Kragujevac Journal of Mathematics
Volume 48(2) (2024), Pages 287–307.

MORE GENERALIZATIONS OF UNION SOFT HYPERIDEALS OF ORDERED SEMIHYPERGROUPS

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Abstract. In this paper, we introduce the notions of (M, N)-union soft hyperideals and (M, N)-union soft interior hyperideals of ordered semihypergroups. Some basic operations are investigated and some related properties are also studied. We present characterizations of ordered semihypergroups in terms of (M, N)-union soft hyperideals and (M, N)-union soft interior hyperideals. We prove that every (M, N)-union soft hyperideal is an (M, N)-union soft interior hyperideal but the converse is not true which is shown with help of an example. However we show that the notions of (M, N)-union soft hyperideals and (M, N)-union soft interior hyperideals coincide in a regular as well as in intra-regular ordered semihypergroups. Moreover we introduce the notion of (M, N)-union soft simple ordered semihypergroups. Finally, we characterize (M, N)-union soft simple ordered semihypergroups by means of (M, N)-union soft hyperideals and (M, N)-union soft interior hyperideals.

1. Introduction

There are many examples in chemistry where the sum of two elements is a set of elements. In this case we have a hyperstructure. Algebraic hyperstructures represent a natural extension of classical algebraic structures and they were originally proposed in 1934 by a French mathematician Marty [8] at the 8^{th} Congress of Scandinavian Mathematicians. One of the main reason which attracts researches towards hyperstructures is its unique property that in hyperstructures composition of two elements is a set, while in classical algebraic structures the composition of two elements is an

Key words and phrases. Regular ordered semihypergroup, intra-regular ordered semihypergroup, (M, N)-union soft hyperideal, (M, N)-union soft interior hyperideal, (M, N)-union soft simple ordered semihypergroup.

2020 *Mathematics Subject Classification.* Primary: 03D40. Secondary: 08Q70, 47D03.

DOI 10.46793/KgJMat2402.287F

Received: April 23, 2020.

Accepted: April 08, 2021.

element. Thus algebraic hyperstructures are natural extension of classical algebraic structures. Since then, hyperstructures are widely investigated from the theoretical point of view and for their applications to many branches of pure and applied mathematics. Especially, semihypergroups are the simplest algebraic hyperstructures which possess the properties of closure and associativity. Nowadays many researchers have studied different aspects of semihypergroups (see [9–15, 18]).

The uncertainty appeared in economics, engineering, environmental science, medical science and social science and so many other applied sciences is too complicated to be solved by traditional mathematical framework. Molodstov [6], introduced soft set theory and it has received much attention since its inception. Soft set theory emphasizes a balanced coverage of both theory and practice. Nowadays, it has promoted a breadth of the discipline of informations sciences with intelligent systems, approximate reasoning, expert and decision support systems, self-adaptation and self-organizational systems, information and knowledge, modeling and computing with words. Soft set theory has been regarded as a new mathematical tool for dealing with uncertainties and it has seen a wide-ranging applications in the mean of algebraic structures such as groups [1], semirings [2], ordered semigroups [4], hemirings [5, 7], and so on. Feng et al. discussed soft relations in semigroups (see [3]) and explored decomposition of fuzzy soft sets with finite value spaces. Khan et al. [17], applied soft set theory to ordered semihypergroups and introduced the notions of uni-soft subsemihypergroups and uni-soft left (resp. right) hyperideals.

In this paper, we study the concepts of union soft interior hyperideals, (M, N)-union soft hyperideals and (M, N)-union soft interior hyperideals in ordered semihypergroups and present some related examples of these concepts. We show that (M, N)-union soft hyperideals and (M, N)-union soft interior hyperideals coincide in regular ordered semihypergroups and intra-regular ordered semihypergroups. We characterize ordered semihypergroups in terms of (M, N)-union soft hyperideals and (M, N)-union soft interior hyperideals. We introduce the concept of (M, N)-union soft simple ordered semihypergroups. Moreover we characterize (M, N)-union soft simple ordered semihypergroups in terms of (M, N)-union soft hyperideals and (M, N)-union soft interior hyperideals.

2. Preliminaries

By an ordered semihypergroup we mean a structure $(S, \circ, \leq)$ in which the following conditions are satisfied:

(1) $(S, \circ)$ is a semihypergroup;

(2) $(S, \leq)$ is a poset;

(3) for all $a, b, x \in S$ $a \leq b$ implies $x \circ a \leq x \circ b$ and $a \circ x \leq b \circ x$.

For $A \subseteq S$, we denote $(A] := \{t \in S : t \leq h \text{ for some } h \in A\}$. For $A, B \subseteq S$, we have $A \circ B := \bigcup\{a \circ b : a \in A,\, b \in B\}$.

A nonempty subset A of an ordered semihypergroup S is called a subsemihypergroup of S if $A^2 \subseteq A$.

A nonempty subset A of S is called a left (resp. right) hyperideal of S if it satisfies the following conditions:

(1) $S \circ A \subseteq A$ (resp. $A \circ S \subseteq A$);

(2) if $a \in A, b \in S$ and $b \leq a$, implying $b \in A$.

By a two sided hyperideal or simply a hyperideal of S we mean a nonempty subset of S which is both a left hyperideal and a right hyperideal of S.

A subsemihypergroup A of S is called an interior hyperideal of S if it satisfies the following conditions:

(1) $S \circ A \circ S \subseteq A$;

(2) if $a \in A, b \in S$ and $b \leq a$, implying $b \in A$.

An ordered semihypergroup $(S, \circ, \leq)$ is called regular if for every $a \in S$ there exists $x \in S$ such that $a \leq a \circ x \circ a$.

An ordered semihypergroup S is called intra-regular if for every $a \in S$, there exist $x, y \in S$ such that $a \leq x \circ a \circ a \circ y$.

3. Soft Sets

In what follows, we take $E = S$ as the set of parameters, which is an ordered semihypergroup, unless otherwise specified.

From now on, U is an initial universe set, E is a set of parameters, $P(U)$ is the power set of U and $A, B, C, \ldots \subseteq E$.

Definition 3.1 (see [6])**.** A *soft set* f_A over U is defined as

$$f_A : E \to P(U) \quad \text{such that} \quad f_A(x) = \emptyset \quad \text{if } x \notin A.$$

Hence, f_A is also called an *approximation function.*

A soft set f_A over U can be represented by the set of ordered pairs

$$f_A = \{(x, f_A(x)) \mid x \in E, f_A(x) \in P(U)\}.$$

It is clear that a soft set is a *parameterized family* of subsets of U. Note that the set of all soft sets over U will be denoted by $S(U)$.

Definition 3.2 (see [6])**.** Let $f_A, f_B \in S(U)$. Then f_A is called a *soft subset* of f_B, denoted by $f_A \tilde{\subseteq} f_B$ if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.

Definition 3.3 (see [6])**.** Two soft sets f_A and f_B are said to be equal soft sets if $f_A \tilde{\subseteq} f_B$ and $f_B \tilde{\subseteq} f_A$ and is denoted by $f_A \tilde{=} f_B$.

Definition 3.4. (see [6]). Let $f_A, f_B \in S(U)$. Then the *soft union* of f_A and f_B, denoted by $f_A \tilde{\cup} f_B = f_{A \cup B}$, is defined by $(f_A \tilde{\cup} f_B)(x) = f_A(x) \cup f_B(x)$ for all $x \in E$.

Definition 3.5 (see [6])**.** Let $f_A, f_B \in S(U)$. Then the *soft intersection* of f_A and f_B, denoted by $f_A \tilde{\cap} f_B = f_{A \cap B}$, is defined by $(f_A \tilde{\cap} f_B)(x) = f_A(x) \cap f_B(x)$ for all $x \in E$.

For $x \in S$, we define $A_x = \{(y, z) \in S \times S \mid x \leq y \circ z\}$.

Definition 3.6 (see [17]). Let f_A and g_B be two soft sets of an ordered semihypergroup S over U. Then, the uni-soft product, denoted by $f_A \tilde{\diamond} g_B$, is defined by

$$f_A \tilde{\diamond} g_B : S \to P(U), \quad x \mapsto (f_A \tilde{\diamond} g_B)(x) = \begin{cases} \bigcap_{(y,z)\in A_x} \{f_A(y) \cup g_B(z)\}, & \text{if } A_x \neq \emptyset, \\ U, & \text{if } A_x = \emptyset, \end{cases}$$

for all $x \in S$.

Definition 3.7 (see [17]). Let $A \subseteq S$. Then the soft characteristic function

$$\chi_A^c : S \to P(U)$$

is defined by

$$\chi_A(x) := \begin{cases} U, & \text{if } x \in A, \\ \emptyset, & \text{if } x \notin A. \end{cases}$$

For the characteristic soft set χ_A over U, the soft set χ_A^c over U given as follows:

$$\chi_A^c(x) := \begin{cases} \emptyset, & \text{if } x \in A, \\ U, & \text{if } x \notin A. \end{cases}$$

For an ordered semihypergroup, the soft sets "$\emptyset_S$" of S over U is defined as follows:

$$\emptyset_S : S \mapsto P(U), \quad x \mapsto \emptyset_S(x) = \emptyset.$$

Definition 3.8 (see [17]). Let f_A be a soft set of an ordered semihypergroup S over U a subset δ such that $\delta \in P(U)$. The δ-exclusive set of f_A is denoted by $e_A(f_A, \delta)$ and defined to be the set

$$e_A(f_A, \delta) = \{x \in S \mid f_A(x) \subseteq \delta\}.$$

Definition 3.9 (see [17])). A soft set f_A of an ordered semihypergroup S over U is called *a union soft subsemihypergroup of* S over U if

$$(\forall x, y \in S) \bigcup_{\alpha \in x \circ y} f_A(\alpha) \subseteq f_A(x) \cup f_A(y).$$

Definition 3.10 (see [17]). Let f_A be a soft set of an ordered semihypergroup S over U. Then f_A is called a union soft left (resp. right) hyperideal of S over U if it satisfies the following conditions:

(1) $(\forall x, y \in S) \bigcup_{\alpha \in x \circ y} f_A(\alpha) \subseteq f_A(y) \left(\text{resp. } \bigcup_{\alpha \in x \circ y} f_A(\alpha) \subseteq f_A(x) \right)$;

(2) $(\forall x, y \in S)\ x \leq y \Rightarrow f_A(x) \subseteq f_A(y)$.

A soft set f_A of an ordered semihypergroup S over U is called a *union soft hyperideal* of S over U if it is both a union soft left hyperideal and a union soft right hyperideal of S over U.

Definition 3.11. A union soft subsemihypergroup f_A of an ordered semihypergroup S over U is called a union soft interior hyperideal of S over U if it satisfies the following conditions:

(1) $(\forall x, y, a \in S) \bigcup\limits_{\alpha \in x \circ a \circ y} f_A(\alpha) \subseteq f_A(a)$;

(2) $(\forall x, y \in S)\ x \leq y \Rightarrow f_A(x) \subseteq f_A(y)$.

Example 3.1. Let $(S, \circ, \leq)$ be an ordered semihypergroup where the hyperoperation and the order relation are defined by:

$\circ$	e_1	e_2	e_3	e_4
e_1	$\{e_1\}$	$\{e_1\}$	$\{e_1\}$	$\{e_1\}$
e_2	$\{e_1\}$	$\{e_1\}$	$\{e_1, e_4\}$	$\{e_1\}$
e_3	$\{e_1\}$	$\{e_1\}$	$\{e_1\}$	$\{e_1\}$
e_4	$\{e_1\}$	$\{e_1\}$	$\{e_1\}$	$\{e_1\}$

,

$$\leq := \{(e_1, e_1), (e_2, e_2), (e_3, e_3), (e_4, e_4), (e_1, e_4)\}.$$

Suppose $U = \{1, 2, 3\}$ and $A = \{e_2, e_3, e_4\}$. Let us define $f_A(e_1) = \emptyset$, $f_A(e_2) = \{1\}$, $f_A(e_3) = \{1, 2, 3\}$ and $f_A(e_4) = \{2, 3\}$. Then f_A is a union soft interior hyperideal of S over U.

4. (M, N)-Union Soft Hyperideals

In this section, we introduce the notions of (M, N)-union soft hyperideal of ordered semihypergroups and investigate some related properties. From now on, $\emptyset \subseteq M \subset N \subseteq U$.

For any soft sets f_A and g_B, we define an order relation $\tilde{\supseteq}_{[M,N]}$ by putting

$$f_A \tilde{\supseteq}_{[M,N]} g_B \Leftrightarrow (f_A(x) \cup M) \cap N \tilde{\supseteq} (g_B(x) \cup M) \cap N,$$

for all $x \in S$.

In case $f_A \tilde{\supseteq}_{[M,N]} g_B$ and $g_B \tilde{\supseteq}_{[M,N]} f_A$ then $f_A =_{[M,N]} g_B$.

Theorem 4.1. *Let $(S, \circ, \leq)$ be an ordered semihypergroup. Then the set*

$$\left(S(U), \tilde{\diamond}, \tilde{\supseteq}_{[M,N]}\right)$$

forms an ordered semihypergroup.

Proof. Obviously, the operation "$\tilde{\diamond}$" is well-defined.

Let f_A, g_B, and $h_C \in S(U)$ and x be any element of S. If $A_x = \emptyset$, then, clearly, $((((f_A \tilde{\diamond} g_B) \tilde{\diamond} h_C)(x)) \cup M) \cap N = (((f_A \tilde{\diamond} (g_B \tilde{\diamond} h_C))(x)) \cup M) \cap N$. Let $A_x \neq \emptyset$, then we have

$$
\begin{aligned}
&((((f_A \tilde{\diamond} g_B) \tilde{\diamond} h_C)(x)) \cup M) \cap N \\
&= \left(\left(\bigcap_{x \leq y \circ z} \{(f_A \tilde{\diamond} g_B)(y) \cup h_C(z)\}\right) \cup M\right) \cap N \\
&= \left(\left(\bigcap_{x \leq y \circ z} \left\{\bigcap_{y \leq u \circ v} \{f_A(u) \cup g_B(v)\} \cup h_C(z)\right\}\right) \cup M\right) \cap N
\end{aligned}
$$

$$
\begin{aligned}
&= \left(\left(\bigcap_{x \leq (u \circ v) \circ z} \{ f_A(u) \cup g_B(v) \cup h_C(z) \} \right) \cup M \right) \cap N \\
&= \left(\left(\bigcap_{x \leq u \circ (v \circ z)} \{ f_A(u) \cup (g_B(v) \cup h_C(z)) \} \right) \cup M \right) \cap N \\
&\supseteq \left(\left(\bigcap_{x \leq u \circ (v \circ z)} \left\{ f_A(u) \cup \left\{ \bigcap_{y \leq v \circ z} (g_B(v) \cup h_C(z)) \right\} \right\} \right) \cup M \right) \cap N \\
&= \left(\left(\bigcap_{x \leq u \circ (v \circ z)} \{ f_A(u) \cup (g_B \widetilde{\diamond} h_C)(v \circ z) \} \right) \cup M \right) \cap N \\
&= (((f_A \widetilde{\diamond} (g_B \widetilde{\diamond} h_C))(x)) \cup M) \cap N.
\end{aligned}
$$

It follows that $((f_A \widetilde{\diamond} g_B) \widetilde{\diamond} h_C) \widetilde{\supseteq}_{[M,N]} (f_A \widetilde{\diamond} (g_B \widetilde{\diamond} h_C))$. Similarly, we can prove that $(f_A \widetilde{\diamond} (g_B \widetilde{\diamond} h_C)) \widetilde{\supseteq}_{[M,N]} ((f_A \widetilde{\diamond} g_B) \widetilde{\diamond} h_C)$. Thus we have proved that $((f_A \widetilde{\diamond} g_B) \widetilde{\diamond} h_C) =_{[M,N]} (f_A \widetilde{\diamond} (g_B \widetilde{\diamond} h_C))$.

Assume that $f_A \widetilde{\supseteq}_{[M,N]} g_B$ and let $A_x = \emptyset$. Then obviously, $(f_A \widetilde{\diamond} h_C) \widetilde{\supseteq}_{[M,N]} (g_B \widetilde{\diamond} h_C)$ and $(h_C \widetilde{\diamond} f_A) \widetilde{\supseteq}_{[M,N]} (h_C \widetilde{\diamond} g_B)$. If $A_x \neq \emptyset$, then

$$
\begin{aligned}
(((f_A \widetilde{\diamond} h_C)(x)) \cup M) \cap N &= \left(\left(\bigcap_{(y,z) \in A_x} \{ f_A(y) \cup h_C(z) \} \right) \cup M \right) \cap N \\
&= \left(\left(\bigcap_{(y,z) \in A_x} \{ f_A(y) \cup h_C(z) \cup M \} \right) \cup M \right) \cap N \\
&\supseteq \left(\left(\bigcap_{(y,z) \in A_x} \{ g_B(y) \cup h_C(z) \cap N \} \right) \cup M \right) \cap N \\
&= \left(\bigcap_{(y,z) \in A_x} \{ g_B(y) \cup h_C(z) \cap N \} \right) \cup (M \cap N) \\
&= \left(\left(\bigcap_{(y,z) \in A_x} \{ g_B(y) \cup h_C(z) \} \right) \cap N \right) \cup M \\
&= \left(\left(\bigcap_{(y,z) \in A_x} \{ g_B(y) \cup h_C(z) \} \right) \cup M \right) \cap N \\
&= (g_B \widetilde{\diamond} h_C)(x).
\end{aligned}
$$

In a similar way, we can show that $(h_C \widetilde{\diamond} f_A) \widetilde{\supseteq}_{[M,N]} (h_C \widetilde{\diamond} g_B)$. Thus, $\left(S(U), \widetilde{\diamond}, \widetilde{\supseteq}_{[M,N]} \right)$ is an ordered semihypergroup.

Definition 4.1. A soft set f_A of an ordered semihypergroup S over U is called an (M,N)-*union subsemihypergroup of* S over U if

$$(\forall x, y \in S)\left(\bigcup_{\alpha \in x \circ y} f_A(\alpha)\right) \cap N \subseteq f_A(x) \cap f_A(y) \cup M.$$

Example 4.1. Let $(S, \circ, \leq)$ be an ordered semihypergroup where the hyperoperation and the order relation are defined by:

$\circ$	p	q	r	s
p	$\{p\}$	$\{p\}$	$\{p\}$	$\{p\}$
q	$\{p\}$	$\{p\}$	$\{p\}$	$\{p\}$
r	$\{p\}$	$\{p\}$	$\{p,q\}$	$\{p\}$
s	$\{p\}$	$\{p\}$	$\{p,q\}$	$\{p,q\}$

,

$$\leq := \{(p,p),(q,q),(r,r),(s,s),(p,q)\}.$$

Suppose $U = \{1,2,3\}$, $A = \{q,r,s\}$, $M = \{2\}$ and $N = \{1,2\}$. Let us define $f_A(p) = \emptyset$, $f_A(q) = \{2\}$, $f_A(r) = \{1,2,3\}$ and $f_A(s) = \{2,3\}$. Then f_A is an (M,N)-union soft subsemihypergroup of S over U.

Theorem 4.2. *A non-empty subset A of an ordered semihypergroup $(S, \circ, \leq)$ is a subsemihypergroup of S if and only if the soft set f_A, defined by*

$$f_A(x) = \begin{cases} \delta_1, & \text{if } x \in A, \\ \delta_2, & \text{if } x \notin A, \end{cases}$$

is an (M,N)-union soft subsemihypergroup of S over U, where $\delta_1, \delta_2 \subseteq U$ such that $M \subseteq \delta_1 \subseteq \delta_2 \subseteq N \subseteq U$.

Proof. Suppose A is a subsemihypergroup of S. Suppose $x, y \in S$. If $x, y \in A$, then $x \circ y \subseteq A$. We have to show that $\bigcup_{\beta \in x \circ y} f_A(\beta) \cap N \subseteq f_A(x) \cap f_A(y) \cup M$. Let $\beta \in x \circ y \subseteq A$. Then $f_A(\beta) = \delta_1$. Also $f_A(x) = \delta_1 = f_A(y)$. So $f_A(\beta) = \delta_1 = f_A(x) \cup f_A(y)$. Hence $\bigcup_{\beta \in x \circ y} f_A(\beta) \cap N = \delta_1 \cap N = \delta_1 = f_A(x) \cup f_A(y) \cup M$. If x or y is not in A, then $x \circ y \subseteq A$ or $x \circ y \nsubseteq A$. If $x \circ y \subseteq A$, then for $\beta \in x \circ y \subseteq A$, we have $f_A(\beta) \cap N = \delta_1 \cap N = \delta_1$. If $x \circ y \nsubseteq A$, then for $\beta \in x \circ y \nsubseteq A$, we have $f_A(\beta) \cap N = \delta_2 \cap N = \delta_2$. But $f_A(x) \cup f_A(y) \cup M = \delta_2 \cup M = \delta_2$. Thus, $\bigcup_{\beta \in x \circ y} f_A(\beta) \cap N \subseteq f_A(x) \cup f_A(y) \cup M$.

Conversely, assume that f_A is an (M,N)-union soft subsemihypergroup of S over U. Let $x, y \in A$. Then $f_A(x) = \delta_1 = f_A(y)$. By our supposition $\bigcup_{\beta \in x \circ y} f_A(\beta) \cap N \subseteq f_A(x) \cup f_A(y) \cup M = \delta_1 \cup M = \delta_1$. But $M \subseteq \delta_1 \subseteq \delta_2 \subseteq N$. So, $f_A(\beta) \subseteq \delta_1$ for every $\beta \in x \circ y$. Thus, $\beta \in A$. This implies that $x \circ y \subseteq A$. Hence, A is subsemihypergroup of S. □

Theorem 4.3. *If f_A and g_B are two (M,N)-union soft subsemihypergroup of S over U, then their union $f_A \cup g_B$ is an (M,N)-union soft subsemihypergroup of S over U.*

Proof. Let $x, y \in S$. Since f_A and g_B are two (M, N)-union soft subsemihypergroup of S over U. Then for every $\alpha \in x \circ y$, we have

$$\begin{aligned}(f_A \cup g_B)(\alpha) \cap N &= (f_A(\alpha) \cup g_B(\alpha)) \cap N \\ &= (f_A(\alpha) \cap N) \cup (g_B(\alpha) \cap N) \\ &\subseteq (f_A(x) \cup f_A(y) \cup M) \cup (g_B(x) \cup g_B(y) \cup M) \\ &= ((f_A(x) \cup g_B(x)) \cup (f_A(y) \cup g_B(y))) \cup M \\ &= (f_A \cup g_B)(x) \cup (f_A \cup g_B)(y) \cup M.\end{aligned}$$

Hence, $\bigcup_{\alpha \in x \circ y} (f_A \cup g_B)(\alpha) \cap N \subseteq (f_A \cup g_B)(x) \cup (f_A \cup g_B)(y) \cup M$. Therefore, $f_A \cup g_B$ is an (M, N)-union soft subsemihypergroup of S over U. □

Definition 4.2. A soft set f_A of an ordered semihypergroup S over U is called an (M, N)-union soft left (resp. right) hyperideal of S over U if it satisfies the following conditions:

(1) $\left(\bigcup_{\alpha \in x \circ y} f_A(\alpha)\right) \cap N \subseteq f_A(y) \cup M$ $\left(\text{resp. } \bigcup_{\alpha \in x \circ y} f_A(\alpha)) \cap N \subseteq f_A(x) \cup M\right)$;

(2) $x \leq y \Rightarrow f_A(x) \cap N \subseteq f_A(y) \cup M$,

for all $x, y \in S$.

A soft set f_A of an ordered semihypergroup S over U is called an (M, N)-*union soft* hyperideal of S over U if it is both an (M, N)-union soft left hyperideal and an (M, N)-union soft right hyperideal of S over U.

Example 4.2. Let $(S, \circ, \leq)$ be an ordered semihypergroup where the hyperoperation and the order relation are defined by:

$\circ$	1	2	3	4
1	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$
2	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$
3	$\{1\}$	$\{1\}$	$\{1\}$	$\{1, 2\}$
4	$\{1\}$	$\{1\}$	$\{1, 2\}$	$\{1, 2, 3\}$

,

$$\leq := \{(1,1), (2,2), (3,3), (4,4), (1,2), (1,3), (1,4), (2,4), (3,4)\}.$$

Suppose $U = \{h_1, h_2, h_3\}$, $A = \{1, 3, 4\}$, $M = \{h_1\}$ and $N = \{h_1, h_3\}$. Let us define $f_A(1) = \emptyset$, $f_A(2) = \{h_1\}$, $f_A(3) = \{h_1, h_2\}$ and $f_A(4) = \{h_1, h_2, h_3\}$. Then f_A is an (M, N)-union soft hyperideal of S over U.

Theorem 4.4. *Let $(S, \circ, \leq)$ be an ordered semihypergroup and $\emptyset \neq A \subseteq S$. Then A is a left (resp. right) hyperideal of S if and only if the soft set χ_A^c of A is an (M, N)-union soft left (resp. right) hyperideal of S over U.*

Proof. Suppose that A is a left hyperideal of S. Let $x, y \in S$. Then

$$\left(\bigcup_{\alpha \in x \circ y} \chi_A^c(\alpha)\right) \cap N \subseteq \chi_A^c(y) \cup M.$$

Indeed, if $y \notin A$ then $\chi_A^c(y) = U$. Since $\chi_A^c(x) \subseteq U$ for all $x \in S$ and $\emptyset \subseteq M \subset N \subseteq U$, we have

$$\left(\bigcup_{\alpha \in x \circ y} \chi_A^c(\alpha)\right) \cap N \subseteq U = \chi_A^c(y) \cup M.$$

Let $y \in A$. Since A is a left hyperideal of S and $x \in S$, we have $x \circ y \subseteq S \circ A \subseteq A$. Thus, in this case $\chi_A^c(\alpha) = \emptyset$ for any $\alpha \in x \circ y$. Hence,

$$\left(\bigcup_{\alpha \in x \circ y} \chi_A^c(\alpha)\right) \cap N = \emptyset \subseteq \chi_A^c(y) \cup M.$$

Let now $x, y \in S$, $x \leq y$. Then $\chi_A^c(x) \cap N \subseteq \chi_A^c(y) \cup M$. In fact, if $y \in A$, then $\chi_A^c(y) = \emptyset$. Since $S \ni x \leq y \in A$, by hypothesis we have $x \in A$, then $\chi_A^c(x) = \emptyset$. Thus $\chi_A^c(x) \cap N = \emptyset \subseteq M = \chi_A^c(y) \cup M$. If $y \notin A$, then $\chi_A^c(y) = U$. Since $x \in S$, $\emptyset \subseteq M \subset N \subseteq U$, we have $\chi_A^c(x) \cap N \subseteq U = \chi_A^c(y) \cup M$. Consequently, χ_A^c is an (M, N)-union soft left hyperideal of S over U.

Conversely, let A be a non-empty subset of S such that χ_A^c is an (M, N)-union soft left hyperideal of S over U. We claim that $S \circ A \subseteq A$. To prove our claim, let $x \in S$ and $y \in A$. By hypothesis,

$$\left(\bigcup_{\alpha \in x \circ y} \chi_A^c(\alpha)\right) \cap N \subseteq \chi_A^c(y) \cup M = \emptyset \cup M = M.$$

Thus, by $\emptyset \subseteq M \subset N \subseteq U$, $\bigcup_{\alpha \in x \circ y} \chi_A^c(\alpha) \cap N \subseteq M$. Hence for any $\alpha \in x \circ y$, $\chi_A^c(\alpha) = \emptyset$, i.e., $\alpha \in A$. It thus follows that $S \circ A \subseteq A$. Furthermore, let $x \in A$, $S \ni y \leq x$. Then $y \in A$. Indeed, it is enough to prove that $\chi_A^c(y) = \emptyset$. By $x \in A$, we have $\chi_A^c(x) = \emptyset$. Since χ_A^c is an (M, N)-union soft left hyperideal of S over U and $y \leq x$, we have $\chi_A^c(y) \cap N \subseteq \chi_A^c(x) \cup M = \emptyset \cup M = M$. Notice that $\emptyset \subseteq M \subset N \subseteq U$, we conclude that $\chi_A^c(y) = \emptyset$. Therefore, A is a left hyperideal of S.

Similarly we can show that χ_A^c is an (M, N)-union soft right hyperideal of S over U, if and only if A is a right hyperideal of S. □

Corollary 4.1. *Let $(S, \circ, \leq)$ be an ordered semihypergroup and $\emptyset \neq A \subseteq S$. Then A is a hyperideal of S if and only if the soft set χ_A^c of A is an (M, N)-union soft hyperideal of S over U.*

Theorem 4.5. *Let f_A be a soft set of an ordered semihypergroup S over U and $\delta \in P(U)$. Then f_A is an (M, N)-union soft hyperideal of S over U if and only if the nonempty δ-exclusive set $e_A(f_A, \delta)$ of f_A is a hyperideal of S and $M \subset \delta \subseteq N$.*

Proof. Assume that f_A is an (M, N)-union soft hyperideal of S over U. Let $x \in e_A(f_A, \delta)$ for $M \subset \delta \subseteq N$ and $y \in S$. Then $f_A(x) \subseteq \delta$. It follows from Definition 4.2, that

$$\left(\bigcup_{\alpha \in x \circ y} f_A(\alpha)\right) \cap N \subseteq f_A(x) \cup M \subseteq \delta \cup M = \delta$$

and

$$\left(\bigcup_{\alpha\in y\circ x} f_A(\alpha)\right)\cap N\subseteq f_A(x)\cup M\subseteq\delta\cup M=\delta.$$

Notice that $\delta\subseteq N$ we can deduce that $\bigcup_{\alpha\in x\circ y} f_A(\alpha)\subseteq\delta$ and $\bigcup_{\alpha\in y\circ x} f_A(\alpha)\subseteq\delta$. Thus it can be easily shown that $x\circ y\subseteq e_A(f_A,\delta)$ and $y\circ x\subseteq e_A(f_A,\delta)$. Furthermore, let $x\in e_A(f_A,\delta)$, $S\ni y\leq x$. Then $y\in e_A(f_A,\delta)$. Indeed, since $x\in e_A(f_A,\delta)$, $f_A(x)\subseteq\delta$ and f_A is an (M,N)-union soft hyperideal of S over U, we have $f_A(y)\cap N\subseteq f_A(x)\cup M\subseteq\delta\cup M=\delta$. By $\delta\subset N$, we have $f_A(y)\subseteq\delta$, i.e., $y\in e_A(f_A,\delta)$. Therefore, $e_A(f_A,\delta)$ is a hyperideal of S.

Conversely, let $e_A(f_A,\delta)\neq\emptyset$ be a hyperideal of S for all $M\subset\delta\subseteq N$. If there exist $x_1,y_1\in S$ such that

$$\left(\bigcup_{\alpha\in x_1\circ y_1} f_A(\alpha)\right)\cap N\supset f_A(y_1)\cup M,$$

then there exists $M\subset\delta\subseteq N$ such that

$$\left(\bigcup_{\alpha\in x_1\circ y_1} f_A(\alpha)\right)\cap N\supset\delta\supseteq f_A(y_1)\cup M$$

and we have $f_A(y_1)\subseteq\delta$ and $\bigcup_{\alpha\in x_1\circ y_1} f_A(\alpha)\supset\delta$. Thus, $y_1\in e_A(f_A,\delta)$ and $x_1\circ y_1\nsubseteq e_A(f_A,\delta)$, which is a contradiction. Hence,

$$\left(\bigcup_{\alpha\in x\circ y} f_A(\alpha)\right)\cap N\subseteq f_A(y)\cup M,$$

for all $x,y\in S$. Moreover if $x\leq y$ then $f_A(x)\cap N\subseteq f_A(y)\cup M$. Indeed, if there exist $x_1,y_1\in S$ such that $x_1\leq y_1$ and $f_A(x_1)\cap N\supset f_A(y_1)\cup M$ then there exists $M\subset\delta\subseteq N$ such that $f_A(x_1)\cap N\supset\delta\supseteq f_A(y_1)\cup M$ and we have $f_A(y_1)\subseteq\delta$ and $f_A(x_1)\supset\delta$. Then $y_1\in e_A(f_A,\delta)$ and $x_1\notin e_A(f_A,\delta)$. This is a contradiction that $e_A(f_A,\delta)$ is a hyperideal of S. Therefore f_A is an (M,N)-union soft left hyperideal of S over U. In a similar way we can show that f_A is an (M,N)-union soft right hyperideal of S over U and thus f_A is an (M,N)-union soft hyperideal of S over U. □

Theorem 4.6. *Let $(S,\circ,\leq)$ be an ordered semihypergroup and f_A be a soft set of S over U. Then f_A is an (M,N)-union soft left hyperideal of S over U if and only if f_A satisfies the following conditions:*

(1) $\emptyset_S\tilde{\diamond}f_A\tilde{\supseteq}_{[M,N]}f_A$;

(2) $(\forall x,y\in S)\ x\leq y\Rightarrow f_A(x)\cap N\subseteq f_A(y)\cup M$.

Proof. Suppose that f_A is an (M,N)-union soft left hyperideal of S over U. Then by Definition 4.2, condition (2) holds. To prove the condition (1) holds, it is enough to prove that $(\emptyset_S\tilde{\diamond}f_A)(x)\cup M\supseteq f_A(x)\cap N$ for any $x\in S$. Indeed, let $x\in S$. If $A_x=\emptyset$, then $(\emptyset_S\tilde{\diamond}f_A)(x)\cup M\supseteq f_A(x)\cap N$. Let $A_x\neq\emptyset$. Then there exist $y,z\in S$ such that

$x \leq y \circ z$ and there exists $v \in y \circ z$ such that $x \leq v$. Since f_A is an (M, N)-union soft left hyperideal of S over U, we have for any $x \leq y \circ z$. Thus,

$$
\begin{aligned}
((\emptyset_S \tilde{\diamond} f_A)(x) \cup M) \cap N &= \left(\left(\bigcap_{(y,z)\in A_x} \{\emptyset_S(y) \cup f_A(z)\}\right) \cup M\right) \cap N \\
&= \left(\left(\bigcap_{(y,z)\in A_x} \{\emptyset \cup f_A(z) \cup M\}\right) \cup M\right) \cap N \\
&= \left(\left(\bigcap_{(y,z)\in A_x} \{f_A(z) \cup M\}\right) \cup M\right) \cap N \\
&\supseteq \left(\left(\bigcap_{(y,z)\in A_x} \{f_A(x) \cap N\}\right) \cup M\right) \cap N \\
&= [\{f_A(x) \cap N\} \cup M] \cap N \\
&= (f_A(x) \cap N) \cup (M \cap N) \\
&= (f_A(x) \cap N) \cup M \\
&= (f_A(x) \cup M) \cap N.
\end{aligned}
$$

Thus, $\emptyset_S \tilde{\diamond} f_A \tilde{\supseteq}_{[M,N]} f_A$ for all $x \in S$.

Conversely, assume that the conditions (1) and (2) hold. Let $y, z \in S$. Then we can prove that $\bigcup_{x\in y\alpha z} f_A(x) \cap N \subseteq f_A(z) \cup M$ for any $x \in y \circ z$. In fact, since $x \in y \circ z$, $x \leq x$, we have $x \leq y \circ z$. Thus by hypothesis, we have

$$
\begin{aligned}
f_A(x) \cap N &\subseteq (f_A(x) \cap N) \cup M \\
&\subseteq ((\emptyset_S \tilde{\diamond} f_A)(x) \cap N) \cup M \\
&= \left(\left(\bigcap_{(p,q)\in A_x} \{\emptyset_S(p) \cup f_A(q)\}\right) \cap N\right) \cup M \\
&\subseteq (\{\emptyset_S(y) \cup f_A(z)\} \cap N) \cup M \\
&= (\{\emptyset \cup f_A(z)\} \cap N) \cup M \\
&= (f_A(z) \cap N) \cup M \\
&= (f_A(z) \cup M) \cap (N \cup M) \\
&= (f_A(z) \cup M) \cap N \\
&\subseteq f_A(z) \cup M.
\end{aligned}
$$

Hence, $\bigcup_{x\in y\alpha z} f_A(x) \cap N \subseteq f_A(z) \cup M$ for any $x \in y \circ z$. Hence, f_A is an (M, N)-union soft left hyperideal of S over U □

Similarly we can prove the following theorem.

Theorem 4.7. *Let $(S,\circ,\leq)$ be an ordered semihypergroup and f_A be a soft set of S over U. Then f_A is an (M,N)-union soft right hyperideal of S over U if and only if f_A satisfies the following conditions:*
(1) $f_A \tilde{\diamond} \emptyset_S \tilde{\supseteq}_{[M,N]} f_A$;
(2) $(\forall x,y\in S)$ $x\leq y \Rightarrow f_A(x)\cap N \subseteq f_A(y)\cup M$.

5. (M,N)-Union Soft Interior Hyperideals

In this section, we introduce the notion of (M,N)-union soft interior hyperideal of ordered semihypergroups and will study some related properties.

Definition 5.1. Let f_A be a soft set of an ordered semihypergroup S over U. Then f_A is called an (M,N)-union soft interior hyperideal of S over U if it satisfies the following conditions:

(1) $(\forall x,y\in S)\left(\bigcup_{\alpha\in x\circ y} f_A(\alpha)\right)\cap N \subseteq f_A(x)\cup f_A(y)\cup M$;

(2) $(\forall x,a,y\in S)\left(\bigcup_{\alpha\in x\circ a\circ y} f_A(\alpha)\right)\cap N \subseteq f_A(a)\cup M$;

(3) $(\forall x,y\in S)\, x\leq y \Rightarrow f_A(x)\cap N \subseteq f_A(y)\cup M$.

Example 5.1. Let $(S,\circ,\leq)$ be an ordered semihypergroup where the hyperoperation and the order relation are defined by:

$\circ$	a	b	c	d	e
a	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$
b	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$
c	$\{a,b\}$	$\{a,b\}$	$\{c\}$	$\{c\}$	$\{e\}$
d	$\{a,b\}$	$\{a,b\}$	$\{c\}$	$\{d\}$	$\{e\}$
e	$\{a,b\}$	$\{a,b\}$	$\{c\}$	$\{c\}$	$\{e\}$

,

$$\leq:=\{(a,a),(b,b),(c,c),(d,d),(e,e),(a,c),(a,d),(a,e),(b,c),(b,d),(b,e),(c,d),(c,e)\}.$$

Let $U=\{1,2,3\}$, $A=\{c,d,e\}$, $M=\{2\}$ and $N=\{1,2\}$. The soft set f_A is defined by

$$f_A=\begin{cases}\emptyset, & \text{if } x\in\{a,b\},\\ U, & \text{if } x\in\{c,d,e\}.\end{cases}$$

Then f_A is an (M,N)-union soft interior hyperideal of S over U.

Theorem 5.1. *Let $(S,\circ,\leq)$ be an ordered semihypergroup and A be a nonempty subset of S. Then A is an interior hyperideal of S if and only if the soft set χ_A^c of A is an (M,N)-union soft interior hyperideal of S over U.*

Proof. Suppose that A is an interior hyperideal of S. Let x, y and a be any elements of S. Then $\left(\bigcup_{\alpha\in x\circ a\circ y} \chi_A^c(\alpha)\right) \cap N \subseteq \chi_A^c(a) \cup M$. Indeed, if $a \in A$, then $\chi_A^c(a) = \emptyset$. Since A is an interior hyperideal of S, we have $\alpha \in x \circ a \circ y \subseteq S \circ A \circ S \subseteq A$ we have $\chi_A^c(\alpha) = \emptyset$ and $\emptyset \subseteq M \subset N \subseteq U$. Thus, $\left(\bigcup_{\alpha\in x\circ a\circ y} \chi_A^c(\alpha)\right) \cap N = \emptyset \subseteq \chi_A^c(a) \cup M$. If $a \notin A$, then $\chi_A^c(a) = U$. Since $\chi_A^c(x) \subseteq U$ for all $x \in S$, thus, $\left(\bigcup_{\alpha\in x\circ a\circ y} \chi_A^c(\alpha)\right) \cap N \subseteq U = \chi_A^c(a) \cup M$. Let $x, y \in S$ with $x \leq y$. Then $\chi_A^c(x) \cap N \subseteq \chi_A^c(y) \cup M$. Indeed, if $y \notin A$, then $\chi_A^c(y) = U$ and $\emptyset \subseteq M \subset N \subseteq U$ so $\chi_A^c(x) \cap N \subseteq U = \chi_A^c(y) \cup M$. If $y \in A$ then $\chi_A^c(y) = \emptyset$. Since $x \leq y$ and A is an interior hyperideal of S, we have $x \in A$ and thus $\chi_A^c(x) \cap N = \emptyset \subseteq \chi_A^c(y) \cup M$. Since A is an interior hyperideal of S., we have, A is a subsemihypergroup of S. Let $x, y \in S$. Then we have $\left(\bigcup_{\alpha\in x\circ y} \chi_A^c(\alpha)\right) \cap N \subseteq \chi_A^c(x) \cup \chi_A^c(y) \cup M$. Indeed, if $x \circ y \nsubseteq A$, then there exists $\alpha \in x \circ y$ such that $\alpha \notin A$, and we have $\bigcup_{\alpha\in x\circ y} \chi_A^c(\alpha) = U$. Besides that $x\circ y \nsubseteq A$ implies that $x \notin A$ or $y \notin A$. Then $\chi_A^c(x) = U$ or $\chi_A^c(y) = U$ and hence $\left(\bigcup_{\alpha\in x\circ y} \chi_A^c(\alpha)\right) \cap N \subseteq U = \chi_A^c(x) \cup \chi_A^c(y) \cup M$. Let $x\circ y \subseteq A$. Then $\chi_A^c(\alpha) = \emptyset$ for any $\alpha \in x\circ y$. It implies that $\bigcup_{\alpha\in x\circ y} \chi_A^c(\alpha) = \emptyset$. Since we have $\chi_A^c(x) \supseteq \emptyset$ for any $x \in A$, it follows, $\left(\bigcup_{\alpha\in x\circ y} \chi_A^c(\alpha)\right) \cap N = \emptyset \subseteq \chi_A^c(x) \cup \chi_A^c(y) \cup M$. Therefore, χ_A^c is an (M, N)-union soft interior hyperideal of S over U.

Conversely, let $\emptyset \neq A \subseteq S$ such that χ_A^c is an (M, N)-union soft interior hyperideal of S over U. We claim that $A \circ A \subseteq A$. To prove the claim, let $x, y \in A$. By hypothesis, $\left(\bigcup_{\alpha\in x\circ y} \chi_A^c(\alpha)\right) \cap N \subseteq \chi_A^c(x) \cup \chi_A^c(y) \cup M$, which implies that $\left(\bigcup_{\alpha\in x\circ y} \chi_A^c(\alpha)\right) \cap N \subseteq \emptyset \cap \emptyset \cup M = M$. Thus by $\emptyset \subseteq M \subset N \subseteq U$, $\bigcup_{\alpha\in x\circ y} \chi_A^c(\alpha) \cap N \subseteq M$. Thus for any $\alpha \in x \circ y$, $\chi_A^c(\alpha) = \emptyset$ implies that $\alpha \in A$. It thus follows that $A \circ A \subseteq A$. Let $\alpha \in S \circ A \circ S$, then there exist $x, y \in S$ and $a \in A$ such that $\alpha \in x \circ a \circ y$. Since $\left(\bigcup_{\alpha\in x\circ a\circ y} \chi_A^c(\alpha)\right) \cap N \subseteq \chi_A^c(a) \cup M$, and $a \in A$ we have $\chi_A^c(a) = \emptyset$. Hence for each $\alpha \in S \circ A \circ S$, we have $\left(\bigcup_{\alpha\in x\circ a\circ y} \chi_A^c(\alpha)\right) \cap N \subseteq \emptyset \cup M = M$. Thus, by $\emptyset \subseteq M \subset N \subseteq U$, $\bigcup_{\alpha\in x\circ a\circ y} \chi_A^c(\alpha) \cap N \subseteq M$. Thus, for any $\alpha \in x \circ a \circ y$, $\chi_A^c(\alpha) = \emptyset$

implies that $\alpha \in A$. Thus $S \circ A \circ S \subseteq A$. Furthermore, let $x \in A$, $S \ni y \leq x$. Then $y \in A$. Indeed, it is enough to prove that $\chi_A^c(y) = \emptyset$. By $x \in A$ we have $\chi_A^c(x) = \emptyset$. Since χ_A^c is an (MN)-union soft interior hyperideal of S over U and $y \leq x$, we have $\chi_A^c(y) \cap N \subseteq \chi_A^c(x) \cup M = \emptyset \cup M = M$. Notice that $\emptyset \subseteq M \subset N \subseteq U$, we conclude that $\chi_A^c(y) = \emptyset$. Hence $y \in A$. Therefore A is a interior hyperideal of S. □

Theorem 5.2. *Let f_A be a soft set of an ordered semihypergroup S over U and $\delta \in P(U)$. Then f_A is an (M,N)-union soft interior hyperideal of S over U if and only if each nonempty δ-exclusive set $e_A(f_A,\delta)$ of f_A is an interior hyperideal of S and $M \subset \delta \subseteq N$.*

Proof. Assume that f_A is an (M,N)-union soft interior hyperideal of S over U. Let $M \subset \delta \subseteq N$ and $e_A(f_A,\delta) \neq \emptyset$. Let $x, y \in e_A(f_A,\delta)$. Then $f_A(x) \subseteq \delta$ and $f_A(y) \subseteq \delta$. By hypothesis, we have $\left(\bigcup_{\alpha \in x \circ y} f_A(\alpha)\right) \cap N \subseteq f_A(x) \cup f_A(y) \cup M \subseteq \delta \cup \delta \cup M = \delta$. Since $M \subset \delta \subseteq N$, we can write as $\bigcup_{\alpha \in x \circ y} f_A(\alpha) \subseteq \delta$. Thus for any $\alpha \in x \circ y$, we have $f_A(\alpha) \subseteq \delta$, implies that $\alpha \in e_A(f_A,\delta)$. It follows that $x \circ y \subseteq e_A(f_A,\delta)$. Hence $e_A(f_A,\delta)$ is a subsemihypergroup of S. Let $y \in e_A(f_A,\delta)$ and $x, z \in S$. Then $f_A(y) \subseteq \delta$. Since f_A is an (M,N)-union soft interior hyperideal of S over U. Thus, $\left(\bigcup_{w \in x \circ y \circ z} f_A(w)\right) \cap N \subseteq f_A(y) \cup M \subseteq \delta \cup M = \delta$. Since $\emptyset \subseteq M \subset \delta \subseteq N \subseteq U$, we can write as $\bigcup_{w \in x \circ y \circ z} f_A(w) \subseteq \delta$. Hence, $f_A(w) \subseteq \delta$ for any $w \in x \circ y \circ z$ implies that $w \in e_A(f_A,\delta)$. Thus, $S \circ e_A(f_A,\delta) \circ S \subseteq e_A(f_A,\delta)$. Furthermore, let $x \in e_A(f_A,\delta)$, $S \ni y \leq x$. Then $y \in e_A(f_A,\delta)$. Indeed, since $x \in e_A(f_A,\delta)$, $f_A(x) \subseteq \delta$ and f_A is an (M,N)-union soft interior hyperideal of S over U, we have $f_A(y) \cap N \subseteq f_A(x) \cup M \subseteq \delta \cup M = \delta$. By $M \subset \delta \subseteq N$, we have $f_A(y) \subseteq \delta$, i.e., $y \in e_A(f_A,\delta)$. Therefore, $e_A(f_A,\delta)$ is an interior hyperideal of S.

Conversely, suppose that $e_A(f_A,\delta) \neq \emptyset$ is an interior hyperideal of S for all $M \subset \delta \subseteq N$. If there exist $x_1, y_1 \in S$ such that $\left(\bigcup_{\alpha \in x_1 \circ y_1} f_A(\alpha)\right) \cap N \supset f_A(x_1) \cup f_A(y_1) \cup M$, then there exists $M \subset \delta \subseteq N$ such that $\left(\bigcup_{\alpha \in x_1 \circ y_1} f_A(\alpha)\right) \cap N \supset \delta \supseteq f_A(x_1) \cup f_A(y_1) \cup M$, and we have $f_A(x_1) \subseteq \delta$, $f_A(y_1) \subseteq \delta$ and $\bigcup_{\alpha \in x_1 \circ y_1} f_A(\alpha) \supset \delta$ which implies that $x_1, y_1 \in e_A(f_A,\delta)$ and $x_1 \circ y_1 \nsubseteq e_A(f_A,\delta)$. It contradicts the fact that $e_A(f_A,\delta)$ is an interior hyperideal of S. Consequently, $\left(\bigcup_{\alpha \in x \circ y} f_A(\alpha)\right) \cap N \subseteq f_A(x) \cup f_A(y) \cup M$ for all $x, y \in S$. Next we show that $\left(\bigcup_{\alpha \in x \circ a \circ y} f_A(\alpha)\right) \cap N \subseteq f_A(a) \cup M$ for all $x, a, y \in S$. If there exist

x_1, a_1, y_1 such that $\left(\bigcup_{\alpha\in x_1\circ a_1\circ y_1} f_A(\alpha)\right)\cap N \supset f_A(a_1)\cup M$, and $M\subset\delta\subseteq N$ such that $\left(\bigcup_{\alpha\in x_1\circ a_1\circ y_1} f_A(\alpha)\right)\cap N \supset \delta \supseteq f_A(a_1)\cup M$, so $f_A(a_1)\subseteq\delta$ and $\bigcup_{\alpha\in x_1\circ a_1\circ y_1} f_A(\alpha)\supset\delta$ then $a_1\in e_A(f_A,\delta)$ and $x_1\circ a_1\circ y_1\nsubseteq e_A(f_A,\delta)$. This is a contradiction that $e_A(f_A,\delta)$ is an interior hyperideal of S. Moreover if $x\le y$, then $f_A(x)\cap N\subseteq f_A(y)\cup M$. Indeed, if there exist $x_1, y_1\in S$ such that $x_1\le y_1$ and $f_A(x_1)\cap N\supset f_A(y_1)\cup M$, then there exists $M\subset\delta\subseteq N$ such that $f_A(x_1)\cap N\supset\delta\supseteq f_A(y_1)\cup M$ and we have $f_A(y_1)\subseteq\delta$ and $f_A(x_1)\supset\delta$. Then $y_1\in e_A(f_A,\delta)$ and $x_1\notin e_A(f_A,\delta)$. This is a contradiction that $e_A(f_A,\delta)$ is an interior hyperideal of S. Thus if $x\le y$ then $f_A(x)\cap N\subseteq f_A(y)\cup M$. □

Theorem 5.3. *Let $(S,\circ,\le)$ be an ordered semihypergroup and f_A be an (M,N)-union soft hyperideal of S over U. Then f_A is an (M,N)-union soft interior hyperideal of S over U.*

Proof. Suppose that f_A is an (M,N)-union soft hyperideal of S over U. Let $x,y\in S$. Then by hypothesis $\left(\bigcup_{\alpha\in x\circ y} f_A(\alpha)\right)\cap N\subseteq f_A(x)\cup M\subseteq f_A(x)\cup f_A(y)\cup M$. Let $x,a,y\in S$. Since f_A is an (M,N)-union soft hyperideal of S over U, then for any $\alpha\in x\circ a\circ y$, and $\emptyset\subseteq M\subset N\subseteq U$ we have

$$\begin{aligned}
\left(\bigcup_{\alpha\in x\circ a\circ y} f_A(\alpha)\right)\cap N &= \left(\left(\bigcup_{\alpha\in x\circ a\circ y} f_A(\alpha)\right)\cap N\right)\cap N\\
&= \left(\left(\bigcup_{\substack{\alpha\in x\circ\beta\\ \beta\in a\circ y}} f_A(\alpha)\right)\cap N\right)\cap N\\
&\subseteq (f_A(\beta)\cup M)\cap N\\
&= (f_A(\beta)\cap N)\cup(N\cap M) = (f_A(\beta)\cap N)\cup M\\
&\subseteq \left(\left(\bigcup_{\beta\in a\circ y} f_A(\beta)\right)\cap N\right)\cup M\\
&\subseteq (f_A(a)\cup M)\cup M\\
&= f_A(a)\cup M.
\end{aligned}$$

Thus,

$$\left(\bigcup_{\alpha\in x\circ a\circ y} f_A(\alpha)\right)\cap N\subseteq f_A(a)\cup M.$$

Therefore, f_A is an (M,N)-union soft interior hyperideal of S over U. □

The converse of above theorem is not true in general. We can illustrate it by the following example.

Example 5.2. Let $(S, \circ, \leq)$ be an ordered semihypergroup where the hyperoperation and the order relation are defined by:

$\circ$	v_1	v_2	v_3	v_4
v_1	$\{v_1\}$	$\{v_1\}$	$\{v_1\}$	$\{v_1\}$
v_2	$\{v_1\}$	$\{v_1\}$	$\{v_1\}$	$\{v_1\}$
v_3	$\{v_1\}$	$\{v_1\}$	$\{v_1, v_2\}$	$\{v_1, v_2\}$
v_4	$\{v_1\}$	$\{v_1\}$	$\{v_1, v_2\}$	$\{v_1\}$

,

$\leq := \{(v_1, v_1), (v_2, v_2), (v_3, v_3), (v_4, v_4), (v_1, v_2), (v_1, v_3), (v_1, v_4), (v_4, v_2), (v_4, v_3)\}$.

Suppose $U = \{x, y, z\}$, $A = \{v_2, v_3\}$, $M = \{y\}$ and $N = \{y, z\}$. Let us define $f_A(v_1) = \emptyset$, $f_A(v_2) = \{x, z\}$, $f_A(v_3) = \{x, y, z\}$ and $f_A(v_4) = \emptyset$. Then f_A is an (M, N)-union soft interior hyperideal of S over U. This is not an (M, N)-union soft left hyperideal as

$$\bigcup_{\alpha \in v_3 \circ v_4 = \{v_1, v_2\}} f_A(\alpha) \cap N = f_A(v_1) \cup f_A(v_2) \cap N = \{z\} \nsubseteq \emptyset \cup \{y\} = \{y\} = f_A(v_4) \cup M.$$

Theorem 5.4. *Let $(S, \circ, \leq)$ be a regular ordered semihypergroup and f_A is an (M, N)-union soft interior hyperideal of S over U. Then f_A is an (M, N)-union soft hyperideal of S over U.*

Proof. Let $x, y \in S$. Since f_A is an (M, N)-union soft interior hyperideal of S over U, then $\left(\bigcup_{\alpha \in x \circ y} f_A(\alpha)\right) \cap N \subseteq f_A(x) \cup M$. Indeed, since S is regular and $x \in S$, then there exists $z \in S$ such that $x \leq x \circ z \circ x$. Then we have $x \circ y \leq (x \circ z \circ x) \circ y = (x \circ z) \circ (x \circ y)$. So, there exist $\alpha \in x \circ y$, $v \in x \circ z$ and $\beta \in v \circ x \circ y$ such that $\alpha \leq \beta$. So $f_A(\alpha) \cap N \subseteq f_A(\beta) \cup M$. Since f_A is an (M, N)-union soft interior hyperideal of S over U, and $\emptyset \subseteq M \subset N \subseteq U$, we have

$$\begin{aligned} f_A(\alpha) \cap N &= (f_A(\alpha) \cap N) \cap N \\ &\subseteq (f_A(\beta) \cup M) \cap N \\ &= (f_A(\beta) \cap N) \cup (N \cap M) = (f_A(\beta) \cap N) \cup M \\ &\subseteq \left(\left(\bigcup_{\beta \in v \circ x \circ y} f_A(\beta)\right) \cap N\right) \cup M \subseteq (f_A(x) \cup M) \cup M \\ &= f_A(x) \cup M. \end{aligned}$$

Thus,

$$\left(\bigcup_{\alpha \in x \circ y} f_A(\alpha)\right) \cap N \subseteq f_A(x) \cup M.$$

Therefore f_A is an (M, N)-union soft right hyperideal of S over U. In a similar way we prove that f_A is an (M, N)-union soft left hyperideal of S over U. $\square$

By Theorem 5.3 and 5.4 we have the following.

Theorem 5.5. *In regular ordered semihypergroups the concepts of* (M, N)*-union soft hyperideals and* (M, N)*-union soft interior hyperideals coincide.*

Theorem 5.6. *Let* $(S, \circ, \leq)$ *be an intra-regular ordered semihypergroup and* f_A *is an* (M, N)*-union soft interior hyperideal of* S *over* U*. Then* f_A *is an* (M, N)*-union soft hyperideal of* S *over* U*.*

Proof. Let $a, b \in S$. Then $\left(\bigcup_{u \in a \circ b} f_A(u)\right) \cap N \subseteq f_A(a) \cup M$. Indeed, since S is intra-regular and $a \in S$, there exist $x, y \in S$ such that $a \leq x \circ a \circ a \circ y$. Then $a \circ b \leq (x \circ a \circ a \circ y) \circ b = x \circ a \circ (a \circ y \circ b)$. So there exist $u \in a \circ b$, $v \in a \circ y \circ b$ and $\alpha \in x \circ a \circ v$ such that $u \leq \alpha$. So $f_A(u) \cap N \subseteq f_A(\alpha) \cup M$. Since f_A is an (M, N)-union soft interior hyperideal of S over U, we have

$$\begin{aligned}
f_A(u) \cap N =& (f_A(u) \cap N) \cap N \\
\subseteq& (f_A(\alpha) \cup M) \cap N \\
=& (f_A(\alpha) \cap N) \cup (N \cap M) = (f_A(\alpha) \cap N) \cup M \\
\subseteq& \left(\left(\bigcup_{\alpha \in x \circ a \circ v} f_A(\alpha)\right) \cap N\right) \cup M \subseteq (f_A(a) \cup M) \cup M \\
=& f_A(a) \cup M.
\end{aligned}$$

Thus,

$$\left(\bigcup_{u \in a \circ b} f_A(u)\right) \cap N \subseteq f_A(a) \cup M.$$

Hence, f_A is an (M, N)-union soft right hyperideal of S over U. Similarly we can prove that f_A is an (M, N)-union soft left hyperideal of S over U. Therefore, f_A is an (M, N)-union soft hyperideal of S over U. □

By Theorem 5.3 and 5.6, we have the following.

Theorem 5.7. *In intra-regular ordered semihypergroups the concepts of* (M, N)*-union soft hyperideals and* (M, N)*-union soft interior hyperideals coincide.*

6. Characterizations of (M, N)-Union Soft Simple Ordered Semihypergroups in Terms of (M, N)-Union Soft Hyperideals and (M, N)-Union Soft Interior Hyperideals

In this section, we introduce the concept of (M, N)-union soft simple ordered semihypergroups and characterize this type of ordered semihypergroups in terms of (M, N)-union soft hyperideals and (M, N)-union soft interior hyperideals.

Definition 6.1 (see [16])**.** An ordered semihypergroup $(S, \circ, \leq)$ is called simple if it has no a proper hyperideal, that is for any hyperideal $A \neq \emptyset$ of S we have $A = S$.

Lemma 6.1 (see [16])**.** *An ordered semihypergroup* $(S, \circ, \leq)$ *is a simple ordered semihypergroup if and only if for every* $a \in S$, $(S \circ a \circ S] = S$.

Definition 6.2. An ordered semihypergroup $(S, \circ, \leq)$ is called (M, N)-union soft simple if for any (M, N)-union soft hyperideal f_A of S over U, we have $f_A(a) \cap N \subseteq f_A(b) \cup M$ for all $a, b \in S$.

Theorem 6.1. *Let be* $(S, \circ, \leq)$ *an ordered semihypergroup. Then* S *is* (M, N)*-union soft simple if and only if for any* (M, N)*-union soft hyperideal* f_A *of* S *over* U, *we have* $e_A(f_A, \delta) = S$ *for all* $\emptyset \subseteq M \subset \delta \subseteq N \subseteq U$ *if* $e_A(f_A, \delta) \neq \emptyset$.

Proof. Suppose that S is an (M, N)-union soft simple ordered semihypergroup and f_A is an (M, N)-union soft hyperideal of S over U. Let $M \subset \delta \subseteq N$ be such that $e_A(f_A, \delta) \neq \emptyset$. We need to prove that $x \in e_A(f_A, \delta)$ for all $x \in S$. Since $e_A(f_A, \delta) \neq \emptyset$, we can suppose that there exits $y \in e_A(f_A, \delta)$, i.e., $f_A(y) \subseteq \delta$. Hence $f_A(x) \cap N \subseteq f_A(y) \cup M \subseteq \delta \cup M = \delta$. Since $M \subset \delta$, we can conclude that $f_A(x) \subseteq \delta$, which implies that $x \in e_A(f_A, \delta)$.

Conversely, for any (M, N)-union soft hyperideal f_A of S over U, suppose that $e_A(f_A, \delta) = S$ for all $\emptyset \subseteq M \subset \delta \subseteq N \subseteq U$ if $e_A(f_A, \delta) \neq \emptyset$. We claim that $f_A(a) \cap N \subseteq f_A(b) \cup M$ for all $a, b \in S$. If there exist $x, y \in S$ such that $f_A(x) \cap N \supset f_A(y) \cup M$, then we have $f_A(x) \cap N \supset \delta \supseteq f_A(y) \cup M$ for some $M \subset \delta \subseteq N$. Thus, $f_A(x) \supset \delta$, i.e., $x \notin e_A(f_A, \delta) = S$, which is a contradiction. Therefore $f_A(a) \cap N \subseteq f_A(b) \cup M$ holds for all $a, b \in S$. Thus, S is (M, N)-union soft simple. □

Let $(S, \circ, \leq)$ be an ordered semihypergroup and $a \in S$, and f_A be a soft set of S over U we denote by I_a the subset of S defines as follows:

$$I_a = \{b \in S \mid f_A(b) \cap N \subseteq f_A(a) \cup M\}.$$

Clearly $I_a \neq \emptyset$, since $a \in I_a$.

Theorem 6.2. *Let* $(S, \circ, \leq)$ *be an ordered semihypergroup and* f_A *is an* (M, N)*-union soft left hyperideals of* S *over* U. *Then the set* I_a *is a left hyperideal of* S *for every* $a \in S$.

Proof. Suppose that f_A is an (M, N)-union soft left hyperideals of S over U. Let $b \in I_a$ and $s \in S$. Then $s \circ b \subseteq I_a$. Indeed, since f_A is an (M, N)-union soft left hyperideal of S over U and $b, s \in S$, we have $\left(\bigcup_{\alpha \in s \circ b} f_A(\alpha)\right) \cap N \subseteq f_A(b) \cup M$. Since $b \in I_a$, we have $f_A(b) \cap N \subseteq f_A(a) \cup M$. Thus,

$$\begin{aligned} f_A(\alpha) \cap N =& (f_A(\alpha) \cap N) \cap N \\ \subseteq& \left(\left(\bigcup_{\alpha \in s \circ b} f_A(\alpha)\right) \cap N\right) \cap N \subseteq (f_A(b) \cup M) \cap N \\ =& (f_A(b) \cap N) \cup (M \cap N) \end{aligned}$$

$$\begin{aligned}&\subseteq (f_A(a) \cup M) \cup M\\&= f_A(a) \cup M.\end{aligned}$$

Thus, $\alpha \in I_a$ and hence $s \circ b \subseteq I_a$. Let $b \in I_a$ and $S \ni s \leq b$. Then $s \in I_a$. Indeed, since f_A is an (M, N)-union soft left hyperideals of S over U, $b, s \in S$ and $s \leq b$, we have $f_A(s) \cap N \subseteq f_A(b) \cup M$. Since $b \in I_a$, we have $f_A(b) \cap N \subseteq f_A(a) \cup M$. Then $f_A(s) \cap N \subseteq f_A(a) \cup M$, so $s \in I_a$. □

In a similar way we prove the following.

Theorem 6.3. *Let $(S, \circ, \leq)$ be an ordered semihypergroup and f_A is an (M, N)-union soft right hyperideals of S over U. Then the set I_a is a right hyperideal of S for every $a \in S$.*

By Theorem 6.2 and 6.3 we have the following.

Theorem 6.4. *Let $(S, \circ, \leq)$ be an ordered semihypergroup and f_A is an (M, N)-union soft hyperideals of S over U. Then the set I_a is a hyperideal of S for every $a \in S$.*

Theorem 6.5. *Let $(S, \circ, \leq)$ be an ordered semihypergroup. Then S is simple if and only if it is (M, N)-union soft simple.*

Proof. Assume that S is a simple ordered semihypergroup. Let f_A is an (M, N)-union soft hyperideal of S over U and $a, b \in S$. By Theorem 6.4, we obtain I_a is a hyperideal of S. Since S is simple, $I_a = S$. Then $b \in I_a$, that is $f_A(b) \cap N \subseteq f_A(a) \cup M$. Therefore, S is (M, N)-union soft simple.

Conversely, suppose that S is (M, N)-union soft simple. Let I be a hyperideal of S. By Corollary 4.1, we obtain the characteristic function χ_I^c is an (M, N)-union soft hyperideal of S over U. We claim that $I = S$. To prove our claim, let $x \in S$. Since S is (M, N)-union soft simple, $\chi_I^c(x) \cap N \subseteq \chi_I^c(y) \cup M$ for all $y \in S$. Since $I \neq \emptyset$, let $a \in I$. Then $\chi_I^c(x) \cap N \subseteq \chi_I^c(a) \cup M = \emptyset \cup M = M$. So, $\chi_I^c(x) \cap N \subseteq M$. Since $M \subset N$, we conclude that $\chi_I^c(x) = \emptyset$, i.e., $x \in I$. Thus, we have shown that $S \subseteq I$, and so, $S = I$. Hence, S is simple. □

Theorem 6.6. *Let $(S, \circ, \leq)$ be an ordered semihypergroup. Then S is a simple if and only if for every (M, N)-union soft interior hyperideal f_A of S over U, we have $f_A(a) \cap N \subseteq f_A(b) \cup M$ for all $a, b \in S$.*

Proof. Suppose that S is a simple ordered semihypergroup. Let f_A be an (M, N)-union soft interior hyperideal of S over U and $a, b \in S$. By Lemma 6.1, we have $S = (S \circ b \circ S]$. Thus by $a \in S$, we have $a \in (S \circ b \circ S]$. Then there exist $x, y \in S$ such that $a \leq x \circ b \circ y$. Then $a \leq \alpha$ for some $\alpha \in x \circ b \circ y$. Since f_A is an (M, N)-union soft interior hyperideal of S over U, we have $f_A(a) \cap N \subseteq f_A(\alpha) \cup M$. Also since $\left(\bigcup_{\alpha \in x \circ b \circ y} f_A(\alpha) \right) \cap N \subseteq f_A(b) \cup M$. Thus,

$$f_A(a) \cap N = (f_A(a) \cap N) \cap N$$

$$\begin{aligned}
&\subseteq (f_A(\alpha) \cup M) \cap N \\
&= (f_A(\alpha) \cap N) \cup (M \cap N) = (f_A(\alpha) \cap N) \cup M \\
&\subseteq \left(\left(\bigcup_{\alpha \in x \circ b \circ y} f_A(\alpha) \right) \cap N \right) \cup M \subseteq (f_A(b) \cup M) \cup M \\
&= f_A(b) \cup M.
\end{aligned}$$

Conversely, assume that for every (M, N)-union soft interior hyperideal f_A of S over U, we have $f_A(a) \cap N \subseteq f_A(b) \cup M$ for all $a, b \in S$. Let f_A be any (M, N)-union soft hyperideal of S over U. Then by Theorem 5.3, f_A is an (M, N)-union soft interior hyperideal of S over U. Hence S is (M, N)-union soft simple by Definition 6.2. It thus follows from Theorem 6.5 that S is a simple ordered semihypergroup. □

As a consequence of Lemma 6.1, Theorem 6.5, and Theorem 6.6, we present characterizations of a simple ordered semihypergroup as the following theorem.

Theorem 6.7. *Let $(S, \circ, \leq)$ be an ordered semihypergroup. Then the following statements are equivalent:*

(1) *S is a simple ordered semihypergroup;*

(2) *$S = (S \circ a \circ S]$ for every $a \in S$;*

(3) *S is (M, N)-union soft simple;*

(4) *for every (M, N)-union soft interior hyperideal of S over U, we have $f_A(a) \cap N \subseteq f_A(b) \cup M$ for all $a, b \in S$.*

7. Conclusion

Ideal theory play a vital role in hyperstructures, in this paper, we introduced the notions of (M, N)-union soft hyperideals and (M, N)-union soft interior hyperideals of ordered semihypergroups and studied them. When $M = \emptyset$ and $N = U$, we meet union soft hyperideals and union soft interior hyperideals. From this view, we say that (M, N)-union soft hyperideals and (M, N)-union soft interior hyperideals are more general concepts than ordinary union soft ones. Moreover we introduced the notion of (M, N)-union soft simple ordered semihypergroup. We characterized (M, N)-union soft simple ordered semihypergroups by means of (M, N)-union soft hyperideals and (M, N)-union soft interior hyperideals. Hopefully that the obtained new characterizations of ordered semihypergroup in terms of (M, N)-union soft hyperideals will be very useful for future study of ordered semihypergroups. In future we will define other (M, N)-union soft hyperideals of ordered semihypergroups and will study their applications.

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