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ON SOME SEQUENCE SPACES OF NON-ABSOLUTE TYPE

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Abstract. In this paper, we introduce the notion of λ_v-convergent and bounded sequences. Further, we introduce the spaces $\ell_\infty^\lambda(\triangle_v)$, $c_0^\lambda(\triangle_v)$ and $c^\lambda(\triangle_v)$, which are BK-spaces of non-absolute type and we prove that these spaces are linearly isomorphic to the spaces ℓ_∞, c_0 and c, respectively. Moreover, we establish some inclusion relations between these spaces.

1. Introduction

A sequence space is defined to be a linear space of real or complex sequences. Let w denote the spaces of all complex sequences. If $x \in w$, then we simply write $x = (x_k)$ instead of $x = (x_k)_{n=0}^\infty$.

Let X be a sequence space. If X is a Banach space and

$$\tau_k : X \to C\ ,\ \tau_k(x) = x_k \quad (k = 1, 2, ...)$$

is a continuous for all k, X is called a BK-space.

We shall write ℓ_∞, c and c_0 for the sequence spaces of all bounded, convergent and null sequences, respectively, which are BK-spaces with the same norm given by

$$\|x\|_\infty = \sup_k |x_k|$$

for all $k \in N$.

M. Mursaleen and A. K. Noman [10] introduced the sequence spaces ℓ_∞^λ, c^λ and c_0^λ as the sets of all λ-bounded, λ-convergent and λ-null sequences, respectively, that is

$$\ell_\infty^\lambda = \left\{ x \in w : \sup_n |\Lambda_n(x)| < \infty \right\},$$

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$$c^{\lambda}=\left\{x\in w:\lim_{n\to\infty}\Lambda_n(x)\,\text{exists}\right\},$$
$$c_0^{\lambda}=\left\{x\in w:\lim_{n\to\infty}\Lambda_n(x)=0\right\},$$

where $\Lambda_n(x)=\frac{1}{\lambda_n}\sum_{k=0}^{n}(\lambda_k-\lambda_{k-1})\,x_k,\ k\in\mathbb{N}$.

H. Ganie and N. A. Sheikh [7] introduced the spaces $c_0\left(\triangle_u^{\lambda}\right)$ and $c\left(\triangle_u^{\lambda}\right)$ as follows:

$$c\left(\Delta_u^{\lambda}\right)=\left\{x\in w:\lim_{n\to\infty}\hat{\Lambda}_n(x)\,\text{exists}\right\},$$
$$c_0\left(\Delta_u^{u}\right)=\left\{x\in w:\lim_{n\to\infty}\hat{\Lambda}_n(x)=0\right\},$$

where $\hat{\Lambda}_n(x)=\frac{1}{\lambda_n}\sum_{k=0}^{n}(\lambda_k-\lambda_{k-1})\,u_k\,(x_k-x_{k-1}),\ \ k\in\mathbb{N}$.

Let $v=(v_k)$ be any fixed sequence of non-zero complex numbers. Colak [6] defined the sequence spaces $\ell_\infty(\Delta_v)$, $c(\Delta_v)$ and $c_0(\Delta_v)$ as follows:

$$\ell_\infty(\Delta_v)=\{x\in w:\Delta_v x_k\in\ell_\infty\},$$
$$c(\triangle_v)=\{x\in w:\Delta_v x_k\in c\},$$
$$c_0(\triangle_v)=\{x\in w:\Delta_v x_k\in c_0\},$$

where $\Delta_v x_k=v_k x_k-v_{k-1}x_{k-1}$.

Several authors have recently introduced new sequence spaces, see for instance [1,2,11].

2. Notion of λ_v-convergent and bounded sequences

Throughout this paper, let $\lambda=(\lambda_k)$ be a strictly increasing sequence of positive reals tending to infinity, $0<\lambda_0<\lambda_1<...$ and $\lambda_k\to\infty$ as $k\to\infty$. We take

$$\tilde{\Lambda}_n(x)=\frac{1}{\lambda_n}\sum_{k=0}^{n}(\lambda_k-\lambda_{k-1})\,\triangle_v\,x_k, \tag{2.1}$$

where $\Delta_v x_k=v_k x_k-v_{k-1}x_{k-1}$.

A sequence $x=(x_k)\in w$ is λ_v-convergent to the number $\ell\in C$, called as the λ_v-limit of x, if $\tilde{\Lambda}_n(x)\to\ell$ as $n\to\infty$. In particular, we say that x is a λ_v-null sequence if $\tilde{\Lambda}_n(x)\to 0$ as $n\to\infty$. Further we say that x is λ_v-bounded if $\sup_n|\tilde{\Lambda}_n(x)|<\infty$. Negative subscript is equal to naught. For instance, $\lambda_{-1}=0$ and $x_{-1}=0$. We have

$$\lim_{n\to\infty}|\tilde{\Lambda}_n(x)-a|=\lim_{n\to\infty}\left|\frac{1}{\lambda_n}\sum_{k=0}^{n}(\lambda_k-\lambda_{k-1})(\Delta_v x_k-a)\right|=0. \tag{2.2}$$

So we can say by (2.2) $\lim_{n\to\infty}\tilde{\Lambda}_n(x)=a$. Hence x is λ_v-convergent to a.

Lemma 2.1. *Every convergent sequence is λ_v-convergent to the same ordinary limit.*

The following result is immediate by Lemma 2.1.

Lemma 2.2. *If a λ_v-convergent sequence converges in the ordinary sense, then it must converge to the same λ_v-limit.*

Let $x = (x_k) \in w$ and $n \geq 1$. Then by using (2.1) we derive that

$$\begin{aligned} \Delta_v x_n - \tilde{\Lambda}_n(x) &= \Delta_v x_n - \frac{1}{\lambda_n} \sum_{k=0}^{n} (\lambda_k - \lambda_{k-1}) \Delta_v x_k \\ &= \frac{1}{\lambda_n} \sum_{k=0}^{n-1} (\lambda_k - \lambda_{k-1}) \sum_{i=k+1}^{n} (\Delta_v x_i - \Delta_v x_{i-1}) \\ &= \frac{1}{\lambda_n} \sum_{i=1}^{n} (\Delta_v x_i - \Delta_v x_{i-1}) \sum_{k=0}^{i-1} (\lambda_k - \lambda_{k-1}) \\ &= \frac{1}{\lambda_n} \sum_{i=1}^{n} \lambda_{i-1} (\Delta_v x_i - \Delta_v x_{i-1}) \end{aligned}$$

Therefore we have for every $x = (x_k) \in w$ that

$$\Delta_v x_n - \tilde{\Lambda}_n(x) = S_n(x) \qquad (n \in \mathbb{N}) \tag{2.3}$$

where the sequence $S(x) = (S_n(x))_{n=0}^{\infty}$ is defined by

$$S_0(x) = 0 \quad \text{and} \quad S_n(x) = \frac{1}{\lambda_n} \sum_{i=1}^{n} \lambda_{i-1} (\Delta_v x_i - \Delta_v x_{i-1}), \quad (n \geq 1). \tag{2.4}$$

Lemma 2.3. *A λ_v-convergent sequence x converges in the ordinary sense if and only if $S(x) \in c_0$.*

Proof. Let $x = (x_n)$ be λ_v-convergent sequence in the ordinary sense. Then, from Lemma 2.2 we have $x = (x_n)$ converges to the same λ_v-limit. We obtain $S(x) \in c_0$ by (2.3). Conversely, let $S(x) \in c_0$. We have

$$\lim_{n\to\infty} \Delta_v x_n = \lim_{n\to\infty} \tilde{\Lambda}_n(x).$$

From the above equation, we deduce that λ_v-convergent sequence x converges in the ordinary sense. □

Lemma 2.4. *Every bounded sequence is λ_v-bounded.*

Lemma 2.5. *A λ_v-bounded sequence x is bounded in the ordinary sense if and only if $S(x) \in \ell_\infty$.*

Proof. We can obtain it directly from Lemma 2.4 and (2.4). □

3. The spaces of λ_v-convergent and bounded sequences

In this section, we introduce the sequence space $\ell_\infty^\lambda(\Delta_v)$, $c^\lambda(\Delta_v)$ and $c_0^\lambda(\Delta_v)$ as the sets of all λ_v-bounded, λ_v-convergent and λ_v-null sequences;

$$\ell_\infty^\lambda(\Delta_v) = \left\{x \in w : \sup \left|\tilde{\Lambda}_n(x)\right| < \infty\right\},$$

$$c^\lambda(\Delta_v) = \left\{x \in w : \lim_{n\to\infty} \tilde{\Lambda}_n(x) \text{ exists}\right\}$$

$$c_0^\lambda(\Delta_v) = \left\{x \in w : \lim_{n\to\infty} \tilde{\Lambda}_n(x) = 0\right\}$$

where $\tilde{\Lambda}_n(x)$ like (2.1).

Theorem 3.1. *The sequence spaces $\ell_\infty^\lambda(\Delta_v)$, $c^\lambda(\Delta_v)$ and $c_0^\lambda(\Delta_v)$ are BK-spaces with the same norm given by*

$$\|x\|_{\ell_\infty^\lambda(\Delta_v)} = \left\|\tilde{\Lambda}_n(x)\right\|_{\ell_\infty} = \sup_n \left|\tilde{\Lambda}_n(x)\right|.$$

Proof. The proof is seen eaisly. So it is omitted. □

Remark 3.1. We can see that the absolute property does not hold on the spaces $\ell_\infty^\lambda(\Delta_v)$, $c^\lambda(\Delta_v)$ and $c_0^\lambda(\Delta_v)$. For at least one sequence x in each of these spaces have that $\|x\|_{\ell_\infty^\lambda(\Delta_v)} \neq \||x|\|_{\ell_\infty^\lambda(\Delta_v)}$, where $|x| = (|x_k|)$. So these spaces are BK-spaces of non-absolute type.

Theorem 3.2. *The sequence spaces $\ell_\infty^\lambda(\Delta_v)$, $c^\lambda(\Delta_v)$ and $c_0^\lambda(\Delta_v)$ are linearly isomorphic to the spaces ℓ_∞, c and c_0.*

Proof. We only consider the case $c_0^\lambda(\Delta_v) \cong c_0$. The cases $c^\lambda(\Delta_v) \cong c$ and $\ell_\infty^\lambda(\Delta_v) \cong \ell_\infty$ can be shown similarly. To prove the theorem, we must show the existence of linear bijection between $c_0^\lambda(\Delta_v)$ and c_0. Consider the transformation T defined, $Tx = \tilde{\Lambda}(x) \in c_0$ for every $x \in c_0^\lambda(\Delta_v)$. The linearity of T is obvious. It is trivial that $x = 0$ whenever $Tx = 0$ and hence T is injective.

To show surjective we define the sequence $x = \{x_k(\lambda)\}$ by

$$x_k(\lambda) = v_k^{-1} \sum_{j=0}^{k} \sum_{i=j-1}^{j} (-1)^{j-i} \frac{\lambda_i}{(\lambda_j - \lambda_{j-1})} y_i \quad \text{for} \quad k \in \mathbb{N}.$$

We have that

$$\tilde{\Lambda}_n(x) = \frac{1}{\lambda_n} \sum_{k=0}^{n} \sum_{i=k-1}^{k} (-1)^{k-i} \lambda_i y_i = y_n. \tag{3.1}$$

We can say that $\tilde{\Lambda}_n(x) = y_n$ from (3.1) and $y \in c_0$, hence $\tilde{\Lambda}_n(x) \in c_0$. We deduce from that $x \in c_0^\lambda(\triangle_v)$ and $Tx = y$. Hence T is surjective.

We have for every $x \in c_0^\lambda(\triangle_v)$ that

$$\|Tx\|_{c_0} = \|Tx\|_{\ell_\infty} = \left\|\tilde{\Lambda}(x)\right\|_{\ell_\infty} = \|x\|_{c_0^\lambda(\triangle_v)}$$

which means that $c_0^\lambda(\triangle_v)$ and c_0 are linearly isomorphic. Similarly we can obtain that $c^\lambda(\Delta_v) \cong c$ and $\ell_\infty^\lambda(\Delta_v) \cong \ell_\infty$. □

4. Some Inclusion Relations

Theorem 4.1. *The inclusions $c_0^\lambda(\Delta_v) \subset c^\lambda(\Delta_v) \subset \ell_\infty^\lambda(\Delta_v)$ strictly hold.*

Proof. It is obvious that the inclusions $c_0^\lambda(\Delta_v) \subset c^\lambda(\Delta_v) \subset \ell_\infty^\lambda(\Delta_v)$ hold. Furthermore, since the inclusion $c_0 \subset c$ is strict, it follows by Lemma 2.1 that the inclusion $c_0^\lambda(\Delta_v) \subset c^\lambda(\Delta_v)$ is also strict. Consider the sequence $x = (x_k)$ defined by

$$x_k = v_k^{-1} \sum_{i=1}^{k} (-1)^i (\lambda_i + \lambda_{i-1}) / (\lambda_i - \lambda_{i-1})$$

for all $k \in \mathbb{N}$. We obtain

$$\Delta_v x_k = (-1)^k (\lambda_k + \lambda_{k-1}) / (\lambda_k + \lambda_{k-1})$$

for $k \in \mathbb{N}$. Then we have for every $n \in \mathbb{N}$ that

$$\tilde{\Lambda}_n(x) = \frac{1}{\lambda_n} \sum_{k=0}^{n} (-1)^k (\lambda_k + \lambda_{k-1}) = (-1)^n.$$

We can say that $\tilde{\Lambda}_n(x) \in \ell_\infty / c$. Hence the sequence x is in $\ell_\infty^\lambda(\Delta_v)$ but not in $c^\lambda(\Delta_v)$. So the inclusion $c^\lambda(\Delta_v) \subset \ell_\infty^\lambda(\Delta_v)$ strictly holds. This completes proof. □

Theorem 4.2. *The inclusion $\ell_\infty(\Delta_v) \subset \ell_\infty^\lambda(\Delta_v)$ holds.*

Proof. Let $x \in \ell_\infty(\Delta_v)$. Then we deduce that

$$\frac{1}{\lambda_n} \sum_{k=0}^{\infty} (\lambda_k - \lambda_{k-1}) |\Delta_v x_k| \leq \frac{1}{\lambda_n} \sup_k |\Delta_v x_k| \sum_{k=0}^{n} (\lambda_k - \lambda_{k-1}) = \sup_k |\Delta_v x_k| < \infty.$$

Hence, $x \in \ell_\infty^\lambda(\Delta_v)$. □

The following result is immediate by the regularity of the matrix $\tilde{\Lambda}$ and by Lemma 2.3.

Lemma 4.1. *The inclusions $c_0 \subset c_0^\lambda(\Delta_v)$ and $c \subset c^\lambda(\Delta_v)$ hold. Furthermore, the equalities hold if and only if $S(x) \in c_0$ for every sequence x in the spaces $c_0^\lambda(\Delta_v)$ and $c^\lambda(\Delta_v)$, respectively.*

Proof. $c_0 \subset c_0^\lambda(\Delta_v)$ and $c \subset c^\lambda(\Delta_v)$ are obvious from Lemma 2.1. To prove second part we suppose firstly equality $c = c_0^\lambda(\Delta_v)$ holds. Then, we have for every $x \in c_0^\lambda(\Delta_v)$ that $x \in c_0$ and hence $S(x) \in c_0$ by Lemma 2.3. Conversely, let $x \in c_0^\lambda(\Delta_v)$. Then, we have that $S(x) \in c_0$. Thus, it follows by Lemma 2.3 and then Lemma 2.2, that $x \in c_0$. This shows that the inclusion $c_0^\lambda(\Delta_v) \subset c_0$ holds. Hence by combining the inclusions $c_0^\lambda(\Delta_v) \subset c_0$ and $c_0 \subset c_0^\lambda(\Delta_v)$, we get the equality $c_0^\lambda(\Delta_v) = c_0$.

We can similarly show the equality $c = c^\lambda(\Delta_v)$ holds if and only if $S(x) \in c_0$ for every $x \in c^\lambda(\Delta_v)$. □

Lemma 4.2. *The inclusion* $\ell_\infty \subset \ell_\infty^\lambda(\Delta_v)$ *holds. Furthermore, the equality* $\ell_\infty = \ell_\infty^\lambda(\Delta_v)$ *holds if and only if* $S(x) \in \ell_\infty$ *for every* $x \in \ell_\infty^\lambda(\Delta_v)$*.*

It can be seen clearly from by Lemma 4.1 we can say that $c_0 \subset c_0^\lambda(\Delta_v) \cap c$. Conversely, it follows by Lemma 2.2 that $c_0^\lambda(\Delta_v) \cap c \subset c_0$. Hence the following result can be derived.

Theorem 4.3. *The equality* $c_0^\lambda(\Delta_v) \cap c = c_0$ *holds.*

Let $x = (x_k) \in w$ and $n \geqslant 1$. Then, from (2.3) and (2.4), we derive that

$$\begin{aligned} S_n(x) &= \frac{1}{\lambda_n}\sum_{k=1}^{n}\lambda_{k-1}(\Delta_v x_k - \Delta_v x_{k-1}) \\ &= \frac{1}{\lambda_n}\left[\sum_{k=1}^{n}\lambda_{k-1}\Delta_v x_k - \sum_{k=1}^{n-1}\lambda_k\Delta_v x_k\right] \\ &= \frac{1}{\lambda_n}\left[\lambda_{n-1}\Delta_v x_n - \sum_{k=0}^{n-1}(\lambda_k - \lambda_{k-1})\Delta_v x_k\right] \\ &= \frac{\lambda_{n-1}}{\lambda_n}\left[\Delta_v x_n - \tilde{\Lambda}_{n-1}(x)\right] \\ &= \frac{\lambda_{n-1}}{\lambda_n}\left[S_n(x) + \tilde{\Lambda}_n(x) - \tilde{\Lambda}_{n-1}(x)\right] \end{aligned}$$

Hence, we have for every $x \in w$ that

$$S_n(x) = \frac{\lambda_{n-1}}{\lambda_n - \lambda_{n-1}}\left[\tilde{\Lambda}_n(x) - \tilde{\Lambda}_{n-1}(x)\right] \qquad (n \in \mathbb{N}). \tag{4.1}$$

Theorem 4.4. *The inclusion* $\ell_\infty \subset \ell_\infty^\lambda(\Delta_v)$ *strictly holds if and only if*

$$\lim_{n\to\infty}\inf\frac{\lambda_{n+1}}{\lambda_n} = 1.$$

Proof. If we suppose that the inclusion $\ell_\infty \subset \ell_\infty^\lambda(\Delta_v)$ is strict, then Lemma 4.2 implies the existence of a sequence $x \in \ell_\infty^\lambda(\Delta_v)$ such that $S(x) = (S(x))_{n=0}^\infty \notin \ell_\infty$. Since $x \in \ell_\infty^\lambda(\Delta_v)$, we have $\tilde{\Lambda}(x) = \left(\tilde{\Lambda}(x)\right)_{n=0}^\infty \in \ell_\infty$ and hence $\left(\tilde{\Lambda}_n(x) - \tilde{\Lambda}_{n-1}(x)\right)_{n=0}^\infty \in \ell_\infty$. Therefore, we deduce from (4.1) that $((\lambda_{n-1}/\lambda_n - \lambda_{n-1}))_{n=0}^\infty \notin \ell_\infty$ and hence $((\lambda_n/\lambda_n - \lambda_{n-1}))_{n=0}^\infty \notin \ell_\infty$. This leads us with part (a) of Lemma 4.5 [see 10] to the consequence that $\lim_{n\to\infty}\inf\lambda_{n+1}/\lambda_n = 1$. To prove the sufficiency, suppose that $\lim_{n\to\infty}\inf\lambda_{n+1}/\lambda_n = 1$. Then, we have by part (a) of Lemma 4.5 [see 10] that $((\lambda_n/\lambda_n - \lambda_{n-1}))_{n=0}^\infty \notin \ell_\infty$. Let us now define the sequence $x = (x_k)$ by $x_k =$

$v_k^{-1} \sum_{i=1}^{k} (-1)^i \lambda_i / (\lambda_i - \lambda_{i-1})$ for all k. Then, we have for every $n \in \mathbb{N}$ that

$$\left|\tilde{\Lambda}_n(x)\right| = \frac{1}{\lambda_n} \left| \sum_{k=0}^{n} (-1)^k \lambda_k \right| \leq \frac{1}{\lambda_n} \sum_{k=0}^{n} (\lambda_k - \lambda_{k-1}) = 1$$

which shows that $\tilde{\Lambda}_n(x) \in \ell_\infty$. Thus, the sequence x is in the $\ell_\infty^\lambda(\Delta_v)$ but not in ℓ_∞. Therefore, by combining this with the fact that the inclusion $\ell_\infty \subset \ell_\infty^\lambda(\Delta_v)$ always holds by Lemma 4.2, we obtain that this inclusion is strict. ☐

Corollary 4.1. *The equality $\ell_\infty^\lambda(\Delta_v) = \ell_\infty$ holds if and only if $\lim_{n\to\infty} \inf \lambda_{n+1}/\lambda_n > 1$.*

Proof. The necessity is immediate by Theorem 4.4. For, if the equalities hold then the inclusions in Theorem 4.4, cannot be strict and hence $\lim_{n\to\infty} \inf \lambda_{n+1}/\lambda_n \neq 1$ which implies that $\lim_{n\to\infty} \inf \lambda_{n+1}/\lambda_n > 1$. Conversely, suppose that $\lim_{n\to\infty} \inf \lambda_{n+1}/\lambda_n > 1$. Then, it follows by part (b) of Lemma 4.5 [see 7] that $(\lambda_n/(\lambda_n - \lambda_{n-1}))_{n=0}^\infty$ and hence $(\lambda_n/(\lambda_n - \lambda_{n-1}))_{n=0}^\infty \in \ell_\infty$. Now, let $x \in \ell_\infty^\lambda(\Delta_v)$ be given. Then we have $\tilde{\Lambda}(x) = \left(\tilde{\Lambda}(x)\right)_{n=0}^\infty \in \ell_\infty$ and hence $\left(\tilde{\Lambda}_n(x) - \tilde{\Lambda}_{n-1}(x)\right)_{n=0}^\infty \in \ell_\infty$. Thus we obtain by (4.1) that $(S_n(x))_{n=0}^\infty \in \ell_\infty$. This shows that $S(x) \in \ell_\infty$ for every $x \in \ell_\infty^\lambda(\Delta_v)$. Consequently, we deduce by Lemma 4.2 that the equality $\ell_\infty^\lambda(\Delta_v) = \ell_\infty$ holds. ☐

References

[1] B. Altay, F. Başar, *Some Euler sequence spaces of non-absolute type*, Ukrainian Math. J. **57(1)** (2005), 1–7.

[2] C. Aydın, F. Başar, *On the new sequence spaces which include the spaces c_0 and c*, Hokkaido Math. J. **33(2)** (2004), 383–398.

[3] C. Bessage and A. Pelczynski, *Selected topics in infinite-dimensional topology*, Warszawa, 1975.

[4] B. Choudhary, S. Nanda, *Functional Analysis with Applications*, John Wiley & Sons Inc. New Delhi, 1989.

[5] R. G. Cooke, *Infinite matrices and sequence spaces*, London: Macmilan and Co. 1950.

[6] R. Çolak, *On some generalized sequence spaces*, Commun. Fac. Sci. Univ. Ank. Series **38** (1989), 35–46.

[7] A. H. Ganie, N. A. Sheikh, *On some new sequence spaces of non-absolute type and matrix transformations*, Journal of Egyptian Math. Society **21** (2013), 108–114.

[8] D. J. H. Garling, *On topological sequence spaces*, Proc. Cambridge Phil. Soc. **63** (1967), 963–981.

[9] I. J. Maddox, *Elements of Functional Analysis*, 2nd ed., The University Press, Cambridge, 1988.

[10] M. Mursaleen, A. K. Noman, *On the spaces of λ-convergent and bounded sequences*, Thai J. Math. **(2)** (2010) 311–329.

[11] M. Şengönül, F. Başar, *Some new Cesaro sequence spaces of non-absolute type which include the spaces c_0 and c*, Soochow J. Math. **31(1)** (2005) 107–119.

[12] A. Wilansky, *Summability Through Functional Analysis*, in: North-Holland Mathematics Studies, Elsevier Science Publishers, Amsterdam, New York, Oxford, 1984.

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