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THE SEMI ORLICZ SPACE $cs \bigcap d_1$

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ABSTRACT. Let Γ denote the space of all entire sequences. Let Λ denote the space of all analytic sequences. In this paper we introduce a new class of sequence spaces namely the semi difference Orlicz space $cs \bigcap d_1$. It is shown that the intersection of all semi difference Orlicz space $cs \bigcap d_1$ is $I \subset cs \bigcap d_1$ and $\Gamma_M(\Delta) \subset I$.

1. INTRODUCTION

A complex sequence, whose k^{th} terms is x_k is denoted by $\{x_k\}$ or simply x. Let w be the set of all sequences and ϕ be the set of all finite sequences. Let ℓ_∞, c, c_0 be the classes of bounded, convergent and null sequence respectively. A sequence $x = \{x_k\}$ is said to be analytic if $\sup_k |x_k|^{1/k} < \infty$. The vector space of all analytic sequences will be denoted by Λ. A sequence x is called entire sequence if $\lim_{k\to\infty} |x_k|^{1/k} = 0$. The vector space of all entire sequences will be denoted by Γ.

Orlicz [9] used the idea of Orlicz function to construct the space (L^M). Lindenstrauss and Tzafriri [5] investigated Orlicz sequence spaces in more detail and proved that every Orlicz sequence space ℓ_M contains a subspace isomorphic to ℓ_p $(1 \leq p < \infty)$. Subsequently different class of sequence spaces defined by Orlicz function were introduced by Parashar and Choudhary [10], Mursaleen et al. [7], Bektas and Altin [1], Tripathy et al. [17], Rao and Subramanian [12] and many others. The Orlicz sequence spaces are the special cases of Orlicz spaces are discussed in detail by Krasnoselskii and Y. B. Rutickii [4].

An Orlicz function is a mapping $M : [0,\infty) \to [0,\infty)$ which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If convexity of Orlicz function M is replaced by sub-additivity, that is

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$M(x+y) \leq M(x) + M(y)$, then this function is called modulus function, defined Nakano [8] and discussed by Ruckle [14] and Maddox [6].

An Orlicz function M is said to satisfy Δ_2-condition for all values of u, if there exists a constant $K > 0$, such that $M(2u) \leq KM(u)$ $(u \geq 0)$ (Krasnoselskii and Rutitsky [4]).

Lindenstrauss and Tzafriri [5] used the idea of Orlicz function to construct Orlicz sequence space

$$\ell_M = \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty, \text{ for some } \rho > 0 \right\}. \tag{1.1}$$

The space ℓ_M with the norm

$$\|x\| = \inf\left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\}, \tag{1.2}$$

becomes a Banach space which is called an Orlicz sequence space. For $M(t) = t^p$, $1 \leq p < \infty$, the space ℓ_M coincide with the classical sequence space ℓ_p. Given a sequence $x = \{x_k\}$ its n^{th} section is the sequence $x^{(n)} = \{x_1, x_2, \ldots, x_n, 0, 0, \ldots\}$ $\delta^{(n)} = (0, 0, \ldots, 1, 0, 0, \ldots)$, 1 in the n^{th} place and zero's elsewhere; and $s^{(k)} = (0, 0, \ldots, 1, -1, 0, \ldots)$, 1 in the n^{th} place, -1 in the $(n+1)^{th}$ place and zero's elsewhere. An *FK*-space (Frechet coordinate space) is a Frechet space which is made up of numerical sequences and has the property that the coordinate functionals $p_k(x) = x_k$ $(k = 1, 2, 3, \ldots)$ are continuous.

We recall the following definitions (one may refer to Wilansky [18]).

An *FK*-space is a locally convex Frechet space which is made up of sequences and has the property that coordinate projections are continuous. A metric space (X, d) is said to have AK (or Sectional convergence) if and only if $d(x^{(n)}, x) \to x$ as $n \to \infty$ (see [18]). The space is said to have AD (or) be an AD space if ϕ is dense in X. We note that AK implies AD by (one may refer to Brown [2]).

If X is a sequence space, we define

(i) $X' =$ the continuous dual of X;
(ii) $X^\alpha = \{a = (a_k) : \sum_{k=1}^{\infty} |a_k x_k| < \infty, \text{for each } x \in X\}$;
(iii) $X^\beta = \{a = (a_k) : \sum_{k=1}^{\infty} a_k x_k \text{ is convergent, for each } x \in X\}$;
(iv) $X^\gamma = \{a = (a_k) : \sup_n |\sum_{k=1}^{n} a_k x_k| < \infty, \text{ for each } x \in X\}$;
(v) Let X be an FK-space $\supset \phi$. Then $X^f = \left\{ f(\delta^{(n)}) : f \in X' \right\}$.

$X^\alpha, X^\beta, X^\gamma$ are called the α- (or Köthe-Töeplitz) dual of X, β- (or generalized Köthe-Töeplitz) dual of X, γ-dual of X. Note that $X^\alpha \subset X^\beta \subset X^\gamma$. If $X \subset Y$ then $Y^\mu \subset X^\mu$, for $\mu = \alpha, \beta$, or γ.

Let $p = (p_k)$ be a positive sequence of real numbers with $0 < p_k \leq \sup_k p_k = G$, $D = \max\left\{1, 2^{G-1}\right\}$. Then it is well known that for all $a_k, b_k \in \mathbb{C}$, ($\mathbb{C}$-the set of complex numbers), and all $k \in \mathbb{N}$,

$$|a_k + b_k|^{p_k} \leq D(|a_k|^{p_k} + |b_k|^{p_k}). \tag{1.3}$$

Lemma 1.1. (Wilansky [18, Theorem 7.2.7]) *Let X be an FK-space $\supset \phi$. Then*

(i) $X^{\gamma} \subset X^{f}$.
(ii) *If X has AK,* $X^{\beta} = X^{f}$.
(iii) *If X has AD,* $X^{\beta} = X^{\gamma}$.

2. Definitions and Preliminaries

Let $\Delta : w \to w$ be the difference operator defined by $\Delta x = (x_k - x_{k+1})_{k=1}^{\infty}$, and M be an Orlicz function, or a modulus function. Let

$$\Gamma_M = \left\{ x \in w : \lim_{k\to\infty} \left(M\left(\frac{|x_k|^{1/k}}{\rho} \right) \right) = 0 \text{ for some } \rho > 0 \right)$$

and

$$\Lambda_M = \left\{ x \in w : \sup_{k} \left(M\left(\frac{|x_k|^{1/k}}{\rho} \right) \right) < \infty \text{ for some } \rho > 0 \right).$$

Define the sets $\Gamma_M(\Delta) = \{x \in w : \Delta x \in \Gamma_M\}$ and $\Lambda_M(\Delta) = \{x \in w : \Delta x \in \Lambda_M\}$. The space $\Gamma_M(\Delta)$ and $\Lambda_M(\Delta)$ is a metric space with the metric

$$d(x, y) = \inf \left\{ \rho > 0 : \sup_{k} \left(M\left(\frac{|\Delta x_k - \Delta y_k|^{1/k}}{\rho} \right) \right) \le 1 \right\}.$$

Because of the historical roots of summability in convergence, conservative space and matrices play a special role in its theory. However, the results seem mainly to depend on a weaker assumption that the spaces are semi conservative (see Wilansky [18]).

Snyder and Wilansky [16] introduced the concept of semi conservative spaces. Snyder [15] studied the properties of semi conservative spaces. Later on, in the year 1996 the semi replete spaces were introduced by Rao and Srinivasalu [13].

In a similar way, in this paper we define semi difference Orlicz space $cs\bigcap d_1$, and show that semi difference Orlicz space $cs\bigcap d_1$ is $I \subset cs\bigcap d_1$ and $\Gamma_M(\Delta) \subset I$.

3. Main Results

Proposition 3.1. $\Gamma \subset \Gamma_M(\Delta)$.

Proof. Let $x \in \Gamma$ and M be an Orlicz function. Then we have

$$|x_k|^{1/k} \to 0, \text{ as } k \to \infty. \tag{3.1}$$

For a $\rho > 0$, we have

$$\begin{aligned} M\left(\frac{|\Delta x_k|^{1/k}}{2\rho} \right) &\le \frac{1}{2} M\left(\frac{|x_k|^{1/k}}{\rho} \right) + \frac{1}{2} M\left(\frac{|x_{k+1}|^{1/k}}{\rho} \right), \text{ by (1.3)} \\ &\to \infty, \text{ as } k \to \infty \text{ by the continuity of } M \text{ and by (3.1).} \end{aligned}$$

This completes the proof. ∎

Proposition 3.2. $\Gamma_M \subset \Gamma_M(\Delta)$ *and the inclusion is strict.*

Proof. Let $x \in \Gamma_M$. Then we have

$$\left(M\left(\frac{|x_k|^{1/k}}{\rho}\right)\right) \to 0 \text{ as } k \to \infty, \text{ for some } \rho > 0 \tag{3.2}$$

which follows from the inequality (3.1) and (3.2).

Inclusion follows from the following example.

Example 3.1. Consider the sequence $e = (1, 1, \ldots)$. Then $e \in \Gamma_M(\Delta)$ but $e \notin \Gamma_M$. Hence the inclusion $\Gamma_M \subset \Gamma_M(\Delta)$ is strict.

□

Lemma 3.1. $A \in (\Gamma, c)$ *if and only if*

$$\lim_{n\to\infty} a_{nk} \text{ exists for each } k \in \mathbb{N}, \tag{3.3}$$

and

$$\sup_{n,k}\left|\sum_{i=0}^{k} a_{ni}\right| < \infty. \tag{3.4}$$

Proposition 3.3. *Define the set* $d_1 = \left\{a = (a_k) \in w : \sup_{n,k\in\mathbb{N}}\left|\sum_{j=0}^{k}\left(\sum_{i=j}^{n} a_i\right)\right| < \infty\right\}$. *Then* $[\Gamma_M(\Delta)]^\beta = cs \bigcap d_1$.

Proof. Consider the equation

$$\sum_{k=0}^{n} a_k x_k = \sum_{k=0}^{n} a_k\left(\sum_{j=0}^{k} y_j\right) = \sum_{k=0}^{n}\left(\sum_{j=k}^{n} a_j\right) y_k = (Cy)_n, \tag{3.5}$$

where $C = (C_{nk})$ is defined by

$$C_{nk} = \begin{cases} \sum_{j=k}^{n} a_j, & \text{if } 0 \le k \le n, \\ 0, & \text{if } k > n, \end{cases} \quad \text{for } n, k \in \mathbb{N}.$$

Thus we deduce from Lemma 3.1 with (3.5) that $ax = (a_k x_k) \in cs$ whenever $x = (x_k) \in \Gamma_M(\Delta)$ if and only if $Cy \in c$ whenever $y = (y_k) \in \Gamma$, that is $C \in (\Gamma, c)$. Thus $(a_k) \in cs$ and $(a_k) \in d_1$ by Lemma 3.1 and (3.3) and (3.4) respectively. This completes the proof. □

Proposition 3.4. $\Gamma_M(\Delta)$ *has AK.*

Proof. Let $x = \{x_k\} \in \Gamma_M(\Delta)$. Then $\left(M\left(\frac{|\Delta x_k|^{1/k}}{\rho}\right)\right) \in \Gamma$. Hence

$$\sup_{k\ge n+1}\left(M\left(\frac{|\Delta x_k|^{1/k}}{\rho}\right)\right) \to 0, \text{ as } k \to \infty. \tag{3.6}$$

$$\begin{aligned} d\left(x, x^{[n]}\right) &= \inf\left\{\rho > 0 : \sup_{k\geq n+1}\left(M\left(\frac{|\Delta x_k|^{1/k}}{\rho}\right)\right) \leq 1\right\} \to 0, \text{ as } n \to \infty \\ & \qquad \text{by using (3.6)} \\ &\Rightarrow x^{[n]} \to x \text{ as } n \to \infty \\ &\Rightarrow \Gamma_M(\Delta) \text{ has AK.} \end{aligned}$$

This completes the proof. □

Proposition 3.5. $\Gamma_M(\Delta)$ *is not solid.*

Proof. The result follows from the following examples.

Example 3.2. Consider $(x_k) = (1) \in \Gamma_M(\Delta)$. Let $\alpha_k = (-1)^k$, for all $k \in \mathbb{N}$ then $(\alpha_k x_k) \notin \Gamma_M(\Delta)$. Hence $\Gamma_M(\Delta)$ is not solid.

Example 3.3. Let $M(x) = x^2$, for $x \in [0, \infty)$. Consider the sequences $(x_k) = e$. Then $e \in \Gamma_M(\Delta)$. Consider the sequence of scalars (α_k) defined by $\alpha_k = (-1)^k$, then $(\alpha_k x_k) \notin \Gamma_M(\Delta)$. Hence $\Gamma_M(\Delta)$ is not solid. This completes the proof.

□

Proposition 3.6. $(\Gamma_M(\Delta))^\mu = cs\bigcap d_1$ *for* $\mu = \alpha, \beta, \gamma, f$.

Proof. (*Step 1*) $\Gamma_M(\Delta)$ has AK by Proposition 3.4. Hence, by Lemma 1.1(ii), we get $(\Gamma_M(\Delta))^\beta = (\Gamma_M(\Delta))^f$. But $(\Gamma_M(\Delta))^\beta = cs\bigcap d_1$. Hence

$$(\Gamma_M(\Delta))^f = cs\bigcap d_1. \tag{3.7}$$

(*Step 2*) Since $AK \Rightarrow AD$, then by Lemma 1.1(iii), we get $(\Gamma_M(\Delta))^\beta = (\Gamma_M(\Delta))^\gamma$. Therefore

$$(\Gamma_M(\Delta))^\gamma = cs\bigcap d_1. \tag{3.8}$$

(*Step 3*) $\Gamma_M(\Delta)$ is not normal by Proposition 3.5. Hence by Proposition 2.7 of Kamthan and Gupta [3] we get

$$(\Gamma_M(\Delta))^\alpha \neq (\Gamma_M(\Delta))^\gamma \neq cs\bigcap d_1. \tag{3.9}$$

From (3.6) and (3.8) we have

$$(\Gamma_M(\Delta))^\beta = (\Gamma_M(\Delta))^\gamma = (\Gamma_M(\Delta))^f = cs\bigcap d_1. \tag{3.10}$$

□

Lemma 3.2. (Wilansky [18, Theorem 8.6.1]) $Y \supset X \Leftrightarrow Y^f \subset X^f$ *where* X *is an AD-space and* Y *an FK-space.*

Proposition 3.7. *Let $Y \supset \phi$ be any FK-space. Then $Y \supset \Gamma_M(\Delta)$ if and only if the sequence $\delta^{(k)}$ is weakly $cs \bigcap d_1$.*

Proof. The following implications establish the result

$$\begin{aligned} Y \supset \Gamma_M(\Delta) &\Leftrightarrow Y^f \subset (\Gamma_M(\Delta))^f, \text{ since } \Gamma_M(\Delta) \text{ has AD by Lemma 3.2} \\ &\Leftrightarrow Y^f \subset cs\bigcap d_1, \text{ since } (\Gamma_M(\Delta))^f = cs\bigcap d_1 \\ &\Leftrightarrow \text{ for each } f \in Y', \text{ the topological dual of } Y, f\left(\delta^{(k)}\right) \in cs\bigcap d_1 \\ &\Leftrightarrow f\left(\delta^{(k)}\right) \text{ is } cs\bigcap d_1 \\ &\Leftrightarrow \delta^{(k)} \text{ is weakly } cs\bigcap d_1. \end{aligned}$$

This completes the proof. ☐

4. Properties of Semi Difference Orlicz Space $cs \bigcap d_1$

Definition 4.1. An FK-space ΔX is called "semi difference Orlicz space $cs \bigcap d_1$" if its dual $(\Delta X)^f \subset cs \bigcap d_1$.

In other words ΔX is semi difference Orlicz space $cs \bigcap d_1$ if $f\left(\delta^{(k)}\right) \in cs \bigcap d_1$ for all $f \in (\Delta X)'$ for some fixed k.

Example 4.1. $\Gamma_M(\Delta)$ is semi difference Orlicz space $cs \bigcap d_1$. Indeed, if $\Gamma_M(\Delta)$ is the space of all difference Orlicz sequence of entire sequences, then by Lemma 4.1, $(\Gamma_M(\Delta))^f = cs \bigcap d_1$.

Lemma 4.1. $(\Gamma_M(\Delta))^f = cs \bigcap d_1$.

Proof. $(\Gamma_M(\Delta))^\beta = cs \bigcap d_1$ by Proposition 3.3. But $(\Gamma_M(\Delta))$ has AK by Proposition 3.4. Hence $(\Gamma_M(\Delta))^\beta = (\Gamma_M(\Delta))^f$. Therefore $(\Gamma_M(\Delta))^f = cs \bigcap d_1$.

This completes the proof. ☐

Lemma 4.2. (Wilansky [18, Theorem 4.3.7]) *Let z be a sequence. Then $\left(z^\beta, P\right)$ is an AK space with $P = (P_k : k = 0, 1, 2, \ldots)$, where $P_0(x) = \sup_m \left|\sum_{k=1}^m z_k x_k\right|$, and $P_n(x) = |x_n|$. For any k such that $z_k \neq 0$, P_k may be omitted. If $z \in \phi$, P_0 may be omitted.*

Proposition 4.1. *Let z be a sequence z^β is a semi difference Orlicz space $cs \bigcap d_1$ if and only if z is $cs \bigcap d_1$.*

Proof. Suppose that z^β is semi difference Orlicz space of $cs \bigcap d_1$. Then z^β has AK by Lemma 4.2. Therefore $z^{\beta\beta} = \left(z^\beta\right)^f$ by [18, Lemma 1]. So z^β is semi difference Orlicz space of $cs \bigcap d_1$ if and only if $z^{\beta\beta} \subset cs \bigcap d_1$. But then $z \in z^{\beta\beta} \subset cs \bigcap d_1$. Hence z is $cs \bigcap d_1$.

Conversely, suppose that z is $cs \bigcap d_1$. Then $z^\beta \supset \{cs \bigcap d_1\}^\beta$ and $z^{\beta\beta} \subset \{cs \bigcap d_1\}^{\beta\beta} = cs \bigcap d_1$. But $\left(z^\beta\right)^f = z^{\beta\beta}$. Hence $\left(z^\beta\right)^f \subset cs \bigcap d_1$. Therefore z^β is semi difference Orlicz space of $cs \bigcap d_1$. This completes the proof. ☐

Proposition 4.2. *Every semi difference Orlicz space $cs \bigcap d_1$ contains Γ_M.*

Proof. Let ΔX be any semi difference Orlicz space of $cs \bigcap d_1$. Hence $(\Delta X)^f \subset cs \bigcap d_1$. Therefore $f\left(\delta^{(k)}\right) \in cs \bigcap d_1$ for all $f \in (\Delta X)'$. So, $\left\{\delta^{(k)}\right\}$ is weakly $cs \bigcap d_1$ with respect to ΔX. Hence $\Delta X \supset \Gamma_M(\Delta)$ by Proposition 3.7. But $\Gamma_M(\Delta) \supset \Gamma_M$. Hence $\Delta X \supset \Gamma_M$. This completes the proof. □

Proposition 4.3. *The intersection of all semi difference Orlicz space $cs \bigcap d_1$, $\{\Delta X_n : n = 1, 2, \ldots\}$, is semi difference Orlicz space of $cs \bigcap d_1$.*

Proof. Let $\Delta X = \bigcap_{n=1}^{\infty} \Delta X_n$. Then ΔX is an *FK*-space which contains ϕ. Also every $f \in (\Delta X)'$ can be written as $f = g_1 + g_2 + \ldots + g_m$, where $g_k \in (\Delta X_n)'$ for some n and for $1 \le k \le m$. But then $f\left(\delta^k\right) = g_1\left(\delta^k\right) + g_2\left(\delta^k\right) + \ldots + g_m\left(\delta^k\right)$. Since $\Delta X_n\,(n = 1, 2, \ldots)$ are semi difference Orlicz space of $cs \bigcap d_1$, it follows that $g_i\left(\delta^k\right) \in cs \bigcap d_1$ for all $i = 1, 2, \ldots, m$. Therefore $f\left(\delta^k\right) \in cs \bigcap d_1$ for all k and for all f. Hence ΔX is semi difference Orlicz space of $cs \bigcap d_1$. This completes the proof. □

Proposition 4.4. *The intersection of all semi difference Orlicz space $cs \bigcap d_1$ is $I \subset (cs \bigcap d_1)^{\beta}$ and $\Gamma_M(\Delta) \subset I$.*

Proof. Let I be of all semi difference Orlicz space of $cs \bigcap d_1$. By Proposition 4.1 we see that

$$I \subset \bigcap \left\{z^{\beta} : z \in cs \bigcap d_1\right\} = \left\{cs \bigcap d_1\right\}^{\beta}. \tag{4.1}$$

By Proposition 4.3 it follows that I is semi difference Orlicz space of $cs \bigcap d_1$. By Proposition 4.2 consequently

$$\Gamma_M = \Gamma_M(\Delta) \subset I. \tag{4.2}$$

From (4.1) and (4.2) we get $I \subset cs \bigcap d_1$ and $\Gamma_M(\Delta) \subset I$. This completes the proof. □

Corollary 4.1. *The smallest semi difference Orlicz space $cs \bigcap d_1$ is $I \subset (cs \bigcap d_1)^{\beta}$ and $\Gamma_M(\Delta) \subset I$.*

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