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# CHARACTERIZATION OF ORDERED SEMIHYPERGROUPS BY COVERED HYPERIDEALS

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ABSTRACT. After introducing the notions of the Green's relation $\mathcal{J}$, hyper $\mathcal{J}$-class and covered hyperideal in an ordered semihypergroup, some important properties of the hyper $\mathcal{J}$-class and covered hyperideals are studied. Then maximal and minimal hyperideals of an ordered semihypergroup are defined and some vital results have been proved. We also define a hyperbase of an ordered semihypergroup and prove the existence of a hyperbase under certain conditions in an ordered semihypergroup. In an ordered semihypergroup, after defining the greatest covered hyperideal and the greatest hyperideal, some results about these hyperideals are proved. Finally, in a regular ordered semihypergroup, we show that, under some conditions, each hyperideal is also a covered hyperideal.

## 1. INTRODUCTION AND PRELINIMARIES

In 1934, Marty [16] introduced the concept of a hyperstructure, in particular, the hypergroup theory in the 8th Congress of Scandinavian Mathematicians. The beauty of hyperstructure is that in hyperstructures, composition of two elements is a set. Thus the notion of algebraic hyperstructures is a generalization of classical notion of algebraic structures. The concept of ordered semihypergroup is a generalization of the concept of ordered semigroup and was introduced by Heidari and Davvaz in [11]. Thereafter it was studied by several authors. Davvaz et al. [1, 2, 11, 17] studied some properties of hyperideals, bi-hyperideals and quasi-hyperideals in ordered semihypergroups. In [7, 9], Fabrici introduced the notion of a covered ideal and, in

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[20], Xie generalized the notion of a covered ideal for ordered semigroups. Thereafter, Thawhat Changphas and Pisan Summaprab [4] discussed the structure of an ordered semigroup containing covered ideals. Later on Saber Omidi and Bijan Davvaz [18] discussed the notion of a covered $\gamma$- hyperideal in an ordered $\gamma$-semihypergroup.

A hyperoperation on a set $S(\neq \emptyset)$ is a map $\circ : S \times S \to \mathcal{P}^\star(S)$, where $\mathcal{P}^\star(S)$ denotes the power set of $S$ except $\{\emptyset\}$. Then $(S, \circ)$ is a hypergroupoid. The image of the pair $(a, b)$ in $S \times S$ is denoted by $a \circ b$.

A hypergroupoid $(S, \circ)$ is called a semihypergroup if for all $x_1, x_2, x_3 \in S$

$$(x_1 \circ x_2) \circ x_3 = x_1 \circ (x_2 \circ x_3).$$

It means that $\bigcup\limits_{t \in x_1 \circ x_2} t \circ x_3 = \bigcup\limits_{r \in x_2 \circ x_3} x_1 \circ r$.

For any $T_1, T_2 \in \mathcal{P}^\star(S)$, we denote

$$T_1 \circ T_2 = \bigcup_{t \in T_1, t' \in T_2} t \circ t'.$$

Instead of $\{x_1\} \circ T_1$ and $T_2 \circ \{x_1\}$ we shall write, in whatever follows, $x_1 \circ T_1$ and $T_2 \circ x_1$, respectively. We shall write $A^n$ for $A \circ A \circ A \circ \cdots \circ A$ ($n$-copies of $A$) in the sequel without further mention.

**Definition 1.1.** Let $\leq$ be an ordered relation on a set $S(\neq \emptyset)$. The triplet $(S, \circ, \leq)$ is called an ordered semihypergroup if $(S, \circ)$ is a semihypergroup and $(S, \leq)$ is a partially ordered set such that: for all $t_1, t_2, t \in S$, $t_1 \leq t_2$ implies $t_1 \circ t \leq t_2 \circ t$ and $t \circ t_1 \leq t \circ t_2$. Here $t_1 \circ t \leq t_2 \circ t$ means that for any $w \in t_1 \circ t$ there exists $w' \in t_2 \circ t$ such that $w \leq w'$.

A subset $H(\neq \emptyset)$ of an ordered semihypergroup $S$ is called a subsemihypergroup of $S$ if $H \circ H \subseteq H$. We note that for every $x, y, z, u, v, w \in S$ such that $x \circ y \leq z \circ w$ and $u \leq v$, we obtain $x \circ y \circ u \leq z \circ w \circ v$.

For $L \subseteq S$, let $(L] = \{t \in S \mid t \leq h \text{ for some } h \in L\}$. Throughout this paper $S$ denotes an ordered semihypergroup until or unless it is mentioned.

**Definition 1.2.** A subset $W(\neq \emptyset)$ of $S$ is called a right (resp. left) hyperideal of $S$ if

(a) $W \circ S \subseteq W$ (resp. $S \circ W \subseteq W$);
(b) $(W] \subseteq W$.

$W$ becomes a hyperideal if it is both a right hyperideal and a left hyperideal of $S$. The set of all hyperideals of $S$ shall be denoted, in whatever follows, by $I^\star$.

**Definition 1.3.** A proper hyperideal $W$ of $S$ is called minimal if $W$ does not contain any hyperideal of $S$. Equivalently, if for any $U \in I^\star$ such that $U \subseteq W$, we have $U = W$. The proper hyperideal $W$ of $S$ is called maximal if for any $V \in I^\star$ such that $W \subset V$, we have $V = S$. Equivalently, if for any $V \in I^\star$ such that $W \subseteq V$, we have $V = W$. Finally, $S$ is called simple if $S$ has no proper hyperideals. The ordered semihypergroup $S$ is called regular if for any $a_1 \in S$ there exists $t \in S$ such that $a_1 \in (a_1 \circ t \circ a_1]$. Equivalently, $W \subseteq (W \circ S \circ W]$ for every $W \subseteq S$.

For an ordered semihypergroup $S$, the hyperideal $J(a)$ generated by the element $a$ of $S$ is equal to $(a \cup S \circ a \cup a \circ S \cup S \circ a \circ S]$.

In Section 2 of the paper, after defining the notions of Green's relation $\mathcal{J}$ and covered hyperideal, some important properties of Green's relation $\mathcal{J}$ and covered hyperideals of an ordered semihypergroup are obtained as the main results while Section 3 deals with the structural properties of ordered semihypergroups containing covered hyperideals.

## 2. Basic Properties of Covered Hyperideals

The Green's relation $\mathcal{J}$ on $S$, an ordered semihypergroup, is defined by, for $t, t' \in S$,

$$t \; \mathcal{J} \; t' \text{ if and only if } J(t) = J(t').$$

For any $t \in S$, $T_t$ is $\mathcal{J}$-hyperclass of $t$. Let $D$ be the collection of all $\mathcal{J}$-hyperclasses of $S$. Define an order '$\preccurlyeq$' on $D$ by: for any $t, t' \in S$,

$$T_t \preccurlyeq T'_t \text{ if and only if } J(t) \subseteq J(t').$$

Then it is easy to verify that $(D, \preccurlyeq)$ is a quasi-ordered set.

The following result easily follows.

**Lemma 2.1.** *Let $t$ be any element of $S$ such that $T(t) \nsubseteq K$ for any principal hyperideal $K$ of $S$. Then the $\mathcal{J}$-hyperclass $T_t$ is maximal.*

**Lemma 2.2.** *Let $K$ be any subset of $S$. Then $K$ is a maximal $\mathcal{J}$-hyperclass of $S$ if and only if $S \setminus K$ is a maximal hyperideal of $S$.*

*Proof.* First we consider that $K$ is a maximal $\mathcal{J}$-hyperclass of $S$. Then $K = T_t$ for some $t \in S$. We now show that $S \setminus T_t$ is a hyperideal of $S$. For this let $h \in S$ and $t' \in S \setminus T_t$, then $t' \notin T_t \Rightarrow J(t) \neq J(t')$. Let $y \in h \circ t'$. Then either $J(y) = J(t)$ or $J(y) \neq J(t)$. If $J(y) \neq J(t)$ then the proof is obvious. If $J(y) = J(t)$, then we have $y \in h \circ t' \subseteq S \circ J(t') \subseteq J(t')$ and $J(y) = J(t) \neq J(t')$. Since $T_{t'}$ and $T_t$ are disjoint $\mathcal{J}$-hyperclasses of $S$, we have $y \notin T_t \Rightarrow y \in S \setminus T_t$. Thus $S \circ S \setminus T_t \subseteq S \setminus T_t$. Similarly, we may show that $(S \setminus T_t) \circ S \subseteq S \setminus T_t$. Let $u \in S \setminus T_t$ and $v \in S$ be such that $v \leq u$. So we have $v \in (v] \subseteq (u] \subseteq J(u)$ and thus, $J(v) \subseteq J(u) \Rightarrow T_v \preccurlyeq T_u$. If $v \in T_t$, since $T_t$ is maximal, so $T_v$ is also maximal $\mathcal{J}$-hyperclass of $S$. Thus we have $T_t = T_u$. So $u \in T_t$, a contradiction. Hence, $v \in S \setminus T_t$ and $S \setminus T_t$ is a hyperideal of $S$. Now it remains to show the maximality of $S \setminus T_t$. For this take any hyperideal $L$ of $S$ such that $S \setminus T_x \subset L$. Then there exists $w \in L \setminus (S \setminus T_t)$. Thus $w \in T_t$. Now, for any $y \in T_t$, we have

$$J(y) = J(x) = J(w) \subseteq L,$$

and, so, $T_t \subseteq L$. Hence, $S = L$. This shows that $S \setminus T_t$ is a maximal hyperideal of $S$.

Conversely suppose that $S \setminus K$ is a maximal hyperideal of $S$. Take $z \in S \setminus (S \setminus K)$. So $z \in K$. If $t \in T_z$, then $J(t) = J(z) \subseteq K$. Thus $t \in K$. Hence, $T_z \subseteq K$. Since $S \setminus K \subset (S \setminus K) \cup J(z)$ and $S \setminus K$ is a maximal hyperideal of $S$, we have $(S \setminus K) \cup J(z) = S$. It now follows that for any $t' \in K$, $J(t') = J(z)$. Thus, for $t' \in K$, $t' \in T_z \Rightarrow K \subseteq T_z$. Hence, $K = T_z$. If $T_z$ is not maximal $\mathcal{J}$-hyperclass of $S$, then there exists $e \in S$ such that $T_z \precneqq T_e$. This implies that $J(z) \subset J(e)$ and, so, by hypothesis, $J(e) \subseteq S \setminus K$. As $e \notin T_z = K \;\Rightarrow\; e \in S \setminus K$. Thus, $z \in S \setminus K$. This is a contradiction as $z \in T_z$. Hence, $T_z$ is a maximal $\mathcal{J}$-hyperclass of $S$. $\square$

**Definition 2.1.** Any proper hyperideal $K$ of an ordered semihypergroup $S$ is called a covered hyperideal of $S$ if $K \subseteq (S \circ (S \setminus K) \circ S]$. The set of all covered hyperideals of $S$ shall be denoted, in whatever follows, by $\mathcal{C}_{\mathcal{H}}$.

*Example* 2.1. Let $S = \{u, v, w, x\}$. Define the hyper operation $(\circ)$ on $S$ by the following table:

| $\circ$ | $u$ | $v$ | $w$ | $x$ |
|---|---|---|---|---|
| $u$ | $\{u\}$ | $\{u,v\}$ | $\{u,w\}$ | $\{u\}$ |
| $v$ | $\{u\}$ | $\{u,v\}$ | $\{u,w\}$ | $\{u\}$ |
| $w$ | $\{u\}$ | $\{u,v\}$ | $\{u,w\}$ | $\{u\}$ |
| $x$ | $\{u\}$ | $\{u,v\}$ | $\{u,w\}$ | $\{u\}$ |

Define order on $S$ as $\leq = \{(u,u),(v,v),(w,w),(x,x),(v,u),(w,u)\}$. Then $(S, \circ, \leq)$ is an ordered semihypergroup. Now, it may easily be verified that $B = \{u, v, w\}$ is a covered hyperideal of $S$.

*Example* 2.2. Let $S = \{u, v, w, x, y\}$. Define the hyper operation $(\circ)$ on $S$ by the following table:

| $\circ$ | $u$ | $v$ | $w$ | $x$ | $y$ |
|---|---|---|---|---|---|
| $u$ | $\{u,v\}$ | $\{u,v\}$ | $\{u,v\}$ | $\{u,v\}$ | $\{u,v\}$ |
| $v$ | $\{u,v\}$ | $\{u,v\}$ | $\{u,v\}$ | $\{u,v\}$ | $\{u,v\}$ |
| $w$ | $\{u,v\}$ | $\{u,v\}$ | $\{w\}$ | $\{w\}$ | $\{y\}$ |
| $x$ | $\{u,v\}$ | $\{u,v\}$ | $\{w\}$ | $\{x\}$ | $\{y\}$ |
| $y$ | $\{u,v\}$ | $\{u,v\}$ | $\{w\}$ | $\{w\}$ | $\{y\}$ |

Define order on $S$ as $\leq = \{(u,u),(v,v),(w,w),(x,x),(y,y),(u,w),(u,x),(u,y),(v,w),$ $(v,x),(v,y),(w,x),(w,y)\}$. Then $(S, \circ, \leq)$ becomes an ordered semihypergroup. One may easily verify that the sets $A_1 = \{u, v\}$ and $A_2 = \{u, v, w, y\}$ are covered hyperideals of $S$.

**Proposition 2.1.** *Let $A_1, A_2$ be different proper hyperideals of $S$ such that $A_1 \cup A_2 = S$. Then none of them is covered hyperideal of $S$.*

*Proof.* On contrary, assume that $A_1$ is a covered hyperideal of $S$. Since $A_1 \cup A_2 = S$, we have $S \setminus A_1 \subseteq A_2$ and $S \setminus A_2 \subseteq A_1$. Thus we have

$$A_1 \subseteq (S \circ (S \setminus A_1) \circ S] \subseteq (S \circ A_2 \circ S] \subseteq A_2.$$

Therefore, $S = A_2$. This is a contradiction. By the similar argument, we may show that if $A_2$ is a covered hyperideal of $S$, then $S = A_1$. This is again a contradiction. Hence, the result hold. $\square$

The following corollary follows easily from Proposition 2.1.

**Corollary 2.1.** *If an ordered semihypergroup $S$ contains two or more maximal hyperideal, then none of them is a covered hyperideal of $S$.*

**Proposition 2.2.** *Let $A_1, A_2$ be covered hyperideals of $S$. Then $A_1 \cup A_2 \in \mathcal{C}_{\mathcal{H}}$.*

*Proof.* Let $A_1$ and $A_2 \in \mathcal{C}_{\mathcal{H}}$. Then $A_1 \subseteq (S \circ (S \setminus A_1) \circ S]$ and $A_2 \subseteq (S \circ (S \setminus A_2) \circ S]$. Clearly $A_1 \cup A_2$ is a hyperideal of $S$. To show that $A_1 \cup A_2 \in \mathcal{C}_{\mathcal{H}}$, take any $z \in (A_1 \cup A_2)$. If $z \in A_1$, then $z \in (s_1 \circ t \circ s_2]$ for some $s_1, s_2 \in S$ and $t \in S \setminus A_1$. In case $t \in S \setminus (A_1 \cup A_2)$, then $z \in (S \circ (S \setminus (A_1 \cup A_2)) \circ S]$. Again, if $t \in (A_1 \cup A_2)$, then $t \in A_2 \subseteq (s'_1 \circ t' \circ s'_2]$ for $s'_1, s'_2 \in S$ and $t' \in S \setminus A_2$. Now $z \in (s_1 \circ t \circ s_2] \subseteq (s_1 \circ (s'_1 \circ t' \circ s'_2] \circ s_2] \subseteq (S \circ S \circ t' \circ S \circ S] \subseteq (S \circ t' \circ S]$. If $t' \in A_1$, then $t \in (s'_1 \circ t' \circ s'_2] \subseteq (S \circ A_1 \circ S] \subseteq A_1$. This is a contradiction. Hence, $t' \in S \setminus (A_1 \cup A_2)$ and, so, $z \in (S \circ (S \setminus (A_1 \cup A_2)) \circ S]$. In a similar way we may show that if $z \in A_2$, then $z \in (S \circ (S \setminus (A_1 \cup A_2)) \circ S]$. Hence, the result follows. □

**Proposition 2.3.** *Let $A_1$ be any hyperideal of $S$ and $A_2 \in \mathcal{C}_{\mathcal{H}}$. Then $A_1 \cap A_2 \in \mathcal{C}_{\mathcal{H}}$.*

*Proof.* First we prove that $A_1 \cap A_2$ is a non-empty hyperideal of $S$. For this, let $t \in A_1$ and $t' \in A_2$, then we have $t \circ t' \subseteq A_1 \circ A_2 \subseteq A_1 \circ S \subseteq A_1$. Also, $t \circ t' \subseteq A_1 \circ A_2 \subseteq S \circ A_2 \subseteq A_2$. Thus, $t \circ t' \subseteq A_1 \cap A_2 \subseteq S$. Clearly, $(A_1 \cap A_2) \circ S \subseteq A_1 \circ S \subseteq A_1$ and $(A_1 \cap A_2) \circ S \subseteq A_2 \circ S \subseteq A_2$. Thus $(A_1 \cap A_2) \circ S \subseteq A_1 \cap A_2$. In a similar way we may show that $S \circ (A_1 \cap A_2) \subseteq A_1 \cap A_2$. Also, as $(A_1 \cap A_2] \subseteq (A_1] = A_1$ and $(A_1 \cap A_2] \subseteq (A_2] = A_2$, we have $(A_1 \cap A_2] \subseteq A_1 \cap A_2$. Now, as $A_1 \cap A_2 \subseteq A_2 \subseteq (S \circ (S \setminus A_2) \circ S] \subseteq (S \circ (S \setminus (A_1 \cap A_2)) \circ S]$, $A_1 \cap A_2$ is a covered hyperideal of $S$. □

**Corollary 2.2.** *If $A_1$ and $A_2 \in \mathcal{C}_{\mathcal{H}}$, then $A_1 \cap A_2 \in \mathcal{C}_{\mathcal{H}}$.*

Combining Proposition 2.2 and Corollary 2.2, we have the following.

**Theorem 2.1.** *For an ordered semihypergroup $S$, $\mathcal{C}_{\mathcal{H}}$ is a sublattice of the lattice of all hyperideals of $S$.*

## 3. Covered Hyperideals in Ordered Semihypergroups

**Theorem 3.1.** *An ordered semihypergroup $S$ contains a covered hyperideal if it is not simple.*

*Proof.* Proof of this theorem is similar to the proof of Theorem 3.10 of [18]. □

**Theorem 3.2.** *If an ordered semihypergroup $S$ contains covered hyperideals, then every covered hyperideal of $S$ is minimal if and only if any two distinct covered hyperideals of $S$ are disjoint.*

*Proof.* Proof of this theorem is similar to the proof of Theorem 3.9 of [18]. □

**Corollary 3.1.** *Let $(S, \circ, \leq)$ be an ordered semihypergroup. If $S$ is not simple, then each covered hyperideal of $S$ is minimal if and only if any two distinct covered hyperideals of $S$ are disjoint.*

**Theorem 3.3.** *Let $K$ be any proper hyperideal of a regular ordered semihypergroup $S$. If for every $J(t) \subseteq K$, there exists $t' \in S \setminus K$ such that $J(t) \subseteq J(t')$, then every proper hyperideal of $S$ is a covered hyperideal of $S$.*

*Proof.* Clearly $(S \circ S] \subseteq S$. As $S$ is regular, $S \subseteq (S \circ S \circ S] \subseteq (S \circ S] \subseteq S \Rightarrow S = (S \circ S]$. Now suppose that for any hyperideal $K$ of $S$ and $t \in K$ such that $J(t) \subseteq K$, there exists $t' \in S \setminus K$ such that $J(t) \subseteq J(t')$. As $S = (S \circ S]$, we get $S = (S \circ S \circ S]$. Then $t' \leq s_1 \circ s_2 \circ s_3$ for some $s_1, s_2, s_3 \in S$. If $s_2 \in K$, then $t' \in (S \circ K \circ S] \subseteq (K] = K$. This is a contradiction. Therefore, $t' \in S \setminus K$. Also, $t' \in (S \circ (S \setminus K) \circ S] \Rightarrow J(t') \subseteq (S \circ (S \setminus K) \circ S]$. Now $t \in J(t) \subseteq J(t') \subseteq (S \circ (S \setminus K) \circ S]$. Hence, $K \in \mathcal{C}_{\mathcal{H}}$. □

The following example illustrates Theorem 3.3.

*Example* 3.1. In Example 2.2, one may easily check that $(S, \circ, \leq)$ is a regular ordered semihypergroup. Consider the subset $K = \{u, v, w, y\}$ of $S$. Then $K$ is a hyperideal of $S$ such that $J(u) \subseteq K$. For $x \in S \setminus K$, $J(t) \subseteq J(x)$ for all $t \in S$. Then, by the hypothesis of the Theorem 3.3, $K$ becomes covered hyperideal of $S$.

**Proposition 3.1.** *Let $K$ be any hyperideal of a regular order semihypergroup $S$. Then any covered hyperideal $L$ of $K$ is also a covered hyperideal of $S$.*

*Proof.* As being a hyperideal of $S$, $K$ is also a subsemihypergroup of $S$. Let $h \in K \subseteq S$. Since $S$ is regular, there exists $t' \in S$ such that $h \leq h \circ t' \circ h \leq h \circ t' \circ (h \circ t' \circ h) = h \circ (t' \circ h \circ t') \circ h$. As $K$ is a hyperideal of $S$, we have $t' \circ h \circ t' \subseteq S \circ K \circ S \subseteq K$. Therefore, $h \in (h \circ K \circ h]$. Hence, $K$ is a regular subsemihypergroup of $S$.

Now we show that $L$ is a hyperideal of $S$. For this, take any $u \in L \subseteq K$ and $s \in S$. Then $u \circ s \subseteq K$. For any $v \in u \circ s \subseteq K$, there exists $h \in K$ such that

$$\begin{aligned} v \leq v \circ h \circ v &\subseteq (u \circ s) \circ h \circ (u \circ s) \\ &\subseteq L \circ (S \circ K \circ S) \circ S \\ &\subseteq L \circ K \circ S \\ &\subseteq L \circ K \subseteq L \quad (\text{ as L is a hyperideal of K}). \end{aligned}$$

Therefore, $u \circ s \subseteq L$. By the similar argument we may show that $s \circ u \subseteq L$. Also, if $l \in L \subseteq K$ and $t \in S$ such that $t \leq l \Rightarrow t \in K$. As $L$ is a hyperideal of $K$, it follows that $t \in L$. Hence $L$ is a hyperideal of $S$. Again, by hypothesis, we have $L \subseteq (K \circ (K \setminus L) \circ K] \subseteq (S \circ (K \setminus L) \circ S] \subseteq (S \circ (S \setminus L) \circ S]$ (since $\phi \neq K \setminus L \subseteq S \setminus L$). Hence, $K \in \mathcal{C}_{\mathcal{H}}$. □

The following example shows that the condition of the Proposition 3.1 on $S$ to be regular ordered semihypergroup is a sufficient condition.

*Example* 3.2. Let $S = \{v, w, x, t\}$. Define a hyper operation $(\circ)$ on $S$ by the following table:

| $\circ$ | $v$ | $w$ | $x$ | $t$ |
|---|---|---|---|---|
| $v$ | $\{v\}$ | $\{v,w\}$ | $\{v,x\}$ | $\{v\}$ |
| $w$ | $\{v\}$ | $\{v,w\}$ | $\{v,x\}$ | $\{v\}$ |
| $x$ | $\{v\}$ | $\{v,w\}$ | $\{v,x\}$ | $\{v\}$ |
| $t$ | $\{v\}$ | $\{v,w\}$ | $\{v,x\}$ | $\{v\}$ |

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Define an order on $S$ as $\leq = \{(v,v),(w,w),(x,x),(t,t),(w,v),(x,v)\}$. Then $(S,\circ,\leq)$ is an ordered semihypergroup but not a regular ordered semihypergroup. For the subsets $K = \{v,w,x\}$, $L_1 = \{w\}$ and $L_2 = \{x\}$ of $S$, one may easily verify that $K$ is a hyperideal of $S$ and each $L_i$ $(i = 1,2)$ is a covered hyperideal of both $K$ and $S$.

**Definition 3.1.** A non-empty subset $H_B$ of $S$ is called a two-sided hyperbase of $S$ if

(a) $S = (H_B \cup H_B \circ S \cup S \circ H_B \cup S \circ H_B \circ S]$;
(b) If $D \subseteq H_B$ such that $S = (D \cup D \circ S \cup S \circ D \cup S \circ D \circ S]$, then $D = H_B$.

Maximal $\mathcal{I}$-hyperclasses of $S$ may be realized as the compliments of maximal hyperideals of $S$. The complement of a maximal hyperideal $H_t$ of $S$, in the sequel, will be denoted by $H^t$.

In the followings, to provide examles of hyperbases of ordered semihypergroups, examples of ordered semihypergroups are taken from [19] and [2], respectively.

*Example* 3.3. Let $S = \{u,v,w,x,y,z\}$. Define a hyper operation $(\circ)$ on $S$ by the following table:

| $\circ$ | $u$ | $v$ | $w$ | $x$ | $y$ | $z$ |
|---|---|---|---|---|---|---|
| $u$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ |
| $v$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ |
| $w$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ |
| $x$ | $\{x,y\}$ | $\{y\}$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ |
| $y$ | $\{x,y\}$ | $\{y\}$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ | $\{x,y\}$ |
| $z$ | $\{u\}$ | $\{v\}$ | $\{w\}$ | $\{x\}$ | $\{y\}$ | $\{z\}$ |

.

Define an order on $S$ as $\leq = \{(u,u),(v,v),(w,w),(x,x),(x,u),(x,w),(x,z),(y,y),$ $(y,u),(y,v),(y,w),(y,x),(y,z),(z,z)\}$. Then $(S,\circ,\leq)$ is an ordered semihypergroup. Consider the subset $H_B = \{z\}$ of $S$. Then, clearly, $S \circ H_B = S$ and, hence, $S = (H_B \cup H_B \circ S \cup S \circ H_B \cup S \circ H_B \circ S]$. So $H_B$ is a hyperbase of $S$.

*Example* 3.4. Let $S = \{u,v,w,x,t\}$. Define a hyper operation $(\circ)$ on $S$ by the following table:

| $\circ$ | $u$ | $v$ | $w$ | $x$ | $t$ |
|---|---|---|---|---|---|
| $u$ | $\{u\}$ | $\{u\}$ | $\{u\}$ | $\{u\}$ | $\{u\}$ |
| $v$ | $\{u\}$ | $\{u,v\}$ | $\{u\}$ | $\{u,x\}$ | $\{u\}$ |
| $w$ | $\{u\}$ | $\{u,t\}$ | $\{u,w\}$ | $\{u,w\}$ | $\{u,t\}$ |
| $x$ | $\{u\}$ | $\{u,v\}$ | $\{u,x\}$ | $\{u,x\}$ | $\{u,v\}$ |
| $t$ | $\{u\}$ | $\{u,t\}$ | $\{u\}$ | $\{u,w\}$ | $\{u\}$ |

.

Define an order on $S$ as $\leq = \{(u,u),(v,v),(w,w),(x,x),(t,t),(u,v),(u,w),(u,x),$ $(u,t)\}$. $(S,\circ,\leq)$ is an ordered semihypergroup may easily be checked. Consider the subsets $H_B = \{v\}$ and $H'_B = \{x\}$ of $S$. It is easy to verify that both $H_B$ and $H'_B$ are hyperbases of $S$.

A covered hyperideal $A$ of an ordered semihypergroup $S$ is called the greatest covered hyperideal of $S$ if it contains every covered hyperideal of $S$. The greatest covered hyperideal $A$ of $S$ will be denoted by $A^g$ in the sequel.

**Theorem 3.4.** *If $S$ is not hypersimple and contains a two-sided hyperbase $H_B$ of $S$, then $S$ has the greatest covered hyperideal $A^g$. Moreover, $A^g = (S^3] \cap \hat{H}$, where $\hat{H} = \bigcap_{t\in\alpha} H_t$, where $\{H_t\}_{t\in\alpha}$ is the family of all maximal hyperideals of $S$.*

*Proof.* Containment of hyperbase $H_B$ implies the existence of maximal hyperideals in $S$ and $H_t = S \setminus H^t$, where $H^t$ is a maximal $\mathcal{J}$-hyperclass. Since $\phi \neq \hat{H} = \bigcap_{t\in\alpha} H_t = \bigcap_{t\in\alpha} S \setminus H^t = S \setminus \bigcup_{t\in\alpha} H^t$. It is easy to verify that $\hat{H}$ and $(S^3]$ are hyperideals of $S$. Let $K = (S^3] \cap \hat{H}$. We show that $K \in \mathcal{C}_{\mathcal{H}}$. For this, let $h \in K$ be any element. Then $h \in (S^3] \Rightarrow h \in (S \circ t' \circ S]$ for some $t' \in S$. If $t' \in H_B$, then $\exists\, c \in H_B$ such that $t' \in J(c)$ and, hence, $t' \in (S \circ c \cup c \circ S \cup S \circ c \circ S]$ i.e. $t'$ is at least in one of the subsets: $(S \circ c]$, $(c \circ S]$, $(S \circ c \circ S]$. Then, for all these subsets, we have $(S \circ t' \circ S] \subseteq (S \circ c \circ S]$ and, hence, $h \in (S \circ c \circ S]$ for $c \in H_B$. Thus, for any $h \in K$, $\exists\, c \in H_B$ such that $h \in (S \circ c \circ S] \subseteq (S \circ H_B \circ S] \subseteq (S \circ (S \setminus \hat{H}) \circ S] \subseteq (S \circ (S \setminus K) \circ S]$. Therefore $K \subseteq (S \circ (S \setminus K) \circ S]$. It now remains to show that $K$ is the greatest covered hyperideal of $S$. To show this, let $L$ be any covered hyperideal of $S$. Then $L \subseteq (S \circ (S \setminus L) \circ S] \subseteq (S^3]$. Since $L \in \mathcal{C}_{\mathcal{H}}$, $L$ can not contain any maximal $\mathcal{J}$-hyperclass. So $L \subseteq S \setminus H^t$ for every $t \in \alpha$. Therefore, $L \subseteq \bigcap_{t\in\alpha} S \setminus H^t = \bigcap_{t\in\alpha} H_t = \hat{H}$. Hence, $L \subseteq (S^3] \cap \hat{H} = K$. Therefore, any covered hyperideal is contained in $K$, i.e., $K = A^g$. $\square$

**Lemma 3.1.** *Let $S$ be any ordered semihypergroup having the greatest covered hyperideal $A^g$. If $A^g \subseteq (S \circ S \circ S]$, then*

(a) *every $\mathcal{J}$-hyperclass in $(S^3] \setminus A^g$ is maximal;*
(b) *$J(t) = (S \circ t \circ S]$ for all $t \in (S^3] \setminus A^g$.*

*Proof.* First we assume that $A^g \subset (S^3]$. Then, we have $(S^3] \setminus A^g \neq \phi$. To show the second part let $t \in (S^3] \setminus A^g$. Since $A^g$ is a hyperideal of $S$, the $\mathcal{J}$-hyperclass $T_t \subseteq (S^3] \setminus A^g$. Thus $t \in (S \circ t' \circ S]$ for some $t' \in S$ and $(S \circ t \circ S] \subseteq (S \circ t' \circ S]$. Since $(S \circ t' \circ S] \subseteq J(t')$, we have $J(t) \subseteq J(t')$. Now suppose to the contrary that $t' \notin T_t$. So $T_t \neq T'_t$. We claim that $t' \in S \setminus J(t)$. For this, if $t' \in J(t)$, then $J(t) = J(t') \Rightarrow T_t = T'_t$, which is impossible. Thus we have $J(t) \subseteq (S \circ (S \setminus J(t)) \circ S]$ and, so, $J(t) \in \mathcal{C}_{\mathcal{H}}$. By Proposition 2.2, $A^g \cup J(t) \in \mathcal{C}_{\mathcal{H}}$. As $t \notin A^g$, we, thus, have $A^g \subset A^g \cup J(t)$. This is a contradiction. Hence, $t' \in T_t$ and $J(t) \subseteq (S \circ t' \circ S] \subseteq J(t') = J(t)$.

Thus, $J(t) = (S \circ t' \circ S] = J(t')$. So, obviously $(S \circ t \circ S] \subseteq J(t)$. Now there are two possibilities: if $t' \leq t$, then $J(t) = (S \circ t' \circ S] \subseteq (S \circ t \circ S] \Rightarrow J(t) \subseteq (S \circ t \circ S]$. If

$t' \leq t$ is not true, then $t' \in (S \circ t \cup t \circ S \cup S \circ t \circ S]$. Now, if $t' \in (S \circ t]$, then we have

$$S \circ t' \circ S \subseteq S \circ (S \circ t] \circ S \subseteq (S \circ (S \circ t] \circ S] \subseteq (S \circ S \circ t \circ S] \subseteq (S \circ t \circ S].$$

Similarly, for $t' \in (t \circ S] \cup (S \circ t \circ S]$, we may show that $(S \circ t' \circ S] \subseteq (S \circ t \circ S]$. Therefore, $J(t) = J(t') = (S \circ t' \circ S] \subseteq (S \circ t \circ S]$.

To prove the reverse part, let $T_t$ be a $\mathcal{J}$-hyperclass in $(S^3] \setminus A^g$. On contrary assume that $T_t$ is not maximal. Then, by Lemma 2.1, $J(t) \subset J(t')$ for some $t' \in S$. So $t \in J(t')$. This implies that $t \in (t'] \cup (S \circ t'] \cup (t' \circ S] \cup (S \circ t' \circ S]$. For such $t$, we may easily prove that $(S \circ t \circ S] \subseteq (S \circ t' \circ S] \Rightarrow J(t) \subseteq (S \circ t' \circ S]$. Now, as $t' \in S \setminus J(t)$, $J(t)$ is a covered hyperideal of $S$. Hence $A^g \subset A^g \cup J(t)$, a contradiction. Therefore, every $\mathcal{J}$-hyperclass in $(S^3] \setminus A^g$ is maximal. □

**Theorem 3.5.** *Let $S$ be any ordered semihypergroup having the greatest covered hyperideal $A^g$. If*

(a) $A^g \subset (S \circ S \circ S]$;

(b) *neither $T_t \preccurlyeq T_{t'}$ nor $T_{t'} \preccurlyeq T_t$ for any $t, t' \in S \setminus (S^2]$,*

*then $S$ contains a hyperbase.*

*Proof.* Suppose that $A^g \subset (S^3]$ and $t, t' \in S \setminus (S^2]$ such that they are incomparable. Since $A^g$ is a covered hyperideal of $S$, we have

$$A^g \subseteq (S \circ (S \setminus A^g) \circ S] \subseteq (S^3] \subseteq (S^2] \subseteq S.$$

Let $C_1 = \{T_t \mid t \in S \setminus (S^2]\}$, $C_2 = \{T_t \mid t \in (S^2] \setminus (S^3]\}$and $C_3 = \{T_t \mid t \in (S^3] \setminus A^g\}$. Let $K$ be the set containing all the elements from the members of $C_1$ and $C_3$. Then, it is easy to verify that $K$ is a hyperbase of $S$. To show that $S = J(K) = (K \cup S \circ K \cup K \circ S \cup S \circ K \circ S]$, we only need to show that $A^g, (S^3] \setminus A^g, (S^2] \setminus (S^3],$ and $S \setminus (S^2]$ are subsets of $J(K)$.

(i) Let $z \in A^g$. Then $z \in (S \circ (S \setminus A^g) \circ S] \Rightarrow z \in (S \circ y \circ S]$ for some $y \in S \setminus A^g$. Clearly $y \in T_t$ for some $t \in (S \setminus (S^2]) \cup ((S^2] \setminus (S^3]) \cup ((S^3] \setminus A^g)$. Now, by the construction of $K$, if $t \in (S \setminus (S^2]) \cup ((S^3] \setminus A^g)$, then we have $y \in J(K)$. Hence $z \in J(K)$. If $t \in (S^2] \setminus (S^3]$, then $t \leq u_1 \circ u_2$ for some $u_1, u_2 \in S$ Since $t \notin (S^3]$, we have $u_1, u_2 \in S \setminus (S^2]$. It implies that $t \in J(K)$ and, so, $y \in J(K)$. Thus, we have $z \in J(K)$.

(ii) If $z \in (S^3] \setminus A^g$. Then there exists $x_1 \in K$ such that $z \in J(x_1)$. Therefore $z \in J(K)$.

(iii) If $z \in (S^2] \setminus (S^3]$, then one may prove in a similar way as in the Case (i).

(iv) If $z \in S \setminus (S^2]$, then there exists $x_2 \in K$ such that $z \in J(x_2) \subseteq J(K)$.

Now, we show the minimality of $K$ satisfying $S = J(K)$. By Lemma 3.1, every $T_t \in C_3$ is maximal. Also every $T_t \in C_1$ is maximal since for any elements $t, t' \in S \setminus (S^2]$, neither $T_t \preccurlyeq T_{t'}$ nor $T_{t'} \preccurlyeq T_t$. Let $L \subset K$ such that $S = (L \cup S \circ L \cup L \circ S \cup S \circ L \circ S]$ and let $z \in K \setminus L$. Then $z \leq z'$ for some $z' \in (L \cup S \circ L \cup L \circ S \cup S \circ L \circ S] \Rightarrow z' \in J(l)$ for some $l \in L$. Thus, $J(z) \subset J(l)$, a contradiction to the construction of $K$. Hence, the proof is completed. □

A hyperideal $A$ of $S$ is called the greatest hyperideal of $S$ if every proper hyperideal of $S$ is contained in $A$. The greatest hyperideal $A$, if exists, will be denoted by $A^\star$ in the sequel.

**Theorem 3.6.** *The greatest hyperideal $A^\star$ of $S$ is a covered hyperideal of $S$ if and only if $(S^2] = (S^3]$.*

*Proof.* First we assume that $A^\star \in \mathcal{C}_{\mathcal{H}}$. So $A^\star \subseteq (S \circ (S \setminus A^\star) \circ S]$. Since $A^\star$ is a maximal hyperideal of $S$, it follows that $S \setminus A^\star = T_a$ is the unique maximal $\mathcal{I}$-hyperclass of $S$. Then either $(S^2] \subset S$ or $(S^2] = S$. If $(S^2] = S$, then the proof is obvious. If $(S^2] \subset S$, then either $(S^3] = (S^2]$ or $(S^3] \subset (S^2]$.

If $(S^3] \subset (S^2]$, then $A^\star \subseteq (S \circ (S \setminus A^\star) \circ S] \subseteq (S^3] \subset (S^2]$. Hence $S \setminus A^\star$ would contain at least two different $\mathcal{I}$-hyperclasses, each from $(S^2] \setminus (S^3]$ and $S \setminus (S^2]$. This is a contradiction to the fact that $S \setminus A^\star$ contains only one maximal $\mathcal{J}$-class. Thus $(S^2] = (S^3]$.

Conversely, suppose that $S$ contains $A^\star$ and $(S^2] = (S^3]$. Then show that $A^\star$ is a covered hyperideal of $S$. For this, take any $z \in A^\star$. Then, for any element $c \in T_a = S \setminus A^\star$, we have $J(c) = S$. Thus $z \in J(c)$. However, $z \in A^\star$ and $c \in T_a = S \setminus A^\star$, hence $z \neq c$. Therefore, $z \in (c \circ S \cup S \circ c \cup S \circ c \circ S]$. If $z \in (c \circ S]$ or $z \in (S \circ c]$, then, clearly $z \in (S^2]$. If $z \in (S \circ c \circ S]$, then $z \in (S^3]$. But, by hypothesis, $(S^2] = (S^3]$. Therefore, $z \in (S^3]$, i.e., $z \in (S \circ d \circ S]$ for some $d \in S = J(c)$. If $d = c$, then, clearly $d \in (S \circ c \circ S]$. If $d \neq c$, then $d \in (c \circ S \cup S \circ c \cup S \circ c \circ S]$. Again, if $d \in (c \circ S]$, then, clearly $(S \circ d \circ S] \subseteq (S \circ c \circ S \circ S] \subseteq (S \circ c \circ S]$. The same relation may be shown if $d \in ((S \circ c]) \cup ((S \circ c \circ S])$. Thus, $z \in (S \circ d \circ S] \subseteq (S \circ c \circ S]$ and $c \in T_a = S \setminus A^\star$. This shows that, for any $z \in A^\star$, we have $z \in (S \circ c \circ S]$ and $c \in T_a = S \setminus A^\star$. Hence, $A^\star \subseteq (S \circ (S \setminus A^\star) \circ S]$ i.e. $A^\star \in \mathcal{C}_{\mathcal{H}}$. ☐

**Theorem 3.7.** *Suppose $S$ has only one maximal hyperideal $K$. If $K \in \mathcal{C}_{\mathcal{H}}$, then $K = A^\star$.*

*Proof.* Let $L$ be any proper hyperideal of $S$. Then it is easy to verify that $L \subseteq K$, otherwise we shall get a contradiction to the Proposition 2.1. Hence, $K = A^\star$. ☐

The following example illustrates that the converse of the Theorem 3.7 is not be true in general.

*Example* 3.5. Let $S = \{u, v, w, x\}$. Define a hyper operation $(\circ)$ on $S$ by the following table:

| $\circ$ | $u$ | $v$ | $w$ | $x$ |
|---|---|---|---|---|
| $u$ | $\{u\}$ | $\{v\}$ | $\{u\}$ | $\{v\}$ |
| $v$ | $\{v\}$ | $\{u\}$ | $\{v\}$ | $\{u\}$ |
| $w$ | $\{u\}$ | $\{v\}$ | $\{u\}$ | $\{v\}$ |
| $x$ | $\{v\}$ | $\{u\}$ | $\{v\}$ | $\{u\}$ |

Define an order on $S$ as $\leq = \{(u,u),(v,v),(w,w),(x,x),(u,w)\}$. The proof that $(S, \circ, \leq)$ is an ordered semihypergroup is an easy exercise. Consider the subset $K =$

$\{u, v, w\}$ of $S$. Then it is easy to verify that $K$ is the only maximal hyperideal of $S$. As any proper hyperideal of $S$ is contained in $K$, Thus, $K = A^\star$. Now $S \setminus A^\star = \{x\}$, $S \circ x \circ S = \{u, v\}$. So, $A^\star \nsubseteq (S \circ (S \setminus A^\star) \circ S]$. Hence, $A^\star$ is not a covered hyperideal of $S$.

**Theorem 3.8.** *If every proper hyperideal of $S$ is a covered hyperideal of $S$, then either of the followings is true:*

(1) *$S$ contains $A^\star$;*
(2) *$S = (S \circ S]$ and for any proper hyperideal $K$ and for every hyperideal $J(t)$ of $S$ such that $J(t) \subseteq K$, there exists $y \in S \setminus K$ such that $J(t) \subset J(y) \subset S$.*

*Proof.* Take any $a, b \in S$. If $T_a$ and $T_b$ are maximal $\mathcal{J}$-hyperclasses of $S$ such that $T_a \neq T_b$, then, by Lemma 2.2, $A_a = S \setminus T_a$ and $A_b = S \setminus T_b$ are maximal proper hyperideals of $S$. So, by Corollary 2.1, none of them is a covered hyperideal of $S$. This is a contradiction. Thus $S$ has no different maximal $\mathcal{J}$-hyperclasses. Hence either $S$ contains one maximal $\mathcal{J}$-hyperclass or $S$ does not contain any maximal $\mathcal{J}$-hyperclass. Let the only maximal $\mathcal{J}$-hyperclass $T_a$ be contained in $S$. Then $A_a = S \setminus T_a$ is a maximal hyperideal of $S$. By hypothesis, $A_a \in \mathcal{C}_{\mathcal{H}}$. Thus, by Theorem 3.7, $A_a = A^\star$.

For the second possibility, suppose that $S$ does not contain any maximal $\mathcal{J}$-hyperclass. We need to show that $S = (S \circ S]$. For this, suppose that $(S \circ S] \subset S$. Then $\exists$ $c \in S \setminus (S \circ S]$. We claim that the principal hyperideal $J(c) \subsetneq S$. If $J(c) = S$, then $S$ has a maximal $\mathcal{J}$-hyperclass which is impossible. Hence $J(c) \subset S$. By hypothesis, $J(c) \in \mathcal{C}_{\mathcal{H}}$, i.e., $J(c) \subseteq (S \circ (S \setminus J(c)) \circ S]$. Then $c \in (S \circ S \circ S] \subseteq (S \circ S]$. This is a contradiction.

Now let $K$ be any proper hyperideal of $S$ and let the principal hyperideal $J(t) \subseteq K$. By hypothesis, $K \subseteq (S \circ (S \setminus K) \circ S]$. So $\exists$ $y \in S \setminus K$ such that $t \in (S \circ y \circ S] \Rightarrow J(t) \subseteq J(y) \subseteq S$. As $y \in S \setminus K$, $J(t) \subset J(y)$. Since $S$ contains no maximal $\mathcal{J}$-hyperclass, we have $J(y) \subset S$, as required. □

**Theorem 3.9.** *Let $(S, \circ, \leq)$ be an ordered semihypergroup. If*

(1) *$S$ contains the greatest hyperideal $A^\star$ such that $A^\star \in \mathcal{C}_{\mathcal{H}}$ or*
(2) *$S = (S^2]$ and for any proper hyperideal $K$ and for every hyperideal $J(t)$ of $S$ such that $J(t) \subseteq K$, there exists $y \in S \setminus K$ such that $J(t) \subseteq J(y)$,*

*then every proper hyperideal of $S$ is a covered hyperideal of $S$.*

*Proof.* Let $K$ be any proper hyperideal of $S$. First, suppose that the condition (1) holds. Then $K \subseteq A^\star$ and $S \setminus A^\star \subseteq S \setminus K$. Since $A^\star \in \mathcal{C}_{\mathcal{H}}$, we have

$$K \subseteq A^\star \subseteq (S \circ (S \setminus A^\star) \circ S] \subseteq (S \circ (S \setminus K) \circ S].$$

Therefore, $K \in \mathcal{C}_{\mathcal{H}}$.

Secondly we assume that $S$ satisfies the condition (2). Let $h \in K \Rightarrow J(h) \subseteq K$. By the condition (2), we have $J(h) \subset J(y)$ for some $y \in S \setminus K$. Since $S = (S^2] \Rightarrow S = (S^3]$. Thus, $y \in (S \circ b \circ S]$ for some $b \in S$. As $y \in S \setminus K$, we thus have $b \in S \setminus K$.

Therefore, $h \in (S \circ b \circ S] \subseteq (S \circ (S \setminus K) \circ S]$ and, so, $K \subseteq (S \circ (S \setminus K) \circ S]$. Hence, $K \in \mathcal{C}_{\mathcal{H}}$. □

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## 4. Open Problems

(1) Is it true that the greatest hyperideal $A^*$ of an ordered semihypergroup $S$ is a covered hyperideal of $S$ if and only if $S = (S \circ S]$?
(2) Suppose an ordered semihypergroup $(S, \circ, \leq)$ contains only one maximal hyperideal $K$. Does $K \in \mathcal{C}_{\mathcal{H}}$ if $K = A^*$, the greatest hyperideal of $S$?

## 5. Conclusion

In ordered semigroups and ordered semihypergroups, ideals and hyperideals, play an important role to discuss the nature of the structure of ordered semigroups and ordered semihypergroups. Nowadays the hyperideal theory has been extensively studied by several authors. In ordered semihypergroups different types of hyperideals such as bi-hyperideals, quasi-hyperideals have been studied. These notions had been widely studied by several authors in different algebraic structures (see [1, 2, 12, 19]). In this paper, we have enhanced the understanding of ordered semihypergroups by introducing the concept of a covered hyperideal in an ordered semihypergroup. We have also introduced the notion of a hyperbase in an ordered semihypergroup and proved some vital results.

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