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ATANASSOV'S INTUITIONISTIC FUZZY IDEALS OF PO-Γ-SEMIGROUPS

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Abstract. Atanassov introduced the notion of intuitionistic fuzzy sets as a generalization of the notion of fuzzy set. In this paper we introduce the concept of Atanassov's intuitionistic fuzzy ideals of po-Γ-semigroups in order to obtain some characterization theorems. Operator po-semigroups of a po-Γ-semigroup have been made to work by obtaining various relationships between intuitionistic fuzzy ideals of a po-Γ-semigroup and that of its operator semigroups.

1. Introduction

Besides several generalizations of fuzzy sets, the intuitionistic fuzzy sets introduced by Atanassov [2][3] have been found to be highly useful to cope with imperfect and/or imprecise information. Atanassov's intuitionistic fuzzy sets are an intuitively straightforward extension of Zadeh's [49] fuzzy sets: while a fuzzy set gives the degree of membership of an element in a given set, an Atanassov's intuitionistic fuzzy set gives both a degree of membership and a degree of non-membership. Many concepts in fuzzy set theory were also extended to intuitionistic fuzzy set theory, such as intuitionistic fuzzy relations, intuitionistic L-fuzzy sets, intuitionistic fuzzy implications, intuitionistic fuzzy logics, the degree of similarity between intuitionistic fuzzy sets, intuitionistic fuzzy rough sets. Atanassov's intuitionistic fuzzy sets as a generalization of fuzzy sets can be useful in situations when description of a problem by a (fuzzy) linguistic variable, given in terms of a membership function only, seems too rough. For example, in decision making problems, particularly in the case of medial diagnosis, sales analysis, new product marketing etc. there is a fair chance of the existence of

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a non-null hesitation part at each moment of evaluation of an unknown object. To be more precise intuitionistic fuzzy sets let us express e.g., the fact that the temperature of a patient changes, and other symptoms are not quite clear.

The topic of investigations about fuzzy semigroups belongs to the theoretical soft computing (fuzzy structures). Indeed, it is well known that semigroups are basic structures in many applicative branches like automata and formal languages, coding theory, finite state machines and others. Due to these possibilities of applications, semigroups and related structures are presently extensively investigated in fuzzy settings. A semigroup is an algebraic structure consisting of a non-empty set S together with an associative binary operation [17]. The formal study of semigroups began in the early 20th century. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis. Nobuaki Kuroki [25][26][27] is the pioneer of fuzzy ideal theory of semigroups. The idea of fuzzy subsemigroup was also introduced by Kuroki [25][27]. In [26], Kuroki characterized several classes of semigroups in terms of fuzzy left, fuzzy right and fuzzy bi-ideals. Others who worked on fuzzy semigroup theory, such as X. Y. Xie [47][48], Y. B. Jun [19][20], are mentioned in the bibliography. X. Y. Xie [47] introduced the idea of extensions of fuzzy ideals in semigroups. N. Kehayopulu and M. Tsingelis [21][22], S. K. Lee [30] worked on po-semigroups. Authors who worked on fuzzy po-semigroup theory are N. Kehayopulu and M. Tsingelis [23][24], X. Y. Xie and F. Yan [48], M. Shabir and A. Khan [44].

The notion of a Γ-semigroup was introduced by Sen and Saha [38] as a generalization of semigroups and ternary semigroup. Γ-semigroup have been analyzed by lot of mathematicians, for instance by Chattopadhyay [5][6], Dutta and Adhikari [1][10][11], Hila [15][16], Chinram [7], Sen et al. [38][39][40][41][42], Seth [43], N. K. Saha [31]. S. K. Sardar and S. K. Majumder [32][33][34][13][14] have introduced the notion of fuzzification of ideals, prime ideals, semiprime ideals and ideal extensions of Γ-semigroups and studied them via its operator semigroups.

Y. I. Kwon and S. K. Lee [28][29], T. K. Dutta and N. C. Adhikari [12], Chinram and K. Tinpun [8], P. Dheena and B. Elavarasan [9], M. Siripitukdet and A. Iampan [18][45] are some authors who worked on po-Γ-semigroup theory. T. K. Dutta and N. C. Adhikari [12] have studied different properties of po-Γ-semigroup by defining operator po-semigroups of such type of po-Γ-semigroups. The purpose of this paper is as stated in the abstract.

In 2007, Uckun Mustafa, Ali Mehmet and Jun Young Bae [46] introduced the notion of intuitionistic fuzzy ideals in Γ-semigroups. Motivated by Kuroki [25][26][27], Mustafa et al. [46], S. K. Sardar et al. have initiated the study of Γ-semigroups in terms of fuzzy subsets [32][33][34] and intuitionistic fuzzy subsets [35][36]. They also initiated the study of po-Γ-semigroups in terms of fuzzy subsets [37]. The purpose of this paper is as mentioned in the abstract.

2. Preliminaries

After the introduction of fuzzy sets by Zadeh [49], several researches were conducted on the generalization of fuzzy sets. As an important generalization of the notion of fuzzy sets on a non-empty set X, Atanassov [2][3] introduced the concept of $IFS(X)$.[1] Now we shall discuss some elementary concepts of $IFS(X)$.

Definition 2.1. ([2]) An $IFS(X)$ is an object having the form

$$A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\},$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$.

Proposition 2.1. ([2][3]) *Let A and B be two $IFS(X)$. Then the following expressions hold:*

(1) $A \subseteq B$ *if and only if* $\mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$,
(2) $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$,
(3) $A^C = \{< x, \nu_A(x), \mu_A(x) >: x \in X\}$,
(4) $A \cap B = \{< x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\} >: x \in X\}$,
(5) $A \cup B = \{< x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\} >: x \in X\}$,
(6) $\Box A = \{< x, \mu_A(x), 1 - \mu_A(x) >: x \in X\}$,
(7) $\Diamond A = \{< x, 1 - \nu_A(x), \nu_A(x) >: x \in X\}$.

Definition 2.2. ([2]) For a non-empty family of $IFS(X)$, $A_i = (\mu_{A_i}, \nu_{A_i})_{i \in I}$, we define $\inf\limits_{i\in I} A_i = (\inf\limits_{i\in I} \mu_{A_i}, \sup\limits_{i\in I} \nu_{A_i})$ and $\sup\limits_{i\in I} A_i = (\sup\limits_{i\in I} \mu_{A_i}, \inf\limits_{i\in I} \nu_{A_i})$ which are also the $IFS(X)$, given as

$$\inf_{i\in I} A_i : X \to \{< a, b >: a, b \in [0,1] \text{ and } a + b \leq 1\}, x \mapsto (\inf_{i\in I} \mu_{A_i}(x), \sup_{i\in I} \nu_{A_i}(x))$$

and

$$\sup_{i\in I} A_i : X \to \{< a, b >: a, b \in [0,1] \text{ and } a + b \leq 1\}, x \mapsto (\sup_{i\in I} \mu_{A_i}(x), \inf_{i\in I} \nu_{A_i}(x)).$$

Remark 2.1. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$.

Now we shall discuss some elementary concepts of po-Γ-semigroup theory which will be required in the sequel.

Definition 2.3. ([41]) Let $S = \{x, y, z, \ldots\}$ and $\Gamma = \{\alpha, \beta, \gamma \ldots\}$ be two non-empty sets. Then S is called a Γ-semigroup if there exists a mapping $S \times \Gamma \times S \to S$ (images to be denoted by $a\alpha b$) satisfying

(1) $x\gamma y \in S$,
(2) $(x\beta y)\gamma z = x\beta(y\gamma z)$

[1] $IFS(X)$ denote the intuitionistic fuzzy sets defined on a non-empty set X.

for all $x, y, z \in S$ and for all $\alpha, \beta, \gamma \in \Gamma$.

Definition 2.4. ([42]) A Γ-semigroup S is said to be a po-Γ-semigroup (partially ordered Γ-semigroup) if

(1) S and Γ are posets,
(2) $a \leq b$ in S implies that $a\alpha c \leq b\alpha c$, $c\alpha a \leq c\alpha b$ in S for all $c \in S$ and for all $\alpha \in \Gamma$,
(3) $\alpha \leq \beta$ in Γ implies that $a\alpha b \leq a\beta b$ in S for all $a, b \in S$.

Remark 2.2. Definition 2.3 and 2.4 are the definitions of one sided Γ-semigroup and one sided po-Γ-semigroup respectively. The following are the definitions of both sided Γ-semigroup [38] given by M. K. Sen and both sided po-Γ-semigroup [12] given by T. K. Dutta and N. C. Adhikari. Throughout this paper unless mentioned otherwise S stands for a both sided po-Γ-semigroup.

Definition 2.5. ([38]) Let S and Γ be two non-empty sets. S is called a Γ-semigroup if there exist mappings from $S \times \Gamma \times S$ to S, written as $(a, \alpha, b) \longrightarrow a\alpha b$, and from $\Gamma \times S \times \Gamma$ to Γ, written as $(\alpha, a, \beta) \longrightarrow \alpha a\beta$ satisfying the following associative laws $(a\alpha b)\beta c = a(\alpha b\beta)c = a\alpha(b\beta c)$ and $\alpha(a\beta b)\gamma = (\alpha a\beta)b\gamma = \alpha a(\beta b\gamma)$ for all $a, b, c \in S$ and for all $\alpha, \beta, \gamma \in \Gamma$.

Definition 2.6. ([12]) A Γ-semigroup S is said to be a po-Γ-semigroup if

(1) S and Γ are posets,
(2) $a \leq b$ in S implies that $a\alpha c \leq b\alpha c, c\alpha a \leq c\alpha b$ in S and $\gamma a\alpha \leq \gamma b\alpha$ in Γ for all $c \in S$ and for all $\alpha, \gamma \in \Gamma$,
(3) $\alpha \leq \beta$ in Γ implies that $\alpha a\gamma \leq \beta a\gamma, \gamma a\alpha \leq \gamma a\beta$ in Γ and $a\alpha b \leq a\beta b$ in S for all $\gamma \in \Gamma$ and for all $a, b \in S$.

Remark 2.3. The partial order relations on S and Γ are denoted by same symbol $\leq$.

Example 2.1. ([12]) Let S be the set of all 2×3 matrices over the set of positive integers and Γ be the set of all 3×2 matrices over same set. Then S is a Γ-semigroup with respect to the usual matrix multiplication. Also S and Γ are posets with respect to $\leq$ defined by $(a_{ik}) \leq (b_{ik})$ if and only if $a_{ik} \leq b_{ik}$ for all i, k. Thus S is a po-Γ-semigroup.

3. Intuitionstic Fuzzy Ideals

Definition 3.1. ([12]) Let S be a po-Γ-semigroup. A non-empty subset I of S is said to be a right ideal(left ideal) of S if

(1) $I\Gamma S \subseteq I$ (resp. $S\Gamma I \subseteq I$),
(2) $a \in I$ and $b \leq a$ imply $b \in I$.

I is said to be an ideal of S if it is a right ideal as well as a left ideal of S.

Definition 3.2. $A = (\mu_A, \nu_A)$, a non-empty IFS of a po-Γ-semigroup S is called an $IFLI(S)$[2] if it satisfies:

(1) $\mu_A(x\gamma y) \geq \mu_A(y)$,
(2) $\nu_A(x\gamma y) \leq \nu_A(y)$,
(3) $x \leq y$ implies $\mu_A(x) \geq \mu_A(y)$ and $\nu_A(x) \leq \nu_A(y)$,

for all $x, y \in S$ and for all $\gamma \in \Gamma$.

Definition 3.3. $A = (\mu_A, \nu_A)$, a non-empty IFS of a po-Γ-semigroup S is called an $IFRI(S)$ if it satisfies:

(1) $\mu_A(x\gamma y) \geq \mu_A(x)$,
(2) $\nu_A(x\gamma y) \leq \nu_A(x)$,
(3) $x \leq y$ implies $\mu_A(x) \geq \mu_A(y)$ and $\nu_A(x) \leq \nu_A(y)$,

for all $x, y \in S$ and for all $\gamma \in \Gamma$.

Definition 3.4. $A = (\mu_A, \nu_A)$, a non-empty IFS of a po-Γ-semigroup S is called an $IFI(S)$ if it is an $IFLI(S)$ as well as an $IFRI(S)$.

REMARK 3.1. For all the results formulated in this paper, we only describe proof for the $IFLI$. For $IFRI$ similar results hold as well.

By routine verification we obtain the following proposition and subsequent lemmas.

Proposition 3.1. *If $\{A_i\}_{i\in\Lambda}$ represents a family of $IFLI(S)(IFRI(S), IFI(S))$, then $\bigcap_{i\in\Lambda} A_i$ is an $IFLI(S)(IFRI(S), IFI(S))$.*

Lemma 3.1. *If $A = (\mu_A, \nu_A)$ is an $IFLI(S)(IFRI(S), IFI(S))$, then so is $\Box A = (\mu_A, \mu_A^c)$.*

Lemma 3.2. *If $A = (\mu_A, \nu_A)$ is an $IFLI(S)(IFRI(S), IFI(S))$, then so is $\Diamond A = (\nu_A^c, \nu_A)$.*

Combining Lemma 3.1 and Lemma 3.2 we obtain the following theorem.

Theorem 3.1. *$A = (\mu_A, \nu_A)$ is an $IFLI(S)(IFRI(S), IFI(S))$, if and only if $\Box A$ and $\Diamond A$ are $IFLI(S)(IFRI(S), IFI(S))$.*

Definition 3.5. ([4]) For any $t \in [0, 1]$ and a fuzzy subset μ of S, the set

$$U(\mu; t) = \{x : x \in S \text{ and } \mu(x) \geq t\} (\text{resp. } L(\mu; t) = \{x : x \in S \text{ and } \mu(x) \leq t\})$$

is called an *upper* (*resp. lower*) *t-level cut of* μ.

Theorem 3.2. *If $A = (\mu_A, \nu_A)$ is an $IFLI(S)(IFRI(S), IFI(S))$, then the upper and lower level cuts $U(\mu_A; t)$ and $L(\nu_A; t)$ are $LI(S)(RI(S), I(S))$, for every $t \in Im(\mu_A) \cap Im(\nu_A)$.*

[2] $IFS(S)$, $IFLI(S)$, $IFRI(S)$, $IFI(S)$, $LI(S)$, $RI(S)$, $I(S)$ denote respectively the intuitionistic fuzzy subset(s), intuitionistic fuzzy left ideal(s), intuitionistic fuzzy right ideal(s), intuitionistic fuzzy ideal(s), left ideals(s), right ideals(s), ideals(s) of a po-Γ-semigroup S.

Proof. Let $t \in Im(\mu_A) \cap Im(\nu_A)$. Then there exists some $\alpha \in S$ such that $\mu_A(\alpha) = t$. Thus $U(\mu; t) \neq \emptyset$. Similarly $L(\nu; t) \neq \emptyset$. Now let $x \in S, \gamma \in \Gamma$ and $y \in U(\mu_A; t)$. Then $\mu_A(y) \geq t$. Since $A = (\mu_A, \nu_A)$ is an $IFLI(S)$, $\mu_A(x\gamma y) \geq \mu_A(y) \geq t$. Consequently, $x\gamma y \in U(\mu_A; t)$. Let $x, y \in S$ be such that $y \leq x$. Let $x \in U(\mu_A; t)$. Then $\mu_A(x) \geq t$. Since $A = (\mu_A, \nu_A)$ is an $IFLI(S)$, so $\mu_A(y) \geq \mu_A(x) \geq t$. Consequently, $y \in U(\mu_A; t)$. Hence $U(\mu_A; t)$ is an $LI(S)$.

Again, let $x \in S, \gamma \in \Gamma$ and $y \in L(\nu_A; t)$. Then $\nu_A(y) \leq t$. Since $A = (\mu_A, \nu_A)$ is an $IFLI(S)$, $\nu_A(x\gamma y) \leq \nu_A(y) \leq t$. Consequently, $x\gamma y \in L(\nu_A; t)$. Let $x, y \in S$ be such that $y \leq x$. Let $x \in L(\nu_A; t)$. Then $\nu_A(x) \leq t$. Since $A = (\mu_A, \nu_A)$ is an $IFLI(S)$, so $\nu_A(y) \leq \nu_A(x) \leq t$. Consequently, $y \in L(\nu_A; t)$. Hence $L(\nu_A; t)$ is an $LI(S)$. □

Theorem 3.3. *If $A = (\mu_A, \nu_A)$ is an $IFS(S)$ such that the non-empty sets $U(\mu_A; t)$ and $L(\nu_A; t)$ are $LI(S)(RI(S), I(S))$, for all $t \in [0, 1]$. Then $A = (\mu_A, \nu_A)$ is an $IFLI(S)(IFRI(S), IFI(S))$.*

Proof. For all $t \in [0, 1]$, let us assume that the non-empty sets $U(\mu_A; t)$ and $L(\nu_A; t)$ are $LI(S)$. Let $y \in S$. Then there exist $t_1, t_2 \in [0, 1]$ with $t_1 + t_2 \leq 1$ such that $\mu_A(y) = t_1$ and $\nu_A(y) = t_2$. Then $y \in U(\mu_A; t_1)$ and $y \in L(\nu_A; t_2)$. Let $x \in S$ and $\gamma \in \Gamma$. Since $U(\mu_A; t_1)$ and $L(\nu_A; t_2)$ are $LI(S)$, $x\gamma y \in U(\mu_A; t_1)$ and $x\gamma y \in L(\nu_A; t_2)$ which implies that $\mu_A(x\gamma y) \geq t_1 = \mu_A(y)$ and $\nu_A(x\gamma y) \leq t_2 = \nu_A(y)$.

Let $x, y \in S$ be such that $x \leq y$. Then there exist $t_1, t_2 \in [0, 1]$ with $t_1 + t_2 \leq 1$ such that $\mu_A(y) = t_1$ and $\nu_A(y) = t_2$. Then $y \in U(\mu_A; t_1)$ and $y \in L(\nu_A; t_2)$. Since $U(\mu_A; t_1)$ and $L(\nu_A; t_2)$ are $LI(S)$, $x \in U(\mu_A; t_1)$ and $x \in L(\nu_A; t_2)$. Then $\mu_A(x) \geq t_1 = \mu_A(y)$ and $\nu_A(x) \leq t_2 = \nu_A(y)$. Consequently, $A = (\mu_A, \nu_A)$ is an $IFLI(S)$. Similarly we can prove the other cases also. □

Proposition 3.2. *Let S be a po-Γ-semigroup and $A = (\mu_A, \nu_A)$ be an $IFI(S)$.*

1. *If ω be a fixed element of S, then $A^\omega = (\mu_A^\omega, \nu_A^\omega) = \{x : x \in S$ and $\mu_A(x) \geq \mu_A(\omega), \nu_A(x) \leq \nu_A(\omega)\}$ is an $I(S)$.*
2. *$U = (\mu_U, \nu_U) = \{x : x \in S$ and $\mu_U(x) = \mu_U(0), \nu_U(x) = \nu_U(0)\}$ is an $I(S)$.*

Proof. The proof is straightforward and so we omit it. □

Theorem 3.4. *For a non-empty subset I of a po-Γ-semigroup S, I is $LI(S)(RI(S), I(S))$ if and only if $A = (\mu, \nu)$ is an $IFLI(S)(IFRI(S), IFI(S))$ where μ and ν are two fuzzy subsets of S defined by*

$$\mu(x) := \begin{cases} \alpha_0 & \text{if } x \in I \\ \alpha_1 & \text{if } x \in S - I \end{cases}$$

and

$$\nu(x) := \begin{cases} \beta_0 & \text{if } x \in I \\ \beta_1 & \text{if } x \in S - I \end{cases}$$

where $0 \leq \alpha_1 < \alpha_0, 0 \leq \beta_0 < \beta_1$ and $\alpha_i + \beta_i \leq 1$ for $i = 0, 1$.

Proof. Let I be a $LI(S)$ and $x, y \in S, \gamma \in \Gamma$. If $y \notin I$, then $\mu(y) = \alpha_1$ and $\nu(y) = \beta_1$ and $\mu(x\gamma y) = \alpha_0$ or α_1 and $\nu(x\gamma y) = \beta_0$ or β_1 according as $x\gamma y \in I$ or $x\gamma y \notin I$. Again if $y \in I$, then $x\gamma y \in I$ and so $\mu(x\gamma y) = \alpha_0 = \mu(y)$ and $\nu(x\gamma y) = \beta_0 = \nu(y)$. Thus we see that $\mu(x\gamma y) \geq \mu(y)$ and $\nu(x\gamma y) \leq \nu(y)$.

Let $x, y \in S$ be such that $x \leq y$. If $y \in I$, then $\mu(y) = \alpha_0$ and $\nu(y) = \beta_0$. Since I is a $LI(S)$, $x \in I$. Then $\mu(x) = \alpha_0 = \mu(y)$ and $\nu(x) = \beta_0 = \nu(y)$. If $y \notin I$, then $\mu(y) = \alpha_1 \leq \mu(x)$ and $\nu(y) = \beta_1 \geq \nu(x)$. Hence $\mu(x) \geq \mu(y)$ and $\nu(x) \leq \nu(y)$. Consequently, (μ, ν) is an $IFLI(S)$. Similarly we can prove the other cases also.

In order to prove the converse, we first observe that by definition of μ and ν, $U(\mu; \alpha_0) = I = L(\nu; \beta_0)$. Then the proof follows from Theorem 3.2. □

Following result is the characteristic function criterion of an $IFLI(S)(IFRI(S), IFI(S))$ which follows as an easy consequence of the above result.

Corollary 3.1. *Let S be a po-Γ-semigroup. Then P is a $LI(S)(RI(S), I(S))$ if and only if $I = (\chi_P, \chi_P^c)$ is an $IFLI(S)(IFRI(S), IFI(S))$ where χ_P be the characteristic function of P.*

Now, we will define a composition of $IFI(S)$ in order to characterize regular po-Γ-semigroups in terms of $IFI(S)$.

Definition 3.6. Let S be a Γ-semigroup. Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B) \in IFLI(S)(IFRI(S), IFI(S))$. Then the product $A \circ B$ of A and B is defined as

$$(\mu_A \circ \mu_B)(x) = \begin{cases} \sup\limits_{x \leq u\gamma v} [\min\{\mu_A(u), \mu_B(v)\} : u, v \in S; \gamma \in \Gamma] \\ 0, \text{ otherwise} \end{cases}$$

and

$$(\nu_A \circ \nu_B)(x) = \begin{cases} \inf\limits_{x \leq u\gamma v} [\max\{\nu_A(u), \nu_B(v)\} : u, v \in S; \gamma \in \Gamma] \\ 1, \text{ otherwise.} \end{cases}$$

Theorem 3.5. *In a po-Γ-semigroup S the following statements are equivalent:*

(1) *$A = (\mu_A, \nu_A)$ is an $IFLI(S)(IFRI(S))$,*
(2) *$S \circ A \subseteq A (A \circ S \subseteq A)$ and $x \leq y$ implies that $\mu_A(x) \geq \mu_A(y)$ and $\nu_A(x) \leq \nu_A(y)$ where $S = (\chi_S, \chi_S^c)$ and χ_S is the characteristic function of S.*

Proof. $(1) \Rightarrow (2)$: Let $A = (\mu_A, \nu_A)$ be an $IFLI(S)$. So from definition $x \leq y$ implies that $\mu_A(x) \geq \mu_A(y)$ and $\nu_A(x) \leq \nu_A(y)$. Let $a \in S$. Suppose there exist $u, v \in S$ and $\delta \in \Gamma$ such that $a \leq u\delta v$. Then, since $A = (\mu_A, \nu_A)$ is an $IFLI(S)$, we have

$$\begin{aligned}(\chi_S \circ \mu_A)(a) &= \sup_{a \leq x\gamma y} [\min\{\chi_S(x), \mu_A(y)\}] \\ &= \sup_{a \leq x\gamma y} [\min\{1, \mu_A(y)\}] = \sup_{a \leq x\gamma y} \mu_A(y)\end{aligned}$$

and

$$\begin{aligned}(\chi_S^c \circ \nu_A)(a) &= \inf_{a \le x\gamma y}[\max\{\chi_S^c(x), \nu_A(y)\}] \\ &= \inf_{a \le x\gamma y}[\max\{0, \nu_A(y)\}] = \inf_{a \le x\gamma y} \nu_A(y).\end{aligned}$$

Now, since $A = (\mu_A, \nu_A)$ is an $IFLI(S)$, $\mu_A(x\gamma y) \ge \mu_A(y)$ and $\nu_A(x\gamma y) \le \nu_A(y)$ for all $x, y \in S$ and for all $\gamma \in \Gamma$. So $\mu_A(y) \le \mu_A(a)$ and $\nu_A(y) \ge \nu_A(a)$ for all $a \le x\gamma y$. Hence $\sup_{a \le x\gamma y} \mu_A(y) \le \mu_A(a)$ and $\inf_{a \le x\gamma y} \nu_A(y) \ge \nu_A(a)$. Thus $\mu_A(a) \ge (\chi_S \circ \mu_A)(a)$ and $\nu_A(a) \le (\chi_S^c \circ \nu_A)(a)$. If there do not exist $x, y \in S, \gamma \in \Gamma$ such that $a \le x\gamma y$ then $(\chi_S \circ \mu_A)(a) = 0 \le \mu_A(a)$ and $(\chi_S^c \circ \nu_A)(a) = 1 \ge \nu_A(a)$. Hence $S \circ A \subseteq A$. By a similar argument we can prove the other case also.

$(2) \Rightarrow (1)$: Let $S \circ A \subseteq A$. Let $x, y \in S$, $\gamma \in \Gamma$ and $a := x\gamma y$. Then clearly $a \le x\gamma y$. So $\mu_A(x\gamma y) = \mu_A(a) \ge (\chi_S \circ \mu_A)(a)$ and $\nu_A(x\gamma y) = \nu_A(a) \le (\chi_S^c \circ \nu_A)(a)$. Now

$$\begin{aligned}(\chi_S \circ \mu_A)(a) &= \sup_{a \le u\alpha v}[\min\{\chi_S(u), \mu_A(v)\}] \ge \min\{\chi_S(x), \mu_A(y)\} \\ &= \min\{1, \mu_A(y)\} = \mu_A(y)\end{aligned}$$

and

$$\begin{aligned}(\chi_S^c \circ \nu_A)(a) &= \inf_{a \le u\alpha v}[\max\{\chi_S^c(u), \nu_A(v)\}] \le \max\{\chi_S^c(x), \nu_A(y)\} \\ &= \max\{0, \nu_A(y)\} = \nu_A(y).\end{aligned}$$

Consequently, $\mu_A(x\gamma y) \ge \mu_A(y)$ and $\nu_A(x\gamma y) \le \nu_A(y)$. Hence $A = (\mu_A, \nu_A)$ is an $IFLI(S)$. By a similar argument we can show that if $A \circ S \subseteq A$, then $A = (\mu_A, \nu_A)$ is an $IFRI(S)$. □

Using the above theorem we deduce the following theorem.

Theorem 3.6. *In a Γ-semigroup S the following statements are equivalent:*
1. *$A = (\mu_A, \nu_A)$ is an $IFI(S)$,*
2. *$S \circ A \subseteq A$ and $A \circ S \subseteq A$ and $x \le y$ implies that $\mu_A(x) \ge \mu_A(y)$ and $\nu_A(x) \le \nu_A(y)$ where $S = (\chi_S, \chi_S^c)$ and χ_S is the characteristic function of S.*

Definition 3.7. ([37]) A po-Γ semigroup S is called regular if for any $x \in S$ there exist $a \in S, \alpha, \beta \in \Gamma$ such that $x \le x\alpha a\beta x$.

Definition 3.8. ([12]) Let A be a subset of a po-Γ semigroup S. Then we define $(A] := \{x : x \in S$ and $x \le y$ for some $y \in A\}$.

Proposition 3.3. ([37]) *In a po-Γ-semigroup S, if A and B are any two non-empty subsets of S, then $(A]\Gamma(B] \subseteq (A\Gamma B]$. Moreover, if A and B are any two ideals (left, right or both sided) of S, then*
1. *$(A] = A, (B] = B$,*
2. *$(A \cap B] = (A] \cap (B]$.*

Theorem 3.7. ([37]) *A po-*Γ *semigroup* S *is regular if and only if* $A \cap B = (A\Gamma B]$ *for any right ideal* A *and for any left ideal* B *of* S.

Proposition 3.4. *Let* $A = (\mu_A, \nu_A)$ *be an* $IFRI(S)$ *and* $B = (\mu_B, \nu_B)$ *be an* $IFLI(S)$. *Then* $A \circ B \subseteq A \cap B$.

Proof. Let $x \in S$. Suppose there exist $u_1, v_1 \in S$ and $\gamma_1 \in \Gamma$ such that $x \leq u_1\gamma_1 v_1$. Then

$$\begin{aligned}(\mu_A \circ \mu_B)(x) &= \sup_{x \leq u\gamma v} \min\{\mu_A(u), \mu_B(v)\} \\ &\leq \sup_{x \leq u\gamma v} \min\{\mu_A(u\gamma v), \mu_B(u\gamma v)\} \\ &\leq \min\{\mu_A(x), \mu_B(x))\} = (\mu_A \cap \mu_B)(x)\end{aligned}$$

and

$$\begin{aligned}(\nu_A \circ \nu_B)(x) &= \inf_{x \leq u\gamma v} \max\{\nu_A(u), \nu_B(v)\} \\ &\geq \inf_{x \leq u\gamma v} \max\{\nu_A(u\gamma v), \nu_B(u\gamma v)\} \\ &\geq \max\{\nu_A(x), \nu_B(x))\} = (\nu_A \cup \nu_B)(x).\end{aligned}$$

Suppose there do not exist $u, v \in S$ such that $x \leq u\gamma v$. Then $(\mu_A \circ \mu_B)(x) = 0 \leq (\mu_A \cap \mu_B)(x)$ and $(\nu_A \circ \nu_B)(x) = 1 \geq (\nu_A \cup \nu_B)(x)$. Hence $A \circ B \subseteq A \cap B$. □

Proposition 3.5. *Let* S *be a regular po-*Γ*-semigroup and* $A = (\mu_A, \nu_A)$ *and* $B = (\mu_B, \nu_B)$ *be two* $IFS(S)$. *Then* $A \circ B \supseteq A \cap B$.

Proof. Let $c \in S$. Since S is regular, then there exists an element $x \in S$ and $\gamma_1, \gamma_2 \in \Gamma$ such that $c \leq c\gamma_1 x\gamma_2 c = c\gamma c$ where $\gamma := \gamma_1 x \gamma_2 \in \Gamma$. Then

$$\begin{aligned}(\mu_A \circ \mu_B)(c) &= \sup_{c \leq u\alpha v} \{\min\{\mu_A(u), \mu_B(v)\}\} \\ &\geq \min\{\mu_A(c), \mu_B(c)\} = (\mu_A \cap \mu_B)(c)\end{aligned}$$

and

$$\begin{aligned}(\nu_A \circ \nu_B)(c) &= \inf_{c \leq u\alpha v} \{\max\{\nu_A(u), \nu_B(v)\}\} \\ &\leq \max\{\nu_A(c), \nu_B(c)\} = (\nu_A \cup \nu_B)(c).\end{aligned}$$

Hence $A \circ B \supseteq A \cap B$. □

We can obtain the following result by routine calculation.

Proposition 3.6. *Let S be a po-Γ-semigroup and $A, B \subseteq S$. Then*
(1) $(A] \subseteq (B]$ *if and only if* $\chi_{(A]} \leq \chi_{(B]}$ *and* $\chi^c_{(A]} \geq \chi^c_{(B]}$,
(2) $\chi_{(A\cap B]} \leq \chi_{(A]} \cap \chi_{(B]}$ *and* $\chi^c_{(A\cap B]} \geq \chi^c_{(A]} \cap \chi^c_{(B]}$,
(3) $\chi_{(A]} \circ \chi_{(B]} = \chi_{(A\Gamma B]}$ *and* $\chi^c_{(A]} \circ \chi^c_{(B]} = \chi^c_{(A\Gamma B]}$,

where $\chi_{(A]}, \chi_{(B]}, \chi_{(A\cap B]}$ *and* $\chi_{(A\Gamma B]}$ *are characteristic functions of* $(A], (B], (A\cap B]$ *and* $(A\Gamma B]$ *respectively.*

To conclude this section we obtain the following characterization of a regular po-Γ-semigroup in terms of intuitionistic fuzzy ideals.

Theorem 3.8. *In a po-Γ-semigroup S the following are equivalent:*
(1) *S is regular,*
(2) $A \circ B = A \cap B$ *where* $A = (\mu_A, \nu_A)$ *is an* $IFRI(S)$ *and* $B = (\mu_B, \nu_B)$ *is an* $IFLI(S)$.

Proof. (1) $\Rightarrow$ (2) : Let S be a regular po-Γ-semigroup. Then by Proposition 3.5, $A \circ B \supseteq A \cap B$. Again by Proposition 3.4, $A \circ B \subseteq A \cap B$. Hence $A \circ B = A \cap B$.

(2) $\Rightarrow$ (1) : Suppose (2) holds. Let L and R be respectively a $LI(S)$ and a $RI(S)$ and $x \in R \cap L$. Then $x \in (R \cap L]$. This together with Proposition 3.3 (2) implies that $x \in (R]$ and $x \in (L]$. Hence $(\chi_{(L]}(x), \chi^c_{(L]}(x)) = (\chi_{(R]}(x), \chi^c_{(R]}(x)) = (1, 0)$(where $\chi_{(L]}(x)$ and $\chi_{(R]}(x)$ are the characteristic functions of $(L]$ and $(R]$ respectively). Thus $(\chi_{(R]} \cap \chi_{(L]})(x) = \min\{\chi_{(R]}(x), \chi_{(L]}(x)\} = 1$ and $(\chi^c_{(R]} \cup \chi^c_{(L]})(x) = \max\{\chi^c_{(R]}(x), \chi^c_{(L]}(x)\}$ $= 0$. Now by Corollary 3.1, $(\chi_{(L]}, \chi^c_{(L]})$ and $(\chi_{(R]}, \chi^c_{(R]})$ are respectively an $IFLI(S)$ and an $IFRI(S)$. Hence by hypothesis, $\chi_{(R]} \circ \chi_{(L]} = \chi_{(R]} \cap \chi_{(L]}$ and $\chi^c_{(R]} \circ \chi^c_{(L]} = \chi^c_{(R]} \cup \chi^c_{(L]}$. So, $(\chi_{(R]} \circ \chi_{(L]})(x) = 1$ and $(\chi^c_{(R]} \circ \chi^c_{(L]})(x) = 0$. Hence, by Proposition 3.6 (3), $\chi_{(R\Gamma L]}(x) = 1$ and $\chi^c_{(R\Gamma L]}(x) = 0$. So $x \in (R\Gamma L]$. Hence $R \cap L \subseteq (R\Gamma L]$. Also clearly $(R\Gamma L] \subseteq R \cap L$. Hence $(R\Gamma L] = R \cap L$. Consequently, S is regular. $\square$

4. Corresponding Intuitionistic Fuzzy Ideals

Many results of po-semigroups could be extended to po-Γ-semigroups directly and via operator po-semigroups [12] (left, right) of a po-Γ-semigroup. In order to make operator po-semigroups of a po-Γ-semigroup work in the context of fuzzy sets as it worked in the study of po-Γ-semigroups [12], we obtain various relationships between intuitionistic fuzzy ideals of a po-Γ-semigroup and that of its operator po-semigroups. Here, among other results we obtain an inclusion preserving bijection between the set of all intuitionistic fuzzy ideals of a po-Γ-semigroup and that of its operator po-semigroups.

Definition 4.1. ([12]) Let S be a Γ-semigroup. Let us define a relation ρ on $S \times \Gamma$ as follows :$(x, \alpha)\rho(y, \beta)$ if and only if $x\alpha s = y\beta s$ for all $s \in S$ and $\gamma x\alpha = \gamma y\beta$ for all $\gamma \in \Gamma$. Then ρ is an equivalence relation. Let $[x, \alpha]$ denote the equivalence class containing (x, α). Let $L = \{[x, \alpha] : x \in S, \alpha \in \Gamma\}$. Then L is a semigroup with respect to the multiplication defined by $[x,\alpha][y,\beta]=[x\alpha y,\beta]$. This semigroup L is called the left operator semigroup of the Γ-semigroup S. Dually the right operator semigroup R of Γ-semigroup S is defined where the multiplication is defined by $[\alpha,a][\beta,b]=[\alpha a\beta,b]$.

Let $((S,\Gamma),\leq)$ be a po-Γ-semigroup. We define a relation $\leq$ on L by $[a,\alpha]\leq[b,\beta]$ if and only if $a\alpha s\leq b\beta s$ for all $s\in S$ and $\gamma a\alpha\leq\gamma b\beta$ for all $\gamma\in\Gamma$. With respect to this relation L becomes a po-semigroup. In a similar way R can be made into a po-semigroup.

If there exists an element $[e,\delta]\in L([\gamma,f]\in R)$ such that $e\delta s=s$ (resp. $s\gamma f=s$) for all $s\in S$ then $[e,\delta]$ (resp. $[\gamma,f]$) is called the left (resp. right) unity of S.

Now let S be a Γ-semigroup with unities and L and R are po-semigroups. We define a relation $\leq$ in S by $a\leq b$ if and only if $[a,\alpha]\leq[b,\alpha]$ in L and $[\alpha,a]\leq[\alpha,b]$ in R for all $\alpha\in\Gamma$. We also define a relation $\leq$ in Γ by $\alpha\leq\beta$ if and only if $[a,\alpha]\leq[a,\beta]$ in L and $[\alpha,a]\leq[\beta,a]$ in R for all $a\in S$. With respect to these relations S becomes a po-Γ-semigroup.

Definition 4.2. ([35]) For an $IFS(R)$, $A=(\mu_A,\nu_A)$, we define an $IFS(S)$, $A^*=(\mu_A,\nu_A)^*=(\mu_A^*,\nu_A^*)$, by $\mu_A^*(a)=\inf\limits_{\gamma\in\Gamma}\mu_A([\gamma,a])$ and $\nu_A^*(a)=\sup\limits_{\gamma\in\Gamma}\nu_A([\gamma,a])$, where $a\in S$. For an $IFS(S)$, $B=(\mu_B,\nu_B)$, we define an $IFS(R)$, $B^{*'}=(\mu_B,\nu_B)^{*'}=(\mu_B^{*'},\nu_B^{*'})$, by $\mu_B^{*'}([\alpha,a])=\inf\limits_{s\in S}\mu_B(s\alpha a)$ and $\nu_B^{*'}([\alpha,a])=\sup\limits_{s\in S}\nu_B(s\alpha a)$, where $[\alpha,a]\in R$. For an $IFS(L)$, $C=(\mu_C,\nu_C)$, we define an $IFS(S)$, $C^+=(\mu_C,\nu_C)^+=(\mu_C^+,\nu_C^+)$, by $\mu_C^+(a)=\inf\limits_{\gamma\in\Gamma}\mu_C([a,\gamma])$ and $\nu_C^+(a)=\sup\limits_{\gamma\in\Gamma}\nu_C([a,\gamma])$, where $a\in S$. For an $IFS(S)$, $D=(\mu_D,\nu_D)$, we define an $IFS(L)$, $D^{+'}=(\mu_D,\nu_D)^{+'}=(\mu_D^{+'},\nu_D^{+'})$, by $\mu_D^{+'}([a,\alpha])=\inf\limits_{s\in S}\mu_D(a\alpha s)$ and $\nu_D^{+'}([a,\alpha])=\sup\limits_{s\in S}\nu_D(a\alpha s)$ where $[a,\alpha]\in L$.

Now we recall the following propositions from [11] which were proved therein for one sided ideals. But the results can be proved to be true for two sided ideals.

Proposition 4.1. ([12]) *Let S be a Γ-semigroup with unities and R be its right operator semigroup. If P is a $LI(R)(I(R))$*[3]*, then P^* is a $LI(S)(I(S))$.*

Proposition 4.2. ([12]) *Let S be a Γ-semigroup with unities and R be its right operator semigroup. If Q is a $LI(S)(I(S))$, then $Q^{*'}$ is a $LI(R)(I(R))$.*

For convenience of the readers, we may note that for a Γ-semigroup S and its left, right operator semigroups L,R respectively four mappings namely $()^+$, $()^{+'}$, $()^*$, $()^{*'}$ occur. They are defined as follows: for $I\subseteq R, I^*=\{s:s\in S \text{ and } \forall\alpha\in\Gamma,\,[\alpha,s]\in I\}$;

[3] $LI(R), I(R), IFLI(R), IFI(R)$ respectively denote left ideal(s) of R, ideal(s) of R, intuitionistic fuzzy left ideal(s) of R, intuitionistic fuzzy ideal(s) of R.

for $P \subseteq S, P^{*'} = \{[\alpha, x] : [\alpha, x] \in R$ and $\forall s \in S,\ s\alpha x \in P\}$; for $J \subseteq L, J^{+} = \{s : s \in S$ and $\forall \alpha \in \Gamma,\ [s, \alpha] \in J\}$; for $Q \subseteq S, Q^{+'} = \{[x, \alpha] : [x, \alpha] \in L$ and $\forall s \in S$, $x\alpha s \in Q\}$.

Proposition 4.3. *Let S be a po-Γ-semigroup with unities, L be its left operator po-semigroup and $A = (\mu_A, \nu_A) \in IFI(L)(IFLI(L))$. Then $A^{+} = (\mu_A, \nu_A)^{+} = (\mu_A^{+}, \nu_A^{+}) \in IFI(S)$ (respectively $IFLI(S)$).*

Proof. Let $a, b \in S$ and $\gamma \in \Gamma$. Then

$$\begin{aligned}\mu_A^{+}(a\gamma b) &= \inf_{\alpha\in\Gamma}\mu_A([a\gamma b, \alpha]) = \inf_{\alpha\in\Gamma}\mu_A([a, \gamma][b, \alpha]) \\ &\geq \inf_{\alpha\in\Gamma}\mu_A([b, \alpha])(\text{since } A \text{ is an } IFLI(L)) \\ &= \mu_A^{+}(b).\end{aligned}$$

Argument is similar for proving $\nu_A^{+}(a\gamma b) \leq \nu_A^{+}(b)$. Similarly we can show that $\mu_A^{+}(a\gamma b) \geq \mu_A^{+}(a)$ and $\nu_A^{+}(a\gamma b) \leq \nu_A^{+}(a)$. Let $a, b \in S$ be such that $a \leq b$. Then $[a, \alpha] \leq [b, \alpha]$ in L for all $\alpha \in \Gamma$. Since $(\mu_A, \nu_A) \in IFI(L)$, $\mu_A([a, \alpha]) \geq \mu_A([b, \alpha])$ and $\nu_A([a, \alpha]) \leq \nu_A([b, \alpha])$ for all $\alpha \in \Gamma$. So

$$\mu_A^{+}(a) = \inf_{\alpha\in\Gamma}\mu_A([a, \alpha]) \geq \inf_{\alpha\in\Gamma}\mu_A([b, \alpha]) = \mu_A^{+}(b)$$

and

$$\nu_A^{+}(a) = \sup_{\alpha\in\Gamma}\nu_A([a, \alpha]) \leq \sup_{\alpha\in\Gamma}\nu_A([b, \alpha]) = \nu_A^{+}(b).$$

Hence $A^{+} = (\mu_A^{+}, \nu_A^{+}) \in IFI(S)$. □

Proposition 4.4. *Let S be a po-Γ-semigroup with unities, L be its left operator po-semigroup and $B = (\mu_B, \nu_B) \in IFI(S)(IFLI(S))$. Then $B^{+'} = (\mu_B, \nu_B)^{+'} = (\mu_B^{+'}, \nu_B^{+'}) \in IFI(L)$ (respectively $IFLI(L)$).*

Proof. Let $[a, \alpha], [b, \beta] \in L$. Then

$$\begin{aligned}\mu_B^{+'}([a, \alpha][b, \beta]) &= \mu_B^{+'}([a\alpha b, \beta]) = \inf_{s\in S}\mu_B(a\alpha b\beta s) = \inf_{s\in S}\mu_B(a\alpha(b\beta s)) \\ &\geq \inf_{s\in S}\mu_B(b\beta s)(\text{since } B \text{ is an } IFLI(S)) \\ &= \mu_B^{+'}([b, \beta]).\end{aligned}$$

Using similar argument we can show that $\mu_B^{+'}([a, \alpha][b, \beta]) \geq \mu_B^{+'}([a, \alpha])$. Similarly we can show that $\nu_B^{+'}([a, \alpha][b, \beta]) \leq \nu_B^{+'}([b, \beta])$ and $\nu_B^{+'}([a, \alpha][b, \beta]) \leq \nu_B^{+'}([a, \alpha])$. Now let $[a, \alpha], [b, \alpha] \in L$ be such that $[a, \alpha] \leq [b, \alpha]$. Then $a\alpha s \leq b\alpha s\ \forall \alpha \in \Gamma, \forall s \in S$. So

$\mu_B(a\alpha s) \geq \mu_B(b\alpha s)$ and $\nu_B(a\alpha s) \leq \nu_B(b\alpha s)$ $\forall \alpha \in \Gamma, \forall s \in S$. Then

$$\begin{aligned} \mu_{_B}^{+'}([a,\alpha]) &= \inf_{s\in S}\mu_{_B}(a\alpha s) \\ &\geq \inf_{s\in S}\mu_{_B}(b\alpha s) \\ &= \mu_{_B}^{+'}([b,\alpha]) \end{aligned}$$

and

$$\begin{aligned} \nu_{_B}^{+'}([a,\alpha]) &= \sup_{s\in S}\nu_B(a\alpha s) \\ &\leq \sup_{s\in S}\nu_{_B}(b\alpha s) \\ &= \nu_{_B}^{+'}([b,\alpha]) \end{aligned}$$

Hence $B^{+'} = (\mu_{_B}^{+'}, \nu_{_B}^{+'}) \in IFI(L)$. $\square$

Theorem 4.1. *Let S be a po-Γ-semigroup with unities and L be its left operator semigroup. Then there exists an inclusion preserving bijection $A \mapsto A^{+'}$ between the set of all $IFI(S)(IFLI(S))^4$ and set of all $IFI(L)$ (resp. $IFLI(L)$), where $A = (\mu_{_A}, \nu_{_A})$ is an $IFI(S)$ (resp. $IFLI(S)$).*

Proof. Let $A \in IFI(S)$. Then by Proposition 4.4, $A^{+'} \in IFI(L)$. So by Proposition 4.3, $(A^{+'})^+ \in IFI(S)$. From Theorem 2.9 [36], it is clear that $(A^{+'})^+ = A$ and $(A^+)^{+'} = A$. Also the inclusion preserving property follows from Theorem 2.9 [36]. $\square$

Remark 4.1. The right operator analogues of Propositions 4.3, 4.4 and Theorem 4.1 are also true.

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[4] $IFLI(L)$ denotes intuitionistic fuzzy left ideal(s) of L.

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