

ON A NEW MODEL OF NONLOCAL MODIFIED GRAVITY

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ABSTRACT. Nonlocal modified gravity without matter, where nonlocality is of the form $R^{-p}\mathcal{F}(\square)R$ is considered from the cosmological point of view. Equations of motion are derived. Cosmological solutions of the form $a(t) = a_0|t - t_0|^\alpha$, for the FLRW metric and curvature constant $k = 0, \pm 1$, are found.

1. INTRODUCTION

General Relativity is the Einstein theory of gravity usually given in the form of the equation of motion for gravitational (metric) field $g_{\mu\nu}$, i.e. $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$, where $R_{\mu\nu}$ is the Ricci curvature tensor, R is scalar curvature, $T_{\mu\nu}$ is the energy-momentum tensor, and speed of light is taken $c = 1$. This Einstein equation can be derived from the Einstein-Hilbert action $S = \frac{1}{16\pi G} \int \sqrt{-g}Rd^4x + \int \sqrt{-g}L_m d^4x$, where $g = \det(g_{\mu\nu})$ and L_m is Lagrangian of matter.

Despite its theoretical beauty and many phenomenological evidences, general relativity is not a complete theory and should be modified. There are many its modifications, which are motivated by quantum gravity, string theory, astrophysics and cosmology (for a review, see [1]). One of very promising directions of research is *nonlocal modified gravity* and its applications to cosmology (as a review, see [2], [3] and [4]). It usually contains an infinite number of spacetime derivatives in the form of some power expansions of the d'Alembert operator $\square = \frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$ or of its inverse \square^{-1} , or a combination of both. We are mainly interested in nonlocality expressed in the form of an analytic function $\mathcal{F}(\square) = \sum_{n=0}^{\infty} f_n \square^n$. For nonlocal gravity with \square^{-1} see also [17, 3, 18, 19]. Nonlocality also improves renormalizability of gravity, see [20, 21]

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and references therein. Note that there have been some investigations with R^{-1} modification of gravity, but they are not nonlocal and they have problems to be confirmed for the Solar System [16].

To solve cosmological Big Bang singularity, nonlocal gravity with replacement $R \rightarrow R + R\mathcal{F}(\square)R$ in the Einstein-Hilbert action was proposed in [5]. This nonlocal model is further elaborated in the series of papers [6, 7, 8, 9, 10, 11, 12].

We are interesting here in nonlocal gravity action without matter in the form

$$(1.1) \quad S = \int \left(\frac{R - 2\Lambda}{16\pi G} + R^{-p}\mathcal{F}(\square)R \right) \sqrt{-g} d^4x,$$

where R is scalar curvature, $p \in \mathbb{N}$, $\mathcal{F}(\square) = \sum_{n=0}^{\infty} f_n \square^n$ is an analytic function of the d'Alembert-Beltrami operator $\square = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu$, $g = \det(g_{\mu\nu})$.

The action (1.1) for $p = 1$ was introduced in [13] as a new approach to nonlocal gravity. Our intention is to present some power-law cosmological solutions as a part of a systematic investigation of nonlocal gravity (1.1). For $p = 1$, in [13, 14] similar power-law cosmological solutions were obtained.

2. EQUATIONS OF MOTION

By variation of action (1.1) with respect to metric $g^{\mu\nu}$ one obtains the equations of motion for $g_{\mu\nu}$

$$(2.1) \quad \begin{aligned} & \frac{1}{16\pi G} (G_{\mu\nu} + \Lambda g_{\mu\nu}) - \frac{1}{2} g_{\mu\nu} R^{-p} \mathcal{F}(\square) R + (R_{\mu\nu} \Phi - K_{\mu\nu} \Phi) \\ & + \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} (g_{\mu\nu} \partial^\alpha \square^l R^{-p} \partial_\alpha \square^{n-1-l} R + g_{\mu\nu} \square^l R^{-p} \square^{n-l} R \\ & - 2\partial_\mu \square^l R^{-p} \partial_\nu \square^{n-1-l} R) = 0, \end{aligned}$$

where $K_{\mu\nu} = \nabla_\mu \nabla_\nu - g_{\mu\nu} \square$, $\Phi = -pR^{-p-1}\mathcal{F}(\square)R + \mathcal{F}(\square)R^{-p}$.

In the case of the FLRW metric, equation (2.1) is equivalent to its trace and 00 component, respectively:

$$(2.2) \quad \begin{aligned} & \frac{1}{16\pi G} (4\Lambda - R) - 2R^{-p} \mathcal{F}(\square) R + (R\Phi + 3\square\Phi) \\ & + \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} (\partial^\mu \square^l R^{-p} \partial_\mu \square^{n-1-l} R + 2\square^l R^{-p} \square^{n-l} R) = 0, \end{aligned}$$

$$\begin{aligned}
 (2.3) \quad & \frac{1}{16\pi G}(G_{00} + \Lambda g_{00}) - \frac{1}{2}g_{00}R^{-p}\mathcal{F}(\square)R + (R_{00}\Phi - K_{00}\Phi) \\
 & + \frac{1}{2}\sum_{n=1}^{\infty}f_n\sum_{l=0}^{n-1}(g_{00}\partial^\alpha\square^l R^{-p}\partial_\alpha\square^{n-1-l}R + g_{00}\square^l R^{-p}\square^{n-l}R \\
 & - 2\partial_0\square^l R^{-p}\partial_0\square^{n-1-l}R) = 0.
 \end{aligned}$$

Equations (2.2) and (2.3) are more suitable for further investigation than (2.1).

In the FLRW metric $ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right)$ one has $R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right)$ and $\square h(t) = -\ddot{h}(t) - 3H\dot{h}(t)$, where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. In the sequel we solve equations of motions (2.2) and (2.3) for cosmological scale factor $a(t)$ and the corresponding R :

$$(2.4) \quad a(t) = a_0|t - t_0|^\alpha,$$

$$(2.5) \quad R(t) = 6\left(\alpha(2\alpha - 1)(t - t_0)^{-2} + \frac{k}{a_0^2}(t - t_0)^{-2\alpha}\right).$$

3. CASE $k = 0$, $\alpha \neq 0$ AND $\alpha \neq \frac{1}{2}$

In this case, there is the following dependence on parameter α :

$$\begin{aligned}
 a &= a_0|t - t_0|^\alpha, & H &= \frac{\alpha}{t - t_0}, \\
 R &= Q(t - t_0)^{-2}, & Q &= 6\alpha(2\alpha - 1), \\
 R_{00} &= 3\alpha(1 - \alpha)(t - t_0)^{-2}, & G_{00} &= 3\alpha^2(t - t_0)^{-2}.
 \end{aligned}$$

Now expressions $\square^n R^{-p}$ and $\square^n R$ become

$$\begin{aligned}
 (3.1) \quad & \square^n R^{-p} = B(n, p)(t - t_0)^{2p-2n}, \quad \square^n R = B(n, -1)(t - t_0)^{-2-2n}, \\
 & B(n, p) = Q^{-p}2^n \prod_{l=1}^n (p - l + 1)(-2p - 3\alpha + 2l - 1), \quad n \geq 1, \\
 & B(n, -1) = Q2^n \prod_{l=1}^n (-l)(1 - 3\alpha + 2l), \quad n \geq 1, \\
 & B(0, p) = Q^{-p}, \quad B(0, -1) = Q.
 \end{aligned}$$

Also, we have

$$\mathcal{F}(\square)(R^{-p}) = \sum_{n=0}^{\infty} f_n B(n, p)(t - t_0)^{2p-2n}.$$

Coefficients $B(n, s)$ we can transform in the following way:

$$(3.2) \quad \begin{aligned} B(n, s) &= Q^{-s}(-4)^n \frac{\Gamma(s+1)\Gamma(s+\eta+1)}{\Gamma(s+1-n)\Gamma(s+\eta+1-n)}, \quad s \neq -1, \\ B(p, -1) &= Q4^p p! \frac{\Gamma(\eta)}{\Gamma(\eta-p)}, \end{aligned}$$

where $\eta = \frac{3\alpha-1}{2}$. Substituting these equations into (2.2) and (2.3) one has

$$(3.3) \quad \begin{aligned} &\sum_{n=0}^{\infty} f_n B(n, p) (Q - 6(p-n)(2p-2n-1) - 18\alpha(p-n)) \tau \\ &+ Q^{-p-1} \sum_{n=0}^{\infty} f_n B(n, -1) ((-2-p)Q + 6p(p-n)(2p-2n-1+3\alpha)) \tau \\ &+ 2 \sum_{n=1}^{\infty} f_n A_n \tau = \frac{1}{16\pi G} (Q(t-t_0)^{-2} - 4\Lambda) \end{aligned}$$

and

$$(3.4) \quad \begin{aligned} &3\alpha \sum_{n=0}^{\infty} f_n B(n, p) (1 - \alpha + 2(p-n)) \tau \\ &+ Q^{-p} \sum_{n=0}^{\infty} f_n B(n, -1) \left(\frac{1}{2} - 3\alpha(1-\alpha)pQ^{-1} - 6\alpha pQ^{-1}(p-n) \right) \tau \\ &- \frac{1}{2} \sum_{n=1}^{\infty} f_n B_n \tau = \frac{1}{16\pi G} (\Lambda - 3\alpha^2(t-t_0)^{-2}), \end{aligned}$$

where

$$(3.5) \quad A_n = \sum_{l=0}^{n-1} B(l, p) (B(n-l, -1) - 2(p-l)(l-n)B(n-l-1, -1)),$$

$$(3.6) \quad \begin{aligned} B_n &= \sum_{l=0}^{n-1} B(l, p) (B(n-l, -1) + 4(p-l)(l-n)B(n-l-1, -1)), \\ \tau &= (t-t_0)^{2p-2n-2}. \end{aligned}$$

From equations (3.3) and (3.4) it follows that if $i \neq p, p-1$ we have $f_i = 0$. Coefficients f_p and f_{p-1} have to satisfy the following systems:

$$(3.7) \quad \begin{aligned} f_p(K_1 + S_1) &= \frac{Q}{16\pi G}, \\ f_p(K_2 + \frac{1}{2}S_2) &= \frac{-3\alpha^2}{16\pi G}, \end{aligned}$$

$$(3.8) \quad \begin{aligned} f_{p-1}(K_3 + S_3) &= \frac{-\Lambda}{4\pi G}, \\ f_{p-1}(K_4 + \frac{1}{2}S_4) &= \frac{\Lambda}{16\pi G}, \end{aligned}$$

where

$$\begin{aligned} K_1 &= (-p-2)Q^{-p}B(p, -1) + QB(p, p), \\ S_1 &= \sum_{l=0}^{p-1} 2B(l, p) (2(l-p)^2B(-l+p-1, -1) + B(p-l, -1)), \\ K_2 &= \left(3(\alpha-1)\alpha p Q^{-1} + \frac{1}{2}\right) Q^{-p}B(p, -1) + 3(1-\alpha)\alpha B(p, p), \\ S_2 &= \sum_{l=0}^{p-1} B(l, p) (4(p-l)^2B(-l+p-1, -1) - B(p-l, -1)), \\ K_3 &= (6(3\alpha+1)p Q^{-1} - p-2) Q^{-p}B(p-1, -1) + QB(p-1, p) + 3B(p, p), \\ S_3 &= \sum_{l=0}^{p-2} 2B(l, p) (2(l-p)(l-p+1)B(-l+p-2, -1) + B(-l+p-1, -1)), p \neq 1, \\ K_4 &= \left(3(\alpha-1)\alpha p - 6\alpha p + \frac{Q}{2}\right) Q^{-p-1}B(p-1, -1) + 3(3-\alpha)\alpha B(p-1, p), \\ S_4 &= \sum_{l=0}^{p-2} B(l, p) (4(l-p)(l-p+1)B(-l+p-2, -1) - B(-l+p-1, -1)), p \neq 1. \end{aligned}$$

For $p = 1$ we have $S_3 = S_4 = 0$.

The systems (3.7) and (3.8) have a solution if and only if

$$\begin{aligned} -3\alpha^2 K_1 - QK_2 &= 3\alpha^2 S_1 + \frac{1}{2}QS_2, \\ K_3 + 4K_4 &= -(S_3 + 2S_4). \end{aligned}$$

Using (3.2) we can show that last conditions are satisfied for all $\alpha \in \mathbb{R} \setminus \{0, \frac{1}{2}\}$ and $p \in \mathbb{N}$.

4. CASE $k = 0$, $\alpha \rightarrow 0$ (MINKOWSKI SPACE)

Substituting (3.1) and (3.5) into trace equation (3.3) we obtain

$$\begin{aligned}
& (6\alpha(2\alpha - 1))^{-p} \left(\sum_{n=0}^{\infty} f_n \tilde{B}(n, p) (6\alpha(2\alpha - 1) - 6(p - n)(2p - 2n - 1) - 18\alpha(p - n)) \tau \right. \\
& + \sum_{n=0}^{\infty} f_n \tilde{B}(n, -1) ((-2 - p)6\alpha(2\alpha - 1) + 6p(p - n)(2p - 2n - 1) + 18\alpha p(p - n)) \tau \\
& \left. + 12\alpha(2\alpha - 1) \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \tilde{B}(l, p) (\tilde{B}(n - l, -1) - 2(p - l)(l - n)\tilde{B}(n - l - 1, -1)) \tau \right) \\
(4.1) \quad & = \frac{1}{16\pi G} (6\alpha(2\alpha - 1)(t - t_0)^{-2} - 4\Lambda),
\end{aligned}$$

where

$$\begin{aligned}
\tilde{B}(n, p) &= 2^n \prod_{l=1}^n (p - l + 1)(-2p - 3\alpha + 2l - 1), \quad n \geq 1, \quad \tilde{B}(0, p) = 1, \\
\tilde{B}(n, -1) &= 2^n \prod_{l=1}^n (-l)(1 - 3\alpha + 2l), \quad n \geq 1, \quad \tilde{B}(0, -1) = 1, \\
\tau &= (t - t_0)^{2p - 2n - 2}.
\end{aligned}$$

Now, if $\alpha \rightarrow 0$ from (4.1) we get

$$\sum_{n=1}^{\infty} f_n D_n 2^{n+1} (p - n)(2p - 2n - 1)(t - t_0)^{2p - 2n - 2} = 0,$$

where

$$D_n = p \prod_{l=1}^n (-l)(1 + 2l) - \prod_{l=1}^n (p - l + 1)(-2p + 2l - 1).$$

From this we conclude

$$f_{p-1}, f_p \in \mathbb{R}, \quad f_i = 0, \quad i \neq p - 1, p.$$

Substituting $f_i = 0$, $i \neq p - 1, p$ into equation (3.4) we obtain the following equations:

$$\begin{aligned}
& \left((-1-p)\tilde{B}(p, -1) + \tilde{B}(p, p) \right. \\
& \quad \left. + \sum_{l=0}^{p-1} \tilde{B}(l, p)(\tilde{B}(p-l, -1) - 4(p-l)^2\tilde{B}(p-l-1, -1)) \right) f_p(t-t_0)^{-2} = 0, \\
& \left((-1-3p)\tilde{B}(p-1, -1) + 3\tilde{B}(p-1, p) \right. \\
& \quad \left. + \sum_{l=0}^{p-2} \tilde{B}(l, p)(\tilde{B}(p-l-1, -1) + 4(p-l)(l-p+1)\tilde{B}(p-l-2, -1)) \right) f_{p-1} = 0.
\end{aligned}$$

When $\alpha \rightarrow 0$ the corresponding solution is

$$f_p \in \mathbb{R}, \quad f_i = 0, \quad i \neq p.$$

Since all $f_n = 0$, $n \neq p$ this is not nonlocal gravity model (1.1). It follows that the above power-law cosmological solutions have not Minkowski space as their background, or in other words, they cannot be obtained as perturbations on Minkowski space.

Remark. For $p \neq 1$, case $k = 0$, $\alpha \rightarrow \frac{1}{2}$ does not yield solution which satisfies the equations of motion. For $p = 1$ we have solution (see [14]).

5. CASE $k \neq 0$

In order to simplify expression (2.5) there are three possibilities: $\alpha = 0$, $\alpha = \frac{1}{2}$ and $\alpha = 1$. For $\alpha = \frac{1}{2}$ we have no solutions which satisfy the equations of motion.

5.1. Case $\alpha = 0$. In the case $\alpha = 0$ we obtain

$$\begin{aligned}
(5.1) \quad & a = a_0, \quad R = \frac{6k}{a_0^2}, \\
& H = 0, \quad \square R = 0.
\end{aligned}$$

We have $\Phi = (1-p)R^{-p}f_0$.

Substituting (5.1) into equations of motion we get the following system of equations:

$$\begin{aligned}
(-p-1)R^{1-p}f_0 &= \frac{R}{16\pi G} - \frac{\Lambda}{4\pi G}, \\
R^{1-p}f_0 &= \frac{\Lambda}{8\pi G} - \frac{R}{16\pi G}, \\
f_i &\in \mathbb{R}, \quad i \neq 0.
\end{aligned}$$

The previous system has solution if and only if

$$32\pi G\Lambda(1-p) = -pR.$$

Obviously, for $p = 1$ the last condition is not satisfied.

5.2. **Case $\alpha = 1$.** In the case $\alpha = 1$ we obtain

$$\begin{aligned} a &= a_0|t - t_0|, & R &= 6\left(1 + \frac{k}{a_0^2}\right)(t - t_0)^{-2}, \\ H &= \frac{1}{t - t_0}, & \square R &= 0, \\ R_{00} &= 0, & G_{00} &= 3\left(1 + \frac{k}{a_0^2}\right)(t - t_0)^{-2}, \end{aligned}$$

and

$$\begin{aligned} \square^n R^{-p} &= E(n, p)(t - t_0)^{2p-2n}, \\ E(n, p) &= \left(6 + \frac{6k}{a_0^2}\right)^{-p} 2^n \prod_{l=1}^n (p - l + 1)(-2p + 2l - 4), \quad n \geq 1, \\ E(0, p) &= \left(6 + \frac{6k}{a_0^2}\right)^{-p}, \\ \square^n R &= E(n, -1)(t - t_0)^{-2-2n} = 0, \quad n \geq 1. \end{aligned}$$

Also, we have

$$\mathcal{F}(\square)(R^{-p}) = \sum_{n=0}^{\infty} f_n E(n, p)(t - t_0)^{2p-2n}.$$

Using the above expressions, from equations of motion it follows that coefficients f_p and f_{p-1} have to satisfy the following systems of equations:

$$\begin{aligned} (4E(p-1, p) + E(p, p))f_p &= \frac{1}{16\pi G}, \\ (5.2) \quad 4E(p-1, p)f_p &= -\frac{1}{16\pi G} \end{aligned}$$

and

$$\begin{aligned} (48\left(1 + \frac{k}{a_0^2}\right)E(p-2, p) + 6\left(1 + \frac{k}{a_0^2}\right)E(p-1, p) + 3E(p, p))f_{p-1} &= -\frac{\Lambda}{4\pi G}, \\ (5.3) \quad 6\left(4\left(1 + \frac{k}{a_0^2}\right)E(p-2, p) + E(p-1, p)\right)f_{p-1} &= \frac{\Lambda}{16\pi G}, \quad p \neq 1. \end{aligned}$$

We obtain

$$\begin{aligned} f_n &= 0, & 0 \leq n \leq p-2 & \quad (p \neq 1), \\ f_n &\in \mathbb{R}, & n \geq p+1. \end{aligned}$$

The systems (5.2) and (5.3) have solution if and only if

$$\begin{aligned} 8E(p-1, p) &= -E(p, p), \\ 48\left(1 + \frac{k}{a_0^2}\right)E(p-2, p) + \left(10 + \frac{2k}{a_0^2}\right)E(p-1, p) + E(p, p) &= 0. \end{aligned}$$

After short calculation we obtain that the last two conditions are satisfied for all $p \neq 1$.

For $p = 1$ system (5.3) (which corresponds to coefficient f_0) becomes:

$$f_0 = \frac{\Lambda}{8\pi G}.$$

Therefore, equations of motion are satisfied for all $p \in \mathbb{N}$.

6. CONCLUDING REMARKS

In this paper we have presented some power-law cosmological solutions of the form $a(t) = a_0|t - t_0|^\alpha$, which are derived from modified gravity with nonlocal term $R^{-p}\mathcal{F}(\square)R$. These solutions do not have Minkowski space background. However, for $p = 1$ there is the de Sitter bounce solution $a(t) = a_0 \exp(\lambda t)$, which in the limit $\lambda \rightarrow 0$ leads to the Minkowski space (see [14, 15]).

It is worth noting that there is solution $a(t) = |t - t_0|$ which corresponds to the Milne universe for $k = -1$.

Note also that all the above presented power-law solutions $a(t) = a_0|t - t_0|^\alpha$ have scalar curvature $R(t) = 6 \left(\alpha(2\alpha - 1)(t - t_0)^{-2} + \frac{k}{a_0^2}(t - t_0)^{-2\alpha} \right)$ (2.5), which satisfies relation $\square R = qR^2$, where parameter q depends on α . This quadratic relation $\square R = qR^2$ was used in [13] as an Ansatz to solve equations of motion for $p = 1$.

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