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COMMON FIXED POINT THEOREMS FOR COMPATIBLE AND SUBSEQUENTIALLY CONTINUOUS MAPPINGS IN FUZZY METRIC SPACES

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Abstract. In this paper, we prove common fixed point theorems for compatible and subsequentially continuous (alternately subcompatible and reciprocally continuous) mappings in fuzzy metric spaces. Some illustrative examples are also furnished which demonstrate the validity of our results. Our results improve and generalize well known results on the topic in the literature.

1. Introduction

In Mathematics, there has been always a tendency to regard the concept of probability as one of the basic mathematical concepts. In fact, the more general (that is, not necessarily probabilistic in nature) concept of "uncertainty" is considered a basic ingredient of some basic mathematical structures. The concept of fuzzy sets [33] constitutes an example, where the concept of uncertainty was introduced in the theory of sets, in a non probabilistic manner. Fuzzy set theory has applications in applied sciences such as mathematical programming, modeling theory, engineering sciences, image processing, control theory, communication etc. In 1975, Kramosil and Michalek [24] introduced the concept of fuzzy metric space, which opened an avenue for further development of analysis in such spaces.

There has been a tendency in metric fixed point theory to improve common fixed point theorems by improving the commutativity conditions. The first attempt was due to Sessa [30], who introduced the notion of weakly commuting mappings. Jungck [19] further enlarged the class of weakly commuting mappings by introducing the notion of compatible mappings and proved common fixed point theorems in presence

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of continuity of at least one of the mappings, completeness of the underlying space and containment of the ranges amongst involved mappings. In [27], Pant showed that in the setting of common fixed point theorems for compatible mappings satisfying contraction conditions, the notion of reciprocal continuity is weaker than the continuity of the mappings. Inspired by the definition of Jungck [19], researchers of this domain have introduced several definitions of compatible-like conditions, such as weakly compatible mappings [20], occasionally weakly compatible mappings [1], and some others (see Singh and Tomar [32]).

In 2009, Bouhadjera and Godet-Thobie [3] introduced new notions of subcompatibility and subsequential continuity which are respectively weaker than occasionally weakly compatibility and reciprocally continuity and proved fixed point theorems using these notions together in metric space. But the results of [3] contain flaws; Imdad et al. [18] noticed these flaws and improved the results of [3]. They showed in [18] that the results of [3] can easily be recovered by replacing subcompatible pairs with compatible pairs or replacing subsequentially continuous pairs with reciprocally continuous pairs. However, after the paper of [18], Bouhadjera and Godet-Thobie [4] improved their results contained in [3].

Various researchers utilized the notions of compatible mappings [26], weakly compatible mappings [31], occasionally weakly compatible mappings [22] and reciprocally continuity [2] in fuzzy metric spaces and proved several fixed point results. Recently, Gopal and Imdad [16] extended the two notions namely subsequential continuity and subcompatibility to fuzzy metric spaces and proved some interesting fixed point results which is based on the idea of [18]. Many authors have proved common fixed point theorems in fuzzy metric spaces for many kinds of generalized contractive conditions. For details, we refer to [5, 6, 7, 8, 9, 10, 11, 12, 13, 28].

The object of this paper is to prove common fixed point theorems in fuzzy metric spaces by using the notions of compatibility and subsequentially continuity (alternately subcompatibility and reciprocally continuity). We furnish some illustrative examples to support our main results. Our results generalize and improve the results of Khan and Sumitra [23], Balasubramaniam et al. [2] and Kutukcu et al. [25] and the references mentioned therein.

2. Preliminaries

Definition 2.1. [29] A mapping $* : [0,1] \times [0,1] \to [0,1]$ is called a t-norm if

(a) $a * 1 = a$,
(b) $a * b = b * a$,
(c) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$,
(d) $(a * b) * c = a * (b * c)$ for all $a, b, c, d \in [0,1]$.

Examples of continuous t-norms are $a * b = \min\{a, b\}$ and $a * b = ab$.

Definition 2.2. [24] A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions, for all $x, y, z \in X, t, s > 0$,

(a) $M(x, y, t) = 0$,
(b) $M(x, y, t) = 1$ if and only if $x = y$,
(c) $M(x, y, t) = M(y, x, t)$,
(d) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$,
(e) $M(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous.

Then M is called a fuzzy metric on X. Then $M(x, y, t)$ denotes the degree of nearness between x and y with respect to t.

In the following example, it is showed that every metric induces a fuzzy metric:

Example 2.1. [15] Let (X, d) be a metric space. Denote $a*b = ab$ (or $a*b = \min\{a, b\}$) for all $a, b \in [0, 1]$ and let M_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M_d(x, y, t) = \frac{t}{(t + d(x, y))}.$$

Then $(X, M_d, *)$ is a fuzzy metric space and the fuzzy metric M induced by the metric d is often referred to as the standard fuzzy metric.

Lemma 2.1. [17] *For all* $x, y \in X, (X, M, \cdot)$ *is non-decreasing function.*

Definition 2.3. [15] Let $(X, M, *)$ be a fuzzy metric space with continuous t-norm $*$. A sequence $\{x_n\}$ in X is said to be

(a) convergent to x in X if for each $\epsilon > 0$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x, t) > 1 - \epsilon$ for all $n \geq n_0$, that is, $M(x_n, x, t) \to 1$ as $n \to \infty$ for all $t > 0$;
(b) Cauchy if for each $\epsilon > 0$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for all $n, m \geq n_0$, that is, $M(x_n, x_m, t) \to 1$ as $n, m \to \infty$ for all $t > 0$.

A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.4. [26] A pair (A, S) of self mappings defined on a fuzzy metric space $(X, M, *)$ is said to be compatible if and only if $M(ASx_n, SAx_n, t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that Ax_n, $Sx_n \to z$ for some $z \in X$ as $n \to \infty$.

Definition 2.5. [20] A pair (A, S) of self mappings of a non-empty set X is said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, that is, if $Az = Sz$ some $z \in X$, then $ASz = SAz$.

Remark 2.1. [20] Two compatible self mappings are weakly compatible, however the converse is not true in general.

Definition 2.6. [1] A pair (A, S) of self mappings of a non-empty set X is occasionally weakly compatible if and only if there is a point $x \in X$ which is a coincidence point of A and S at which A and S commute.

In a recent paper of Đorić et al. [14], it is showed that in the presence of a unique point of coincidence (and a unique common fixed point) of the given single-valued mappings, occasionally weak compatibility actually reduces to weak compatibility. Thus, no generalization can be obtained by replacing weak compatibility with occasionally weak compatibility.

Definition 2.7. [16] A pair (A, S) of self mappings defined on a fuzzy metric space $(X, M, *)$ is said to be subcompatible if and only if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$ and $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ for all $t > 0$.

Two occasionally weakly compatible mappings are subcompatible, however the converse is not true in general (see [4, Example 1.2]).

Definition 2.8. [2] A pair (A, S) of self mappings defined on a fuzzy metric space $(X, M, *)$ is said to be reciprocally continuous if for a sequence $\{x_n\}$ in X, $\lim_{n\to\infty} ASx_n = Az$ and $\lim_{n\to\infty} SAx_n = Sz$, whenever $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

If two self mappings are continuous, then they are obviously reciprocally continuous but converse is not true. Moreover, in the setting of common fixed point theorems for compatible pair of self mappings satisfying contractive conditions, continuity of one of the mappings implies their reciprocal continuity but not conversely (see [27]).

Definition 2.9. [16] A pair (A, S) of self mappings defined on a fuzzy metric space $(X, M, *)$ is said to be subsequentially continuous if and only if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$ and $\lim_{n\to\infty} ASx_n = Az$ and $\lim_{n\to\infty} SAx_n = Sz$.

If two self mappings are continuous or reciprocally continuous, then they are naturally subsequentially continuous. However, there exist subsequentially continuous pair of mappings which are neither continuous nor reciprocally continuous (see [4, Example 1.4]).

Throughout this paper, $(X, M, *)$ is considered to be a fuzzy metric space with condition $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$.

3. Results

Theorem 3.1. *Let A, B, S and T be self mappings of a fuzzy metric space $(X, M, *)$ with $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$. If the pairs (A, S) and (B, T) are compatible and subsequentially continuous mappings, then*

(a) *the pair (A, S) has a coincidence point,*
(b) *the pair (B, T) has a coincidence point.*

(c) *Further, the mappings A, B, S and T have a unique common fixed point in X provided the involved maps satisfy the inequality*

$$M^2(Ax, By, t) * [M(Sx, Ax, t).M(Ty, By, t)] \geq [pM(Sx, Ax, t) + qM(Sx, Ty, t)] M(Sx, By, t) \quad (3.1)$$

for all $x, y \in X$ and $t > 0$, where $0 < p, q < 1$ and $p + q = 1$.

Proof. Since the pair (A, S) (also (B, T)) is subsequentially continuous and compatible mappings, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z, \text{ for some } z \in X$$

and

$$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = M(Az, Sz, t) = 1, \quad \text{for all } t > 0.$$

Then we have $Az = Sz$, whereas in respect of the pair (B, T), there exists a sequence $\{y_n\}$ in X such that

$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = w, \text{ for some } w \in X$$

and

$$\lim_{n\to\infty} M(BTy_n, TBy_n, t) = M(Bw, Tw, t) = 1, \text{ for all } t > 0.$$

Then we get $Bw = Tw$. Hence z is a coincidence point of the pair (A, S) whereas w is a coincidence point of the pair (B, T).

First we prove that $z = w$. Putting $x = x_n$ and $y = y_n$ in inequality (3.1), we have

$$M^2(Ax_n, By_n, t) * [M(Sx_n, Ax_n, t).M(Ty_n, By_n, t)]$$
$$\geq [pM(Sx_n, Ax_n, t) + qM(Sx_n, Ty_n, t)]M(Sx_n, By_n, t).$$

Taking the limit as $n \to \infty$, we get

$$\begin{aligned} M^2(z, w, t) * [M(z, z, t).M(w, w, t)] &\geq [pM(z, z, t) + qM(z, w, t)]M(z, w, t) \\ M^2(z, w, t) &\geq [p + qM(z, w, t)]M(z, w, t) \\ M(z, w, t) &\geq [p + qM(z, w, t)] \\ M(z, w, t) &\geq \left(\frac{p}{1-q}\right) \\ &= 1, \end{aligned}$$

for all $t > 0$. Thus we have $z = w$. We show that $Az = z$. Putting $x = z$ and $y = y_n$ in inequality (3.1), we get

$$M^2(Az, By_n, t) * [M(Sz, Az, t).M(Ty_n, By_n, t)]$$
$$\geq [pM(Sz, Az, t) + qM(Sz, Ty_n, t)]M(Sz, By_n, t).$$

Taking the limit as $n \to \infty$, we get

$$M^2(Az, w, t) * [M(Sz, Az, t).M(w, w, t)] \geq [pM(Sz, Az, t) + qM(Sz, w, t)]M(Sz, w, t),$$

and so

$$\begin{aligned} M^2(Az, z, t) &\geq [p + qM(Az, z, t)]M(Az, z, t) \\ M(Az, z, t) &\geq [p + qM(Az, z, t)] \\ M(Az, z, t) &\geq \left(\frac{p}{1-q}\right) \\ &= 1, \end{aligned}$$

for all $t > 0$. Thus we have $Az = z$. Therefore, $Az = Sz = z$. Now we assert that $Bz = z$. Putting $x = x_n$ and $y = z$ in inequality (3.1), we have

$$\begin{aligned} &M^2(Ax_n, Bz, t) * [M(Sx_n, Ax_n, t).M(Tz, Bz, t)] \\ &\geq [pM(Sx_n, Ax_n, t) + qM(Sx_n, Tz, t)]M(Sx_n, Bz, t). \end{aligned}$$

Taking the limit as $n \to \infty$, we get

$$\begin{aligned} M^2(z, Bz, t) * [M(z, z, t).M(Bz, Bz, t)] &\geq [pM(z, z, t) + qM(z, Bz, t)]M(z, Bz, t) \\ M^2(z, Bz, t) &\geq [p + qM(z, Bz, t)]M(z, Bz, t) \\ M(z, Bz, t) &\geq [p + qM(z, Bz, t)] \\ M(z, Bz, t) &\geq \left(\frac{p}{1-q}\right) \\ &= 1, \end{aligned}$$

for all $t > 0$, we have $Bz = Sz = z$. Thus in all, $z = Az = Sz = Bz = Tz$, that is, z is a common fixed point of the mappings A, B, S and T.

Uniqueness of the common fixed point is an easy consequence of inequality (3.1). This completes the proof of the theorem. □

Theorem 3.2. *Let A, B, S and T be self mappings of a fuzzy metric space $(X, M, *)$ with $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$. If the pairs (A, S) and (B, T) are subcompatible and reciprocally continuous mappings, then*

(a) *the pair (A, S) has a coincidence point,*
(b) *the pair (B, T) has a coincidence point.*
(c) *Further, the mappings A, B, S and T have a unique common fixed point in X provided the involved maps satisfy the inequality (3.1) of Theorem 3.1.*

Proof. Since the pair (A, S) (also (B, T)) is subcompatible and reciprocally continuous, therefore there exists a sequences $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z, \text{ for some } z \in X$$

and

$$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = \lim_{n\to\infty} M(Az, Sz, t) = 1, \text{ for all } t > 0$$

whereas in respect of the pair (B, T), there exists a sequence $\{y_n\}$ in X with

$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = w, \text{ for some } w \in X$$

and

$$\lim_{n\to\infty} M(BTx_n, TBx_n, t) = \lim_{n\to\infty} M(Bz, Tz, t) = 1, \text{ for all } t > 0.$$

Therefore, $Az = Sz$ and $Bw = Tw$, that is, z is a coincidence point of the pair (A, S) whereas w is a coincidence point of the pair (B, T).

The rest of the proof can be completed on the lines of Theorem 3.1. □

Remark 3.1. It is clear that the conclusion of Theorem 3.1 remains valid if we replace compatibility with subcompatibility and subsequential continuity with reciprocally continuity, besides retaining the rest of the hypothesis (see [18]).

By setting $A = B$ in Theorem 3.1, we obtain the following result for three mappings.

Corollary 3.1. *Let A, S and T be self mappings of a fuzzy metric space $(X, M, *)$ with $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$. If the pairs (A, S) and (A, T) are compatible and subsequentially continuous mappings, then*

(a) *the pair (A, S) has a coincidence point,*
(b) *the pair (A, T) has a coincidence point.*
(c) *Further, the mappings A, S and T have a unique common fixed point in X provided the involved mappings satisfy the inequality*

$$M^2(Ax, Ay, t) * [M(Sx, Ax, t).M(Ty, Ay, t)] \geq [pM(Sx, Ax, t) + qM(Sx, Ty, t)] M(Sx, Ay, t), \tag{3.2}$$

for all $x, y \in X$ and $t > 0$, where $0 < p, q < 1$ and $p + q = 1$.

Alternately, by setting $S = T$ in Theorem 3.1, we also get another corollary.

Corollary 3.2. *Let A, B and S be self mappings of a fuzzy metric space $(X, M, *)$ with $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$. If the pairs (A, S) and (B, S) are compatible and subsequentially continuous mappings (alternately subcompatible and reciprocally continuous mappings), then*

(a) *the pair (A, S) has a coincidence point,*
(b) *the pair (B, S) has a coincidence point.*
(c) *Further, the mappings A, B and S have a unique common fixed point in X provided the involved mappings satisfy the inequality*

$$M^2(Ax, By, t) * [M(Sx, Ax, t).M(Sy, By, t)] \geq [pM(Sx, Ax, t) + qM(Sx, Sy, t)] M(Sx, By, t), \tag{3.3}$$

for all $x, y \in X$ and $t > 0$, where $0 < p, q < 1$ and $p + q = 1$.

By taking $A = B$ and $S = T$ in Theorem 3.1, we get the following result.

Corollary 3.3. *Let A and S be self mappings of a fuzzy metric space $(X, M, *)$ with $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$. If the pair (A, S) is compatible as well as subsequentially continuous mappings, then*

(a) *the pair (A, S) has a coincidence point.*

(b) *Further, the mappings A and S have a unique common fixed point in X provided the involved mappings satisfy the inequality*

$$M^2(Ax, Ay, t) * [M(Sx, Ax, t).M(Sy, Ay, t)] \geq [pM(Sx, Ax, t) + qM(Sx, Sy, t)] M(Sx, Ay, t), \tag{3.4}$$

for all $x, y \in X$ and $t > 0$, where $0 < p, q < 1$ and $p + q = 1$.

Example 3.1. Let $X = [0, \infty)$ and d be the usual metric on X and for each $t \in [0, 1)$, define

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0, \end{cases}$$

for all $x, y \in X$. Clearly $(X, M, *)$ be a fuzzy metric space with $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$. Define the self mappings A and S on X by

$$A(X) = \begin{cases} \frac{x}{4}, & \text{if } x \in [0, 1]; \\ 5x - 4, & \text{if } x \in (1, \infty). \end{cases} \quad S(X) = \begin{cases} \frac{x}{5}, & \text{if } x \in [0, 1]; \\ 4x - 3, & \text{if } x \in (1, \infty). \end{cases}$$

Consider a sequence $\{x_n\} = \left\{\frac{1}{n}\right\}_{n\in\mathbb{N}}$ in X. Then

$$\lim_{n\to\infty} A(x_n) = \lim_{n\to\infty} \left(\frac{1}{4n}\right) = 0 = \lim_{n\to\infty} \left(\frac{1}{5n}\right) = \lim_{n\to\infty} S(x_n).$$

Next,

$$\lim_{n\to\infty} AS(x_n) = \lim_{n\to\infty} A\left(\frac{1}{5n}\right) = \lim_{n\to\infty} \left(\frac{1}{20n}\right) = 0 = A(0),$$
$$\lim_{n\to\infty} SA(x_n) = \lim_{n\to\infty} S\left(\frac{1}{4n}\right) = \lim_{n\to\infty} \left(\frac{1}{20n}\right) = 0 = S(0),$$

and

$$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1,$$

for all $t > 0$. Consider another sequence $\{x_n\} = \left\{1 + \frac{1}{n}\right\}_{n\in\mathbb{N}}$ in X. Then

$$\lim_{n\to\infty} A(x_n) = \lim_{n\to\infty} \left(5 + \frac{5}{n} - 4\right) = 1 = \lim_{n\to\infty} \left(4 + \frac{4}{n} - 3\right) = \lim_{n\to\infty} S(x_n).$$

Also,

$$\lim_{n\to\infty} AS(x_n) = \lim_{n\to\infty} A\left(1 + \frac{4}{n}\right) = \lim_{n\to\infty} \left(5 + \frac{20}{n} - 4\right) = 1 \neq A(1),$$
$$\lim_{n\to\infty} SA(x_n) = \lim_{n\to\infty} S\left(1 + \frac{5}{n}\right) = \lim_{n\to\infty} \left(4 + \frac{20}{n} - 3\right) = 1 \neq S(1),$$

but $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$. Thus the pair (A, S) is compatible as well as subsequentially continuous but not reciprocally continuous. Therefore all the conditions of

Corollary 3.3 are satisfied. Here, 0 is a coincidence as well as unique common fixed point of the pair (A, S). It is noted that this example cannot be covered by those fixed point theorems which involve compatibility and reciprocal continuity both or by involving conditions on completeness (or closedness) of underlying space (or subspaces). Also, in this example neither X is complete nor any subspace $A(X) = \left[0, \frac{1}{4}\right] \cup (1, \infty)$ and $S(X) = \left[0, \frac{1}{5}\right] \cup (1, \infty)$ are closed. It is noted that this example cannot be covered by those fixed point theorems which involve compatibility and reciprocal continuity both (e.g. [2, 25, 27]).

Example 3.2. Let $X = \mathbb{R}$ (set of real numbers) and d be the usual metric on X and for each $t \in [0, 1)$, define

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0, \end{cases}$$

for all $x, y \in X$. Clearly $(X, M, *)$ be a fuzzy metric space with $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$. Define the self mappings A and S on X by

$$A(X) = \begin{cases} \frac{x}{4}, & \text{if } x \in (-\infty, 1); \\ 5x - 4, & \text{if } x \in [1, \infty). \end{cases} \quad S(X) = \begin{cases} x + 3, & \text{if } x \in (-\infty, 1); \\ 4x - 3, & \text{if } x \in [1, \infty). \end{cases}$$

Consider a sequence $\{x_n\} = \left\{1 + \frac{1}{n}\right\}_{n \in \mathbb{N}}$ in X. Then

$$\lim_{n\to\infty} A(x_n) = \lim_{n\to\infty} \left(5 + \frac{5}{n} - 4\right) = 1 = \lim_{n\to\infty} \left(4 + \frac{4}{n} - 3\right) = \lim_{n\to\infty} S(x_n).$$

Also,

$$\lim_{n\to\infty} AS(x_n) = \lim_{n\to\infty} A\left(1 + \frac{4}{n}\right) = \lim_{n\to\infty} \left(5 + \frac{20}{n} - 4\right) = 1 = A(1),$$

$$\lim_{n\to\infty} SA(x_n) = \lim_{n\to\infty} S\left(1 + \frac{5}{n}\right) = \lim_{n\to\infty} \left(4 + \frac{20}{n} - 3\right) = 1 = S(1),$$

and

$$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1,$$

for all $t > 0$. Consider another sequence $\{x_n\} = \left\{\frac{1}{n} - 4\right\}_{n \in \mathbb{N}}$ in X. Then

$$\lim_{n\to\infty} A(x_n) = \lim_{n\to\infty} \left(\frac{1}{4n} - 1\right) = -1 = \lim_{n\to\infty} \left(\frac{1}{n} - 4 + 3\right) = \lim_{n\to\infty} S(x_n).$$

Next,

$$\lim_{n\to\infty} AS(x_n) = \lim_{n\to\infty} A\left(\frac{1}{n} - 1\right) = \lim_{n\to\infty} \left(\frac{1}{4n} - \frac{1}{4}\right) = -\frac{1}{4} = A(-1),$$

$$\lim_{n\to\infty} SA(x_n) = \lim_{n\to\infty} S\left(\frac{1}{4n} - 1\right) = \lim_{n\to\infty} \left(\frac{1}{4n} - 1 + 3\right) = 2 = S(-1),$$

and $\lim_{n\to\infty} M(ASx_n, SAx_n, t) \neq 1$. Thus the pair (A, S) is reciprocally continuous as well as subcompatible but not compatible. Therefore, all the conditions of Corollary 3.3 are satisfied. Thus, 1 is a coincidence as well as unique common fixed point of the pair (A, S). It is also noted that this example cannot be covered by those fixed point theorems which involve compatibility and reciprocal continuity both (e.g. [2, 25, 27]).

4. Conclusion

Theorem 3.1 and Theorem 3.2 generalize and improve the results of Khan and Sumitra [23], Balasubramaniam et al. [2] and Kutukcu et al. [25] without any requirement on completeness (or closedness) of the underlying space (or subspaces), containment of ranges amongst involved mappings and continuity in respect of any one of the involved mapping.

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