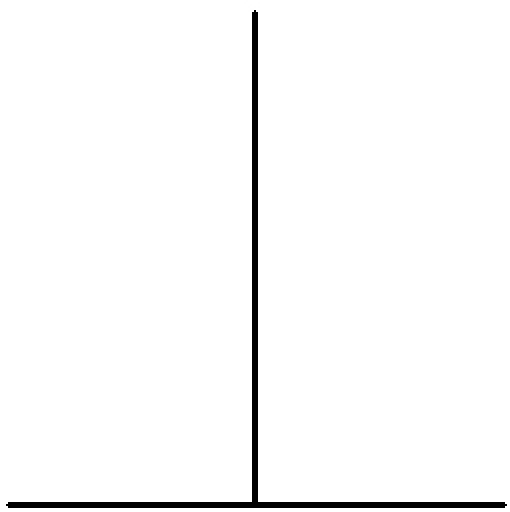


Figure 1. The horizontal-vertical illusion. The vertical line is perceived longer than the horizontal line although both lines are geometrically of equal length.

For most visual perceptions, this amendment to the isotropic phenomenology of human visual sensations is not of crucial importance from practical points of view, but, still, for quite a number of visual perceptions it is rather significant to properly take into account the above anisotropy. In any case, in Section 1, from [5] we will briefly recall some elements concerning the nature of visual observation following J. Koenderink and A. van Doorn. This presentation will not be done in a subtle way. We will take the most naive possible approach to this matter, referring the reader who is in need for a more serious treatment to [5] [6] [7]. Next, in Section 2, we will at first recall some basic facts related to visual anisotropies due to M. Borisavljević [10], and in this context also will consider some studies of G. Fechner and A. Fick; (in these respects, [11] could be mentioned here as a reference with many references showing some psychologists' kinds of ways of relating to the golden numbers $\varphi = (\sqrt{5}-1)/2$ and $\Phi = (\sqrt{5}+1)/2$). And, then, we will introduce *a scientific description of visual sensation which, at least qualitatively and at present, seems to be most realistic and natural.* This new geometrical model for human visual sensation was inspired by the results of some experiments from Ons whereby the horizontal-vertical visual effect plays a role of crucial importance, as is reported on in [15].

2010 Mathematics Subject Classification: 91E99, 92C20, 91E30, 53B25.

Key words: Human visual sensation, Perception, Anisotropic smoothing, Casorati curvature.

1. ON THE NATURE OF OBSERVATION FOLLOWING KOENDERINK AND VAN DOORN

The following could well be considered as essentially being concerned with a basic scientific phenomenology of several of the human sensations in general. Our present study being exclusively limited to discussing visual sensation and perception, what is needed further on from the more general work by Koenderink and van Doorn will be specifically formulated right from the start for visual sensation and perception only.

For *images* in an (x, y)–plane R^2, let the *intensity* I be considered in a z–direction R as a scalar field on R^2. Typically, the (x, y)–plane R^2 is regarded as a Euclidean plane E^2, but, since the z and the (x, y) co–ordinates of the resulting $3D$ space $R^3 = R^2 \times R$ are obviously *incommensurable*, the relief which is the graph of the image intensity I in R^3 should geometrically not be treated as a surface in 3–dimensional Euclidean space E^3: rather the z–direction might measure a logarithm of I (cfr. also the classical studies of Plateau, Fechner, Weber, Stevens,... [8] [9], and, most in particular for vision, see [7]) and instead of using Euclidean geometry in the study of this graph–surface in the ambient $3D$ space R^3, priority might rather be given to the $(2+1)D$– or (in the sense of Strübecker and Sachs isotropic, i.e.) the *degenerate* or *null–Riemannian geometry* determined in the chart (x, y, z) of R^3 by the metrical fundamental form $ds^2 = dx^2 + dy^2$. However, for the main point that we want to make in the present article, this matter is only of secondary importance at this stage; and so, moreover in the knowledge that this point will have more chances of getting across much better when thinking of the ambient $3D$ space $R^3 = R^2 \times R$ basically as a $3D$–Euclidean space, hereafter we will yet do so indeed.

For reasons well explained in [5] [6], the evident blurring of the image intensities $I(x, y; s)$, whereby s is the *scale parameter*, (related to the level of resolution d by $s = d^2$), which one needs to take into account in order that scalar fields could be defined after all, essentially and necessarily, is done through *convolution with Gaussian kernels*

$$G_0(x, y; s) = \frac{1}{4\pi s}\, e^{\frac{-(x^2+y^2)}{4s}}, \tag{1}$$

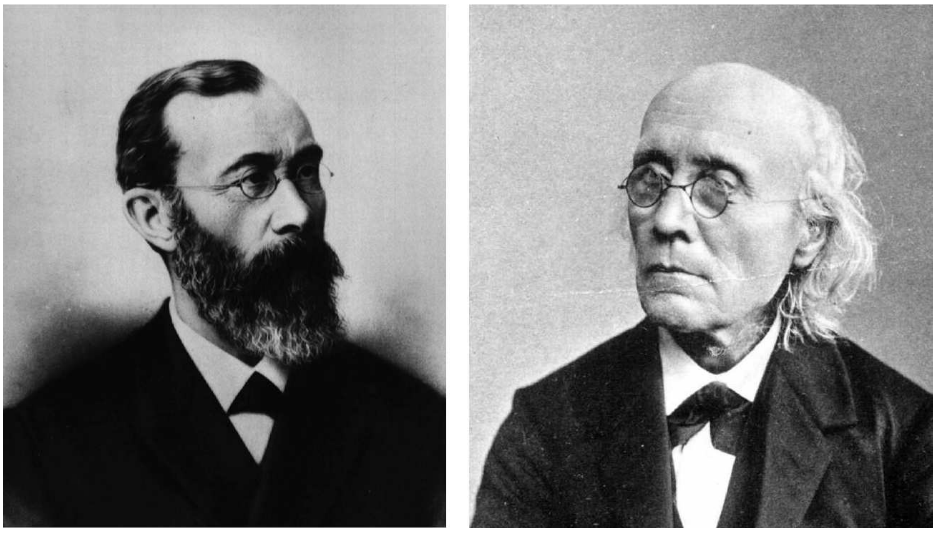
when considering the (x, y)–plane R^2 geometrically as a Euclidean plane E^2. In the context of the *scale space setting of observations*, accordingly:

$$I(x, y; s) = I(x, y) \otimes G_0(x, y; s), \tag{2}$$

as one finds appropriately discussed in [5] and [6].

2. ON THE HORIZONTAL–VERTICAL VISUAL EFFECT AND ITS IMPACT ON A SCIENTIFIC DEFINITION OF VISUAL SENSATION

In his 1954 book on "the golden number", M. Borisavljević in particular aims to offer scientific explanations of the universal taste of beauty of "golden rectangles" (i.e. of rectangles for which the proportion width : length equals the proportion $\varphi : 1$), and from the 1958 English version [10] of the book comes the following quotation: "*The visual field for both eyes represents an oval shape exactly inscribed in a (horizontal) golden rectangle. ... The beautiful is what corresponds to our nature (here to the nature of our eyes) and this fact explains the beauty of the golden rectangle*". In agreement with this fact concerning the horizontal–vertical anisotropy of the human visual field, (and as is very well known, in contrast with the former), our eyes are not equally accommodated for seeing horizontally and for seeing vertically, (and, with a little sense for some imagination, one could well think of some cultural evolutional origin for this matter). And in the course of time, many interesting quantitative studies of this phenomenon have been made; besides with respect to the above mentioned Figure 1 of Wundt, for instance, we further mention some studies by G. Fechner and A. Fick relating a.o. to the distances from which visual observations of paintings and of buildings are made, referring to [10] and [11], respectively, and the references therein; for more recent work relating to this phenomenon, see e.g. also [12] [13] [14]. In any case, such studies illustrate the varying ways in which our visual system may deal with visual measures of objects depending on their eventual orientations.



Wilhelm Wundt and Gustav Fechner

To take this fact into account in a geometric, i.e. in a scientific description of human vision, we next propose the following amendment in this respect to the above recalled model for visual sensation of Koenderink and van Doorn. Staying in tune with the earlier made comments of describing hereby "things" exclusively in a Euclidean way and remaining consistent with the previous notations, we define a visual sensation at a *scale* (a_h, a_v) by

$$I(x, y; a_h, a_v) = I(x, y) \otimes G_0(x, y; a_h, a_v), \tag{3}$$

whereby *now convolutions are made with "elliptical" Gaussians*

$$G_0(x, y; a_h, a_v) = \frac{1}{2\pi a_h a_v} \, e^{-\frac{1}{2}\left((\frac{x}{a_h})^2 + (\frac{y}{a_v})^2\right)}, \tag{4}$$

having essentially $0 < a_h < a_v$ corresponding to the general experience of the horizontal–vertical effect, i.e.: a_h and a_v denoting respectively the horizontal and the vertical axis of the ellipses involved, (versus the former convolutions with "circular" Gaussians $G_0(x, y; s)$). At least at this stage, and in accordance with the announced non–subtle way of presentation, hereby we take the kind of "direct" approach rather than, for instance, to consider solving diffusion equations for light intensities I on Riemann–Finsler (x, y) planes with a metrical indicatrix related to the shape of the human visual field, etcetera. The directness of this approach seems well justified, at least in principle and at present, both from the points of view of naturalness and of effectiveness, respectively with respect to what was stated before and to what will follow later. So, *in summary* and fundamentally referring to [5] (in particular pp. 77–81): *visual sensation is defined via elliptical Gaussian apertures determined by axes* $a_h < a_v$.

3. AN ILLUSTRATION: HUMAN VISUAL SENSATIONS AND THEIR PERCEPTIONS OF ELLIPTICAL GAUSSIAN BLOBS

It is trivially irresistible here and now to concretely illustrate the above in case that *the surfaces* $z = I(x, y)$ themselves are *Gaussians which are essentially elliptical.* In subsequent papers we intend to treat a variety of other images as well, of course. So, hereafter we thus consider such Gaussians of which the main axis in each case makes an angle of 35° with the x–axis, but of which the ratio's of the length of the main axis versus the length of the minor axis varies: for a given length of the main axis, the lengths of the smaller axes become bigger and bigger, cfr. Figure 2. In [15], the *illusory orientation tilts of such Gaussian luminance blobs* are studied. More precisely, the participants to an experiment were presented several essentially elliptical Gaussian stimuli as visual images and they were asked to indicate their perception of orientation of the main axis of these blobs. The discrepancies of these *experienced orientations* in comparison to the *physically "real" orientations* of these Gaussians were measured and a natural account on how such tilt biases do emerge in human visual sensation of such stimuli was then provided via anisotropic smoothing. In Figure 2, the illusory perceptual tilt is put on view to provide the reader a concrete sense of how anisotropic smoothing can affect the perception of a stimulus with a smooth luminance gradient, i.e. a blob with a Gaussian luminance descent. The less steep the descent, the more a convolution with an elliptical Gaussian kernel will affect the perception of the orientation of the stimuli. Perceptually, in each row of Figure 2, any stimulus seems to be tilted away from the horizontal direction more than the one on its left, although, physically, the *main axes* of all these elliptical Gaussians have the same orientation.

Figure 2. In the upper row and in the lower row, three Gaussian luminance profiles are directed $35°$ clockwise and counterclockwise from the horizontal axis. See the main text for an explanation in more detail.

In Figure 3, isotropic (middle column) and anisotropic (right column) smoothing will be contrasted with each other in case of the elliptical Gaussian blob $I(x, y)$ depicted as an image in the middle panel of the first row in Figure 2 and shown explicitly as a surface in R^3 in the upper row, left column of Figure 3. In the middle and the right column, the surfaces in the upper row represent the same Gaussian blob, but smoothed isotropically and anisotropically, respectively. We refer to these surfaces as the sensations $I(x, y; s)$ and $I(x, y; a_h, a_v)$ resulting from a convolution with an isotropic and anisotropic kernel G_0 of the luminance surface $I(x, y)$ depicted at the left. Equal scales were used for isotropic and anisotropic Gaussian kernels, however, in order to demonstrate a clear contrast the ratio of the horizontal and the vertical axis of the anisotropic kernel $a_h : a_v$ was chosen rather drastically as $3 : 1$.

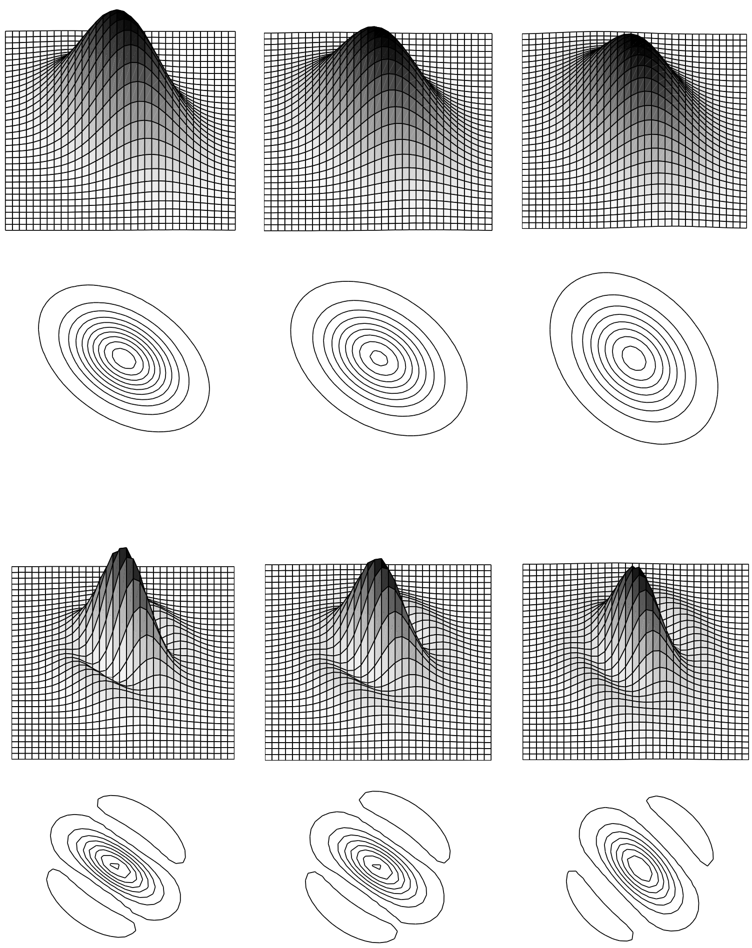


Figure 3. No smoothing (left column), isotropic smoothing (middle column) and anisotropic smoothing (right column). See main text for an explanation in more detail.

The second row shows a topographical representation of the same luminance surfaces and maybe provides a better view on the orientation differences between the two convolved luminance functions with respect to the initial luminance surface, i.e. the anisotropic smoothed luminance profile is more tilted to the vertical direction while the isotropic smoothed luminance profile maintains the same orientation.

In the top half of Figure 3 are presented graphical views on what we intuitively call a sensation. In our geometrical approach, sensation is defined by smoothing the luminance surface by elliptical Gaussian kernels. In previous work, cfr. [1] [2] [3] [4], perception has been defined geometrically via the Casorati curvature of sensation, i.e. the most intuitive surface curvature of the smoothed luminance surface. In the third row of Figure 3, the Casorati curvature surfaces of the surfaces of the first row, and so, also of *the sensation surfaces* shown in the upper row are depicted, and finally in the fourth row, the surfaces of the third row are presented topographically.

Besides the reason to consider here elliptical Gaussians for the images $I(x,y)$ which was indicated at the very beginning of the section, another one is that, qualitatively, in these examples there is no distinction between the orientations presented by the anisotropically smoothed surfaces $I(x,y) \otimes G_0(x,y;a_h,a_v)$ and the orientations of the surfaces determined by their Casorati curvatures, or, still, for these examples, there is no difference between the orientations of the corresponding visual sensations and of their perceptions; (with the aim that eventual readers who are not acquainted with what actually are perceptions in terms of curvature at the time of their first reading of the present paper, could maybe focus somewhat better on the novelty of the present paper itself, i.e. on a scientific definition of visual sensation, rather than to have to get acquainted with scientific definitions of visual sensation and of perception at the same time).

In any case, at least in our opinion, all kinds of so–called illusions, the static as well as the dynamic ones which are pertaining to early vision, do loose the mystery they may have offered and pretty certainly will continue to offer to those who look at them in a state of scientific ignorance. The last paragraph in [16] goes as follows: "*The physical sciences take immense trouble to avoid errors. Here we seek out* *and study errors for understanding how we see and to suggest something of how the brain works. The weird and wonderful errors of illusions are not trivial. They are truly phenomenal phenomena, central to art and a major reason for the experimental methods of science*". These phenomenal phenomena, after all, turn out to be very natural phenomena indeed, if only one cares to reflect scientifically on sensation and perception: "*Nature likes to be looked at with geometers' eyes and brains*".

References

[1] P. Verstraelen, *A remark on visual illusions and neuroscience: "Basically, we only see what there is to be seen"*, Psychological Reports, Laboratory of Experimental Psychology, K. U. Leuven (2003).

[2] P. Verstraelen, *The geometry of eye and brain*, Soochow J. Math. **30** (2004), 367-376.

[3] P. Verstraelen, *A geometrical descripition of visual perception–The Leuven Café Erasmus model and Bristol Café wall illusion*, Kragujevac J. Math. **28** (2005), 7–17.

[4] P. Verstraelen, *Geometry and Vision*, Balkan J. Geom. and Appl. **10(1)** (2005), 95–98.

[5] J. J. Koenderink, A. J. van Doorn, *The structure of relief*, Adv. Imag. Electr. Physics **103** (1998), 65-150.

[6] J. J. Koenderink, *The structure of images*, Biol. Cybern. **50** (1984) 363–370.

[7] J. J. Koenderink, A. J. van Doorn, *Image processing done right*, Lecture Notes in Computer Science, **2350** (2002), 158–172.

[8] H. R. Schiffman, *Sensation and Perception*, 4th edition, J. Wiley and Sons Inc, New York (1995).

[9] M. W. Levine : Levine and Shefner, *Fundamentals of sensation and perception*, Oxford University Press, Oxford (2002).

[10] M. Borissavliévitch, *The Golden Number and the scientific aesthetics of architecture*, Alex Tiranti Ltd., London (1958).

[11] C. D. Green, *All that glitters: a review of Psychological Research in the Aesthetics of the Golden Section*, Perception **24** (1995), 937–968.

[12] M. Livio, *The Golden Ratio: The Story of Phi, The World's Most Astonishing Number*, New York: Brodway Books (2002).

[13] A. Seckel, *Incredible Visual Illusions*, Arcturus Publ. Ltd., London (2003).

[14] Scientific American Reports (Ed. M. DiChristina), *105 Mind–Bending Illusions*, Scientific American, **18(2)** (2008).

[15] B. Ons, P. Verstraelen, J. Wagemans, *A Theory on Visual Anisotropy*, (in preparation).

[16] R. L. Gregory, *Eye and Brain*, Oxford University Press, Oxford (1998).

P.S. The authors and the members of the Redaction Committee are pleased that a picture of Milutin Borisavljević (born in Kragujevac in 1888) could be presented in the previous article thanks to the kindness of Professor Koichi Yoshida of the Yokohama National University, who received the photo of Milutin Borisavljević from Madame Borisavljević, and showed it in Part 1 of his article on the life and work of Milutin Borisavljević:

K. Yoshida, *Essay on Miloutine Borissavlievitch, Part 1, his life and his work*, Transactions of the Architectural Institute of Japan, No. **225** (1977), 109–116, (in Japanese).