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COUPLED FIXED POINT THEOREMS FOR MONOTONE MAPPINGS IN PARTIALLY ORDERED METRIC SPACES

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Abstract. In this paper, by reducing of coincidence and coupled fixed point results in ordered metric spaces to the respective results for mappings with one variable, some recent results established by T. G. Bhaskar and V. Lakshmikantham [T. G. Bhaskar, V. Lakshmikantham, Fixed point theorems in partially ordered metric spaces and applications, Nonlinear Analysis 65 (2006) 1379-1393], V. Lakshmikantham and L. Ćirić [V. Lakshmikantham, L. Ćirić, Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces, Nonlinear Analysis 70 (2009) 4341–4349] are extended, generalized, unified and improved by using mappings with monotonicity instead of with mixed monotone property. Moreover, two examples are given to support these improvements.

1. Introduction and preliminaries

It is well known that the metric fixed point theory is still very actual, important and useful in all areas of Mathematics. It can be applied, for instance, in variational inequalities, optimization, dynamic programming, approximation theory and so on.

The fixed point theorems in partially ordered metric spaces play a major role in proving the existence and uniqueness of solutions for some differential and integral equations. One of the most interesting fixed point theorems in ordered metric spaces was investigated by T. G. Bhaskar and V. Lakshmikantham [1] thereof applied their results to the existence and uniqueness of solutions for a periodic boundary value problem. For some questions from linear and nonlinear differential equations the reader can see the recent paper of J. Nieto and R. R. Lopez [12]. Afterwards, many authors obtained several interesting results in ordered metric spaces.

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We start out with listing some notations and preliminaries in order to express our results. In the sequel $(X, d, \preceq)$ always denotes a partially ordered metric space where $(X, \preceq)$ is a partially ordered set and (X, d) is a metric space.

In this paper we do not use mappings with the mixed monotone property as in [1] and [11]. In the following, similarly as in [2] and [3], we introduce some important and basic notions as follows:

Definition 1.1. Let $(X, \preceq)$ be a partially ordered set and $F : X \times X \to X$ be a mapping. We say that F is a monotone if $F(x, y)$ is monotone nondecreasing in x and y, that is, for any $x, y \in X$,

$$x_1, x_2 \in X, x_1 \preceq x_2 \Rightarrow F(x_1, y) \preceq F(x_2, y),$$

and

$$y_1, y_2 \in X, y_1 \preceq y_2 \Rightarrow F(x, y_1) \preceq F(x, y_2).$$

An element $(x, y) \in X \times X$ is called a coupled fixed point of F if $F(x, y) = x$ and $F(y, x) = y$. It is clear that (x, y) is a coupled fixed point of F if and only if (y, x) is such.

Definition 1.2. Let $(X, \preceq)$ be a partially ordered set, $F : X \times X \to X$ and $g : X \to X$ be two mappings.

(i) We say that F has the g-monotone property if $F(x, y)$ is g-monotone nondecreasing in x and y, that is, for any $x, y \in X$,

$$x_1, x_2 \in X, g(x_1) \preceq g(x_2) \Rightarrow F(x_1, y) \preceq F(x_2, y),$$

and

$$y_1, y_2 \in X, g(y_1) \preceq g(y_2) \Rightarrow F(x, y_1) \preceq F(x, y_2).$$

(ii) An element $(x, y) \in X \times X$ is called a coupled coincidence point of F and g if $F(x, y) = gx, F(y, x) = gy$. Moreover, (x, y) is called a coupled common fixed point of F and g if

$$F(x, y) = gx = x, F(y, x) = gy = y.$$

(iii) The mappings F and g are called commutative if for all $x, y \in X$ it holds $g(F(x, y)) = F(gx, gy)$.

(iv) Let (X, d) be a metric space. The mappings $F : X \times X \to X$ and $g : X \to X$ are called compatible if

$$\begin{aligned} \lim_{n\to\infty} d(gF(x_n, y_n), F(gx_n, gy_n)) &= 0, \text{ and} \\ \lim_{n\to\infty} d(gF(y_n, x_n), F(gy_n, gx_n)) &= 0, \end{aligned}$$

hold whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X such that

$$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} gx_n, \lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} gy_n.$$

Definition 1.3. [1] Let $(X, \preceq)$ be an ordered set and d be a metric on X. We say that $(X, d, \preceq)$ is regular if it has the following properties:

(i) if nondecreasing sequence $\{x_n\}$ holds $d(x_n, x) \to 0$, then $x_n \preceq x$ for all n;
(ii) if nonincreasing sequence $\{y_n\}$ holds $d(y_n, y) \to 0$, then $y_n \succeq y$ for all n.

The proof of the following Lemma is immediate.

Lemma 1.1. *The following statements hold.*

(i) *Let $(X, d, \preceq)$ be a partially ordered metric space. If relation $\sqsubseteq$ is defined on $X^2 = X \times X$ by*

$$Y \sqsubseteq V \Leftrightarrow x \preceq u \wedge y \preceq v, \;\; Y = (x, y), V = (u, v) \in X^2,$$

and $d_{\max} : X^2 \times X^2 \to \mathbb{R}^+$ is given by

$$d_{\max}(Y, V) = \max\{d(x, u), d(y, v)\}, Y = (x, y), V = (u, v) \in X^2,$$

then $(X^2, \sqsubseteq, d_{\max})$ is an ordered metric space. The space $(X^2, d_{\max})$ is complete if and only if (X, d) is complete. Also, the space $(X^2, d_{\max}, \sqsubseteq)$ is regular if and only if $(X, d, \preceq)$ is regular.

(ii) *If $F : X \times X \to X$ and $g : X \to X$ and if F has the g-monotone property, then a mapping $T_F : X \times X \to X \times X$ given by*

$$T_F(Y) = (F(x, y), F(y, x)), \;\; Y = (x, y) \in X^2$$

is T_g-nondecreasing with respect to $\sqsubseteq$, that is,

$$T_g(Y) \sqsubseteq T_g(V) \Rightarrow T_F(Y) \sqsubseteq T_F(V),$$

where $T_g(Y) = T_g((x, y)) = (gx, gy)$.

(iii) *The mappings F and g are continuous (resp. compatible) if and only if T_F and T_g are continuous (resp. compatible).*

(iv) *$F(X^2)$ (resp. $g(X^2)$) is complete in the metric spaces (X, d) if and only if $T_F(X^2)$ (resp. $T_g(X^2)$) is complete in the space $(X^2, d_{\max})$.*

(v) *The mappings $F : X^2 \to X$ and $g : X \to X$ have a coupled coincidence point if and only if the mappings T_F and T_g have a coincidence point in X^2.*

(vi) *The mappings $F : X^2 \to X$ and $g : X \to X$ have a coupled common fixed point if and only if mappings T_F and T_g have a common fixed point in X^2.*

Be similar to the following lemma, some assertions were used in the frame of metric spaces in the course of proofs of several fixed point results in various papers.

Lemma 1.2. [13] *Let (X, d) be a metric space and let $\{y_n\}$ be a sequence in X such that*

$$\lim_{n\to\infty} d(y_n, y_{n+1}) = 0.$$

If $\{y_n\}$ is not a Cauchy sequence in (X, d), then there exist $\varepsilon > 0$ and two sequences $\{m(k)\}$ and $\{n(k)\}$ of positive integers such that $m(k) > n(k) > k$ and the following four sequences tend to ε^+ as $k \to \infty$:

$$d\left(y_{m(k)}, y_{n(k)}\right), d\left(y_{m(k)}, y_{n(k)+1}\right), d\left(y_{m(k)-1}, y_{n(k)}\right), d\left(y_{m(k)-1}, y_{n(k)+1}\right).$$

In [1] Blaskar and Lakshmikantham proved the following theorem and formulated as Theorem 2.1.

Theorem 1.1. *Let $F : X \times X \to X$ be a continuous mapping having the mixed monotone property on X. Assume that there exists a $k \in [0,1)$ with*

$$d(F(x,y), F(u,v)) \leq \frac{k}{2}[d(x,u) + d(y,v)], \forall x \succeq u, y \preceq v.$$

If there exist $x_0, y_0 \in X$ such that

$$x_0 \preceq F(x_0,y_0) \text{ and } y_0 \succeq F(y_0,x_0),$$

then there exist $x, y \in X$ such that

$$x = F(x,y) \text{ and } y = F(y,x).$$

Also, in [1] Blaskar and Lakshmikantham proved the following theorem and formulated as Theorem 2.1.

Theorem 1.2. *Let $(X, \preceq)$ be a partially ordered set and suppose that there exists a metric d in X such that (X,d) is a complete metric space. Assume that X has the following properties:*

(i) *if a nondecreasing sequence $\{x_n\} \to x$, then $x_n \preceq x$, for all $n \in \mathbb{N}$;*
(ii) *if a nonincreasing sequence $\{y_n\} \to y$, then $y \preceq y_n$, for all $n \in \mathbb{N}$.*

Let $F : X \times X \to X$ be a mapping having the mixed monotone property on X. Assume that there exists a $k \in [0,1)$ with

$$d(F(x,y), F(u,v)) \leq \frac{k}{2}[d(x,u) + d(y,v)], \forall x \succeq u, y \preceq v.$$

If there exist $x_0, y_0 \in X$ such that

$$x_0 \preceq F(x_0,y_0) \text{ and } y_0 \succeq F(y_0,x_0),$$

then there exist $x, y \in X$ such that

$$x = F(x,y) \text{ and } y = F(y,x).$$

In [11] Ćirić proved the following result and formulated as Theorem 2.1.

Theorem 1.3. *Let $(X, \preceq)$ be a partially ordered set and suppose that there is a metric d on X such that (X,d) is a complete metric space. Assume that there is a function $\varphi : [0,+\infty) \to [0,+\infty)$ with $\varphi(t) < t$ and $\lim_{r \to t^+} \varphi(r) < t$ for each $t > 0$ and also suppose that $F : X \times X \to X$ and $g : X \to X$ are both mappings such that F has the mixed g-monotone property and*

$$d(F(x,y), F(u,v)) \leq \varphi\left(\frac{d(g(x), g(u)) + d(g(y), g(v))}{2}\right) \tag{1.1}$$

for all $x, y, u, v \in X$ and $gx \preceq gu$ and $gy \succeq gv$. Suppose that $F(X \times X) \subseteq g(X)$, g is continuous and commutes with F and also suppose that

(a) *either F is continuous or*
(b) *X has the following properties:*
  (i) *if a nondecreasing sequence $\{x_n\} \to x$, then $x_n \preceq x$ for all n;*
  (ii) *if a nonincreasing sequence $\{y_n\} \to y$, then $y_n \preceq y$ for all n.*
  If there exist $x_0, y_0 \in X$ such that $g(x_0) \preceq F(x_0, y_0)$ and $g(y_0) \succeq F(y_0, x_0)$, then there exist $x, y \in X$ such that $g(x) = F(x, y)$ and $g(y) = F(y, x)$, that is, F and g have a coupled coincidence.

2. Main results

Firstly, let us consider the set of functions

$$\Phi = \left\{ \varphi \mid \varphi : [0, +\infty) \to [0, +\infty) \text{ with } \varphi(t) < t,\ t > 0 \text{ and } \lim_{r \to t^+} \varphi(r) < t,\ t > 0 \right\}.$$

If $\varphi \in \Phi$, then $\lim_{n\to\infty} \varphi^n(t) = 0$ for each $t > 0$.

Our first result is the following lemma which generalizes Theorem 2.1 of [11]. After that, inspired by Theorem 1.3, we formulate the result which is more general than it.

Lemma 2.1. (see also [20]) Let $(X, d, \preceq)$ be a partially ordered metric space and f and g be two self mappings on X. Assume that there exists $\varphi \in \Phi$ such that

$$d(fx, fy) \le \varphi(d(gx, gy)) \tag{2.1}$$

for all $x, y \in X$ with $gx \preceq gy$ or $gx \succeq gy$. If the following conditions hold:

(i) f is $g-$nondecreasing with respect to $\preceq$ and $f(X) \subseteq g(X)$;
(ii) there exists $x_0 \in X$ such that $gx_0 \preceq fx_0$;
(iii) f and g are continuous and compatible and (X, d) is complete, or
(iv) $(X, d, \preceq)$ is regular and one of $f(X)$ and $g(X)$ is complete; then f and g have a coincidence point in X.

Proof. Because of $f(X) \subseteq g(X)$ and (**ii**) we can define Jungck sequence as follows:

$$y_n = f(x_n) = g(x_{n+1}) \text{ for } n = 0, 1, 2, ...$$

It can be proved by induction that $y_n \preceq y_{n+1}$. If $y_n = y_{n+1}$ for some n, then x_n is a coincidence point. So, we will suppose that $y_n \ne y_{n+1}$ for all n.

Now, applying (2.1) to $x = x_n$ and $y = x_{n+1}$, we obtain

$$d(y_n, y_{n+1}) \le \varphi(d(y_{n-1}, y_n)) < d(y_{n-1}, y_n). \tag{2.2}$$

From (2.2) it follows that the sequence $d(y_n, y_{n+1})$ is monotone decreasing. Therefore, $d(y_n, y_{n+1}) \to \varepsilon \ge 0$ as $n \to \infty$.

We now prove that $\varepsilon = 0$. Assume, on the contrary, that $\varepsilon > 0$. If we take limit in (2.2) as $n \to \infty$, we obtain that

$$\varepsilon \le \lim_{n\to\infty} \varphi(d(y_{n-1}, y_n)) = \lim_{d(y_{n-1}, y_n) \to \varepsilon^+} \varphi(d(y_{n-1}, y_n)) < \varepsilon,$$

which is a contradiction. Hence, $\lim_{n\to\infty} d(y_n, y_{n+1}) = 0$.

We can prove that $\{y_n\}$ is a Cauchy sequence in the space (X, d) by using Lemma 1.2. Indeed, suppose that $\{y_n\}$ is not a Cauchy sequence. Then, Lemma 1.2. implies that there exist $\varepsilon > 0$ and two sequences $\{m(k)\}$ an $\{n(k)\}$ of positive integers such that the sequences

$$d\left(y_{m(k)}, y_{n(k)}\right), d\left(y_{m(k)}, y_{n(k)+1}\right), d\left(y_{m(k)-1}, y_{n(k)}\right), d\left(y_{m(k)-1}, y_{n(k)+1}\right)$$

tend to ε (from above) as $k \to \infty$. Now, using (2.1) with $x = x_{m(k)-1}$ and $y = x_{n(k)}$, we get

$$d\left(y_{m(k)}, y_{n(k)+1}\right) \leq \varphi\left(d\left(y_{m(k)-1}, y_{n(k)}\right)\right). \tag{2.3}$$

Taking limit in (2.3) as $k \to \infty$, we get

$$\varepsilon \leq \lim_{k\to\infty} \varphi\left(d\left(y_{m(k)-1}, y_{n(k)}\right)\right) = \lim_{d\left(y_{m(k)-1}, y_{n(k)}\right)\to\varepsilon^+} \varphi\left(d\left(y_{m(k)-1}, y_{n(k)}\right)\right) < \varepsilon,$$

which is a contradiction. Hence, $\{y_n\}$ is a Cauchy sequence.

Now, from (iii) ((X, d) is complete) it follows that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z \in X.$$

Further, since f and g are continuous and compatible we get that

$$\lim_{n\to\infty} fgx_n = fz, \lim_{n\to\infty} gfx_n = gz \text{ and } \lim_{n\to\infty} d\left(fgx_n, gfx_n\right) = 0.$$

We shall show that $fz = gz$. Indeed, we have

$$d(fz, gz) \leq d(fz, fgx_n) + d(fgx_n, gfx_n) + d(gfx_n, gz) \to 0 \ (n \to \infty),$$

which follows that $fz = gz$. It means that f and g have a coincidence point.

From (iv) it follows that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gu$$

for some $u \in X$. Because of regularity, we have $gx_n \preceq gu$. Applying (2.1) to $x = x_n, y = u$ we have

$$d(fx_n, fu) \leq \varphi(d(gx_n, gu)) < d(gx_n, gu), \tag{2.4}$$

(because $d(gx_n, gu) > 0$ for each $n \in \mathbb{N}$). If we take limit in (2.4) as $n \to \infty$, we obtain $d(gu, fu) \leq d(gu, gu) = 0$. Hence, f and g have a coincidence point $u \in X$. □

The following theorem generalizes and extends all three theorems 1.1, 1.2 and 1.3.

Theorem 2.1. *Let $(X, \preceq)$ be a partially ordered set and suppose that there is a metric d on X such that (X, d) is a metric space. Let $F : X \times X \to X$ and $g : X \to X$ be both mappings such that F has the g-monotone property. Assume that there is a function $\varphi \in \Phi$ such that*

$$\begin{aligned} &\max\{d(F(x, y), F(u, v)), d(F(y, x), F(v, u))\} \\ &\leq \varphi(\max\{d(gx, gu), d(gy, gv)\}) \end{aligned} \tag{2.5}$$

for all $x, y, u, v \in X$ with $gx \preceq gu$ and $gy \preceq gv$ or $gx \succeq gu$ and $gy \succeq gv$. Suppose that $F(X \times X) \subseteq g(X)$, and also suppose that either F and g are continuous and compatible and (X, d) is a complete, or $(X, d, \preceq)$ is regular and one of $F(X \times X)$ and $g(X)$ is complete. If there exist $x_0, y_0 \in X$ such that

$$x_0 \preceq F(x_0, y_0) \text{ and } y_0 \preceq F(y_0, x_0, y_0) \text{ or}$$

$$x_0 \succeq F(x_0, y_0, z_0) \text{ and } y_0 \succeq F(y_0, x_0, y_0),$$

then there exist $x, y \in X$ such that

$$F(x, y) = gx \text{ and } F(y, x) = gy.$$

Proof. According to Lemma 1.1(i)-(iv), the condition (2.5) implies the following contractive condition:

$$d_{\max}(T_F(Y), T_F(V)) \leq \varphi(d_{\max}(T_g(Y), T_g(V))),$$

for all $Y, V \in X \times X$ with $T_g(Y) \sqsubseteq T_g(V)$ or $T_g(Y) \sqsupseteq T_g(V)$. Further, the proof follows from Lemma 2.1 and Lemma 1.1(v). □

Remark 2.1. It is clear that condition (1.1) implies the following

$$d(F(x, y), F(u, v)) \leq \varphi(\max\{d(gx, gu), d(gy, gv)\}), \tag{2.6}$$

for all $x, y, u, v \in X$ with $gx \preceq gu$ and $gy \preceq gv$ or $gx \succeq gu$ and $gy \succeq gv$. Further, the condition (2.6) implies (2.5).

Remark 2.2. If we suppose that (X, d) is complete in Theorem 2.1, then it is sufficient that one of $F(X \times X)$ and $g(X)$ is closed.

Remark 2.3. Theorem 2.1 of [11] is true as well as in the context of monotone mappings.

Remark 2.4. Theorems 2.1 and 2.2 of [1] are true as well as in the context of monotone mappings.

Theorem 2.1 is more general than Theorem 1.3. The following example illustrates our claim.

Example 2.1. (a) Let $X = [0, 1]$ with the usual metric and order. Consider the mappings $F : X \times X \to X$ and $g : X \to X$ defined by

$$F(x, y) = \frac{x^2 + 2y^2}{4} \text{ and } g(x) = x^2.$$

Then all conditions of Theorem 2.1 are satisfied. In particular, we will check that F and g are compatible. Indeed, let $\{x_n\}$ and $\{y_n\}$ be two sequences in X such that

$$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} gx_n = a \text{ and } \lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} gy_n = b.$$

Then $a^2 = \frac{a^2+2b^2}{4}$ and $b^2 = \frac{b^2+2a^2}{4}$, from which it follows that $a = b = 0$.

Further, we have

$$d\left(gF\left(x_n,y_n\right),F\left(gx_n,gy_n\right)\right)=\left|\left(\frac{x_n^2+2y_n^2}{4}\right)^2-\frac{x_n^2+2y_n^2}{4}\right|\to 0\ \ (n\to\infty).$$

And similarly, $d\left(gF\left(y_n,x_n\right),F\left(gy_n,gx_n\right)\right)\to 0\ (n\to\infty)$.

Also, it follows immediately that the contractive condition (2.5) is satisfied with $\varphi(t)=\frac{3}{4}t$.

The functions F and g do not commute and therefore coupled coincidence point of F and g cannot be obtained by Theorem 1.3. This means that Theorem 2.1 is a proper generalization of Theorem 1.3. □

(b) Let $X=\mathbb{R}$ also with the usual metric and order. Take the mappings $F: X\times X\to X$ and $g: X\to X$ defined by

$$F(x,y)=\frac{3}{20}x+\frac{1}{20}y+1 \text{ and } g(x)=\frac{1}{2}x-1.$$

Then all conditions of Theorem 2.1 are satisfied. Note that F and g are not compatible, but $(X,d,\preceq)$ is complete and regular. It is not hard to verify that the contractive condition (2.5) is satisfied with $\varphi(t)=\frac{4}{5}t$. This example also shows that Theorem 2.1 is a genuine generalization of Theorem 1.3.

Remark 2.5. We note that the function $F(x,y)=\frac{x^2+2y^2}{4}$ has not g-mixed monotone property. Hence coupled coincidence point $(0,0)$ of F and g cannot be obtained by Theorem 1.3. However, it has the g-monotone property where $g(x)=x^2$. The same conclusion is as well as for the functions $F(x,y)=\frac{3}{20}x+\frac{1}{20}y+1$ and $g(x)=\frac{1}{2}x-1$.

For similar approaches also see [10], [14]-[20]. Also, for some new applications in fixed point theory, see [4-9].

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