

**A SURVEY ON STRONGLY REGULAR GRAPHS WITH  $m_2 = qm_3$   
AND  $m_3 = qm_2$**

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**ABSTRACT.** We say that a regular graph  $G$  of order  $n$  and degree  $r \geq 1$  (which is not the complete graph) is strongly regular if there exist non-negative integers  $\tau$  and  $\theta$  such that  $|S_i \cap S_j| = \tau$  for any two adjacent vertices  $i$  and  $j$ , and  $|S_i \cap S_j| = \theta$  for any two distinct non-adjacent vertices  $i$  and  $j$ , where  $S_k$  denotes the neighborhood of the vertex  $k$ . Let  $\lambda_1 = r$ ,  $\lambda_2$  and  $\lambda_3$  be the distinct eigenvalues of a connected strongly regular graph. Let  $m_1 = 1$ ,  $m_2$  and  $m_3$  denote the multiplicity of  $r$ ,  $\lambda_2$  and  $\lambda_3$ , respectively. We here survey results related to the parameters  $n$ ,  $r$ ,  $\tau$  and  $\theta$  for strongly regular graphs with  $m_2 = qm_3$  and  $m_3 = qm_2$  for  $q = 2, 3, \dots, 12$ .

1. INTRODUCTION

Let  $G$  be a simple graph of order  $n$  with vertex set  $V(G) = \{1, 2, \dots, n\}$ . The spectrum of  $G$  consists of the eigenvalues  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$  of its  $(0,1)$  adjacency matrix  $A$  and is denoted by  $\sigma(G)$ . We say that a regular graph  $G$  of order  $n$  and degree  $r \geq 1$  (which is not the complete graph  $K_n$ ) is strongly regular if there exist non-negative integers  $\tau$  and  $\theta$  such that  $|S_i \cap S_j| = \tau$  for any two adjacent vertices  $i$  and  $j$ , and  $|S_i \cap S_j| = \theta$  for any two distinct non-adjacent vertices  $i$  and  $j$ , where  $S_k \subseteq V(G)$  denotes the neighborhood of the vertex  $k$ . We know that a regular connected graph  $G$  is strongly regular if and only if it has exactly three distinct eigenvalues [1] (see also [3]). Let  $\lambda_1 = r$ ,  $\lambda_2$  and  $\lambda_3$  denote the distinct eigenvalues of a connected strongly regular graph  $G$ . Let  $m_1 = 1$ ,  $m_2$  and  $m_3$  denote the multiplicity of  $r$ ,  $\lambda_2$  and  $\lambda_3$ . Further, let  $\bar{r} = (n - 1) - r$ ,  $\bar{\lambda}_2 = -\lambda_3 - 1$  and  $\bar{\lambda}_3 = -\lambda_2 - 1$  denote the distinct

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eigenvalues of the strongly regular graph  $\overline{G}$ , where  $\overline{G}$  denotes the complement of  $G$ . Then  $\overline{\tau} = n - 2r - 2 + \theta$  and  $\overline{\theta} = n - 2r + \tau$ , where  $\overline{\tau} = \tau(\overline{G})$  and  $\overline{\theta} = \theta(\overline{G})$ .

*Remark 1.1.* (i) If  $G$  is a disconnected strongly regular graph of degree  $r$ , then  $G = mK_{r+1}$ , where  $mH$  denotes the  $m$ -fold union of the graph  $H$ .

(ii)  $G$  is a disconnected strongly regular graph if and only if  $\theta = 0$ .

*Remark 1.2.* (i) A strongly regular graph  $G$  of order  $n = 4k + 1$  and degree  $r = 2k$  with  $\tau = k - 1$  and  $\theta = k$  is called a conference graph.

(ii) A strongly regular graph is a conference graph if and only if  $m_2 = m_3$ .

(iii) If  $m_2 \neq m_3$ , then  $G$  is an integral graph.

Note. We say that a connected or disconnected graph  $G$  is integral if its spectrum  $\sigma(G)$  consists only of integral values.

We have recently started to investigate strongly regular graphs with  $m_2 = qm_3$  and  $m_3 = qm_2$ , where  $q$  is a positive integer [4]. In the same work we have described the parameters  $n, r, \tau$  and  $\theta$  for strongly regular graphs with  $m_2 = qm_3$  and  $m_3 = qm_2$  for  $q = 2, 3, 4$ . In particular, we have described in [5], [6] and [7] the parameters  $n, r, \tau$  and  $\theta$  for strongly regular graphs with  $m_2 = qm_3$  and  $m_3 = qm_2$  for  $q = 5, 6, \dots, 12$ . We here survey results related to the parameters of strongly regular graphs with  $m_2 = qm_3$  and  $m_3 = qm_2$  for  $q = 2, 3, \dots, 12$ , as follows.

**Proposition 1.1** ([2]). *Let  $G$  be a connected or disconnected strongly regular graph of order  $n$  and degree  $r$ . Then,*

$$(1.1) \quad r^2 - (\tau - \theta + 1)r - (n - 1)\theta = 0.$$

**Proposition 1.2** ([2]). *Let  $G$  be a connected strongly regular graph of order  $n$  and degree  $r$ . Then,*

$$(1.2) \quad 2r + (\tau - \theta)(m_2 + m_3) + \delta(m_2 - m_3) = 0,$$

where  $\delta = \lambda_2 - \lambda_3$ .

*Remark 1.3* ([4]). Using the same procedure applied in [4] we can establish the parameters  $n, r, \tau$  and  $\theta$  for strongly regular graphs with  $m_2 = qm_3$  and  $m_3 = qm_2$  for any fixed value  $q \in \mathbb{N}$ , as follows. First, let  $m_3 = p$ ,  $m_2 = qp$  and  $n = (q + 1)p + 1$ , where  $q \in \mathbb{N}$ . Using (1.2) we obtain  $r = p(|\lambda_3| - q\lambda_2)$ . Let  $|\lambda_3| - q\lambda_2 = t$  where  $t = 1, 2, \dots, q$ . Let  $\lambda_2 = k$  where  $k$  is a positive integer. Then, (i)  $\lambda_3 = -(qk + t)$ ; (ii)  $\tau - \theta = -((q - 1)k + t)$ ; (iii)  $\delta = (q + 1)k + t$ ; (iv)  $r = pt$  and (v)  $\theta = pt - qk^2 - kt$ . Using (ii), (iv) and (v) we can easily see that (1.1) is reduced to

$$(1.3) \quad (p + 1)t^2 - ((q + 1)p + 1)t + q(q + 1)k^2 + 2qkt = 0.$$

Second, let  $m_2 = p$ ,  $m_3 = qp$  and  $n = (q + 1)p + 1$ , where  $q \in \mathbb{N}$ . Using (1.2) we obtain  $r = p(q|\lambda_3| - \lambda_2)$ . Let  $q|\lambda_3| - \lambda_2 = t$ , where  $t = 1, 2, \dots, q$ . Let  $\lambda_3 = -k$ , where  $k$  is a positive integer. Then, (i)  $\lambda_2 = qk - t$ ; (ii)  $\tau - \theta = (q - 1)k - t$ ; (iii)  $\delta = (q + 1)k - t$ ;

(iv)  $r = pt$  and (v)  $\theta = pt - qk^2 + kt$ . Using (ii), (iv) and (v) we can easily see that (1.1) is reduced to

$$(1.4) \quad (p + 1)t^2 - ((q + 1)p + 1)t + q(q + 1)k^2 - 2qkt = 0.$$

Using (1.3) and (1.4) we can obtain for  $t = 1, 2, \dots, q$  the corresponding classes of strongly regular graphs with  $m_2 = qm_3$  and  $m_3 = qm_2$ , respectively.

## 2. MAIN RESULTS

*Remark 2.1.* Since  $m_2(\overline{G}) = m_3(G)$  and  $m_3(\overline{G}) = m_2(G)$  we note that if  $m_2(G) = qm_3(G)$ , then  $m_3(\overline{G}) = qm_2(\overline{G})$ .

*Remark 2.2.* In Theorems 2.1, 2.2,  $\dots$ , 2.11 the complements of strongly regular graphs appear in pairs in  $(k^0)$  and  $(\overline{k}^0)$  classes, where  $k$  denotes the corresponding number of a class.

*Remark 2.3.*  $\overline{\alpha K_\beta}$  is a strongly regular graph of order  $n = \alpha\beta$  and degree  $r = (\alpha - 1)\beta$  with  $\tau = (\alpha - 2)\beta$  and  $\theta = (\alpha - 1)\beta$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -\beta$  with  $m_2 = \alpha(\beta - 1)$  and  $m_3 = \alpha - 1$ .

In order to demonstrate a method which is applied to describe the parameters  $n, r, \tau$  and  $\theta$  for strongly regular graphs with  $m_2 = qm_3$  and  $m_3 = qm_2$ , we shall establish the parameters of strongly regular graphs with  $m_2 = 2m_3$  and  $m_3 = 2m_2$ , as follows.

**Proposition 2.1.** *Let  $G$  be a connected strongly regular graph of order  $n$  and degree  $r$  with  $m_2 = 2m_3$ . Then  $G$  belongs to the class  $(\overline{2}^0)$  or  $(3^0)$  represented in Theorem 2.1.*

**Proof.** Let  $m_3 = p, m_2 = 2p$  and  $n = 3p + 1$  where  $p \in \mathbb{N}$ . Using (1.2) we obtain  $r = p(|\lambda_3| - 2\lambda_2)$ . Let  $|\lambda_3| - 2\lambda_2 = t$  where  $t = 1, 2$ . Let  $\lambda_2 = k$  where  $k$  is a positive integer. Then according to Remark 1.3 we have (i)  $\lambda_3 = -(2k + t)$ ; (ii)  $\tau - \theta = -(k + t)$ ; (iii)  $\delta = 3k + t$ ; (iv)  $r = pt$  and (v)  $\theta = pt - 2k^2 - kt$ . In this case we can easily see that (1.3) is reduced to

$$(2.1) \quad (p + 1)t^2 - (3p + 1)t + 6k^2 + 4kt = 0.$$

CASE 1. ( $t = 1$ ). Using (i), (ii), (iii), (iv) and (v) we find that  $\lambda_2 = k$  and  $\lambda_3 = -(2k + 1), \tau - \theta = -(k + 1), \delta = 3k + 1, r = p$  and  $\theta = p - 2k^2 - k$ . Using (2.1) we find that  $p = k(3k + 2)$ . So, we obtain that  $G$  is a strongly regular graph of order  $n = (3k + 1)^2$  and degree  $r = k(3k + 2)$  with  $\tau = k^2 - 1$  and  $\theta = k(k + 1)$ .

CASE 2. ( $t = 2$ ). Using (i), (ii), (iii), (iv) and (v) we find that  $\lambda_2 = k$  and  $\lambda_3 = -(2k + 2), \tau - \theta = -(k + 2), \delta = 3k + 2, r = 2p$  and  $\theta = 2p - 2k^2 - 2k$ . Using (2.1) we find that  $p = (k + 1)(3k + 1)$ . Replacing  $k$  with  $k - 1$  we arrive at  $p = k(3k - 2)$ . So, we obtain that  $G$  is a strongly regular graph of order  $n = (3k - 1)^2$  and degree  $r = 2k(3k - 2)$  with  $\tau = (k - 1)(4k + 1)$  and  $\theta = 2k(2k - 1)$ .  $\square$

**Proposition 2.2.** *Let  $G$  be a connected strongly regular graph of order  $n$  and degree  $r$  with  $m_3 = 2m_2$ . Then  $G$  belongs to the class  $(2^0)$  or  $(\bar{3}^0)$  represented in Theorem 2.1.*

**Proof.** Let  $m_2 = p$ ,  $m_3 = 2p$  and  $n = 3p + 1$  where  $p \in \mathbb{N}$ . Using (1.2) we obtain  $r = p(2|\lambda_3| - \lambda_2)$ . Let  $2|\lambda_3| - \lambda_2 = t$  where  $t = 1, 2$ . Let  $\lambda_3 = -k$  where  $k$  is a positive integer. Then according to Remark 1.3 we have (i)  $\lambda_2 = 2k - t$ ; (ii)  $\tau - \theta = k - t$ ; (iii)  $\delta = 3k - t$ ; (iv)  $r = pt$  and (v)  $\theta = pt - 2k^2 + kt$ . In this case we can easily see that (1.4) is reduced to

$$(2.2) \quad (p+1)t^2 - (3p+1)t + 6k^2 - 4kt = 0.$$

CASE 1. ( $t = 1$ ). Using (i), (ii), (iii), (iv) and (v) we find that  $\lambda_2 = 2k - 1$  and  $\lambda_3 = -k$ ,  $\tau - \theta = k - 1$ ,  $\delta = 3k - 1$ ,  $r = p$  and  $\theta = p - 2k^2 + k$ . Using (2.2) we find that  $p = k(3k - 2)$ . So we obtain that  $G$  is a strongly regular graph of order  $n = (3k - 1)^2$  and degree  $r = k(3k - 2)$  with  $\tau = k^2 - 1$  and  $\theta = k(k - 1)$ .

CASE 2. ( $t = 2$ ). Using (i), (ii), (iii), (iv) and (v) we find that  $\lambda_2 = 2k - 2$  and  $\lambda_3 = -k$ ,  $\tau - \theta = k - 2$ ,  $\delta = 3k - 2$ ,  $r = 2p$  and  $\theta = 2p - 2k^2 + 2k$ . Using (2.2) we find that  $p = (k - 1)(3k - 1)$ . Replacing  $k$  with  $k + 1$  we arrive at  $p = k(3k + 2)$ . So we obtain that  $G$  is a strongly regular graph of order  $n = (3k + 1)^2$  and degree  $r = 2k(3k + 2)$  with  $\tau = (k + 1)(4k - 1)$  and  $\theta = 2k(2k + 1)$ .  $\square$

**Theorem 2.1.** *Let  $G$  be a connected strongly regular graph of order  $n$  and degree  $r$  with  $m_2 = 2m_3$  or  $m_3 = 2m_2$ . Then,  $G$  is one of the following strongly regular graphs.*

- (1<sup>0</sup>)  $G$  is the complete bipartite graph of order  $n = 4$  and degree  $r = 2$  with  $\tau = 0$  and  $\theta = 2$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -2$  with  $m_2 = 2$  and  $m_3 = 1$ .
- (2<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (3k - 1)^2$  and degree  $r = k(3k - 2)$  with  $\tau = k^2 - 1$  and  $\theta = k(k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = 2k - 1$  and  $\lambda_3 = -k$  with  $m_2 = k(3k - 2)$  and  $m_3 = 2k(3k - 2)$ .
- ( $\bar{2}^0$ )  $G$  is a strongly regular graph of order  $n = (3k - 1)^2$  and degree  $r = 2k(3k - 2)$  with  $\tau = (k - 1)(4k + 1)$  and  $\theta = 2k(2k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = k - 1$  and  $\lambda_3 = -2k$  with  $m_2 = 2k(3k - 2)$  and  $m_3 = k(3k - 2)$ .
- (3<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (3k + 1)^2$  and degree  $r = k(3k + 2)$  with  $\tau = k^2 - 1$  and  $\theta = k(k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = k$  and  $\lambda_3 = -(2k + 1)$  with  $m_2 = 2k(3k + 2)$  and  $m_3 = k(3k + 2)$ .
- ( $\bar{3}^0$ )  $G$  is a strongly regular graph of order  $n = (3k + 1)^2$  and degree  $r = 2k(3k + 2)$  with  $\tau = (k + 1)(4k - 1)$  and  $\theta = 2k(2k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 2k$  and  $\lambda_3 = -(k + 1)$  with  $m_2 = k(3k + 2)$  and  $m_3 = 2k(3k + 2)$ .

*Proof.* First, according to Remark 2.3 we have  $\alpha(\beta - 1) = 2(\alpha - 1)$ , from which we find that  $\alpha = 2$ ,  $\beta = 2$ . In view of this we obtain the strongly regular graph represented in Theorem 2.1 (1<sup>0</sup>). Next, according to Proposition 2.1 it turns out that  $G$  belongs to the class  $(\bar{2}^0)$  or  $(3^0)$  if  $m_2 = 2m_3$ . According to Proposition 2.2 it turns out that  $G$  belongs to the class  $(2^0)$  or  $(\bar{3}^0)$  if  $m_3 = 2m_2$ .  $\square$

**Theorem 2.2** ([4]). *Let  $G$  be a connected strongly regular graph of order  $n$  and degree  $r$  with  $m_2 = 3m_3$  or  $m_3 = 3m_2$ . Then,  $G$  is one of the following strongly regular graphs.*

- (1<sup>0</sup>)  $G$  is the strongly regular graph  $\overline{3K_3}$  of order  $n = 9$  and degree  $r = 6$  with  $\tau = 3$  and  $\theta = 6$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -3$  with  $m_2 = 6$  and  $m_3 = 2$ .
- (2<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (4k - 1)^2$  and degree  $r = 2k(2k - 1)$  with  $\tau = k^2 + k - 1$  and  $\theta = k(k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = 3k - 1$  and  $\lambda_3 = -k$  with  $m_2 = 2k(2k - 1)$  and  $m_3 = 6k(2k - 1)$ .
- (2<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (4k - 1)^2$  and degree  $r = 6k(2k - 1)$  with  $\tau = 9k^2 - 5k - 1$  and  $\theta = 3k(3k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = k - 1$  and  $\lambda_3 = -3k$  with  $m_2 = 6k(2k - 1)$  and  $m_3 = 2k(2k - 1)$ .
- (3<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (4k + 1)^2$  and degree  $r = 2k(2k + 1)$  with  $\tau = k^2 - k - 1$  and  $\theta = k(k + 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = k$  and  $\lambda_3 = -(3k + 1)$  with  $m_2 = 6k(2k + 1)$  and  $m_3 = 2k(2k + 1)$ .
- (3<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (4k + 1)^2$  and degree  $r = 6k(2k + 1)$  with  $\tau = 9k^2 + 5k - 1$  and  $\theta = 3k(3k + 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = 3k$  and  $\lambda_3 = -(k + 1)$  with  $m_2 = 2k(2k + 1)$  and  $m_3 = 6k(2k + 1)$ .

**Theorem 2.3** ([4]). *Let  $G$  be a connected strongly regular graph of order  $n$  and degree  $r$  with  $m_2 = 4m_3$  or  $m_3 = 4m_2$ . Then,  $G$  is one of the following strongly regular graphs.*

- (1<sup>0</sup>)  $G$  is the complete bipartite graph  $K_{3,3}$  of order  $n = 6$  and degree  $r = 3$  with  $\tau = 0$  and  $\theta = 3$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -3$  with  $m_2 = 4$  and  $m_3 = 1$ .
- (2<sup>0</sup>)  $G$  is the strongly regular graph  $\overline{4K_4}$  of order  $n = 16$  and degree  $r = 12$  with  $\tau = 8$  and  $\theta = 12$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -4$  with  $m_2 = 12$  and  $m_3 = 3$ .
- (3<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (5k - 1)^2$  and degree  $r = k(5k - 2)$  with  $\tau = k^2 + 2k - 1$  and  $\theta = k(k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = 4k - 1$  and  $\lambda_3 = -k$  with  $m_2 = k(5k - 2)$  and  $m_3 = 4k(5k - 2)$ .
- (3<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (5k - 1)^2$  and degree  $r = 4k(5k - 2)$  with  $\tau = 16k^2 - 7k - 1$  and  $\theta = 4k(4k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = k - 1$  and  $\lambda_3 = -4k$  with  $m_2 = 4k(5k - 2)$  and  $m_3 = k(5k - 2)$ .
- (4<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (5k + 1)^2$  and degree  $r = k(5k + 2)$  with  $\tau = k^2 - 2k - 1$  and  $\theta = k(k + 1)$ , where  $k \geq 3$ . Its eigenvalues are  $\lambda_2 = k$  and  $\lambda_3 = -(4k + 1)$  with  $m_2 = 4k(5k + 2)$  and  $m_3 = k(5k + 2)$ .
- (4<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (5k + 1)^2$  and degree  $r = 4k(5k + 2)$  with  $\tau = 16k^2 + 7k - 1$  and  $\theta = 4k(4k + 1)$ , where  $k \geq 3$ . Its eigenvalues are  $\lambda_2 = 4k$  and  $\lambda_3 = -(k + 1)$  with  $m_2 = k(5k + 2)$  and  $m_3 = 4k(5k + 2)$ .
- (5<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 6(5k - 1)^2$  and degree  $r = 2(30k^2 - 12k + 1)$  with  $\tau = 24k^2 - 15k + 1$  and  $\theta = 6k(4k - 1)$ , where  $k \in \mathbb{N}$ . Its

eigenvalues are  $\lambda_2 = 3k - 1$  and  $\lambda_3 = -(12k - 2)$  with  $m_2 = 4(30k^2 - 12k + 1)$  and  $m_3 = 30k^2 - 12k + 1$ .

- ( $\bar{5}^0$ )  $G$  is a strongly regular graph of order  $n = 6(5k - 1)^2$  and degree  $r = 3(30k^2 - 12k + 1)$  with  $\tau = 18k(3k - 1)$  and  $\theta = 3(3k - 1)(6k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 12k - 3$  and  $\lambda_3 = -3k$  with  $m_2 = 30k^2 - 12k + 1$  and  $m_3 = 4(30k^2 - 12k + 1)$ .
- ( $6^0$ )  $G$  is a strongly regular graph of order  $n = 6(5k + 1)^2$  and degree  $r = 2(30k^2 + 12k + 1)$  with  $\tau = 24k^2 + 15k + 1$  and  $\theta = 6k(4k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 12k + 2$  and  $\lambda_3 = -(3k + 1)$  with  $m_2 = 30k^2 + 12k + 1$  and  $m_3 = 4(30k^2 + 12k + 1)$ .
- ( $\bar{6}^0$ )  $G$  is a strongly regular graph of order  $n = 6(5k + 1)^2$  and degree  $r = 3(30k^2 + 12k + 1)$  with  $\tau = 18k(3k + 1)$  and  $\theta = 3(3k + 1)(6k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 3k$  and  $\lambda_3 = -(12k + 3)$  with  $m_2 = 4(30k^2 + 12k + 1)$  and  $m_3 = 30k^2 + 12k + 1$ .

**Theorem 2.4** ([5]). *Let  $G$  be a connected strongly regular graph of order  $n$  and degree  $r$  with  $m_2 = 5m_3$  or  $m_3 = 5m_2$ . Then,  $G$  is one of the following strongly regular graphs.*

- ( $1^0$ )  $G$  is the strongly regular graph  $\overline{5K_5}$  of order  $n = 25$  and degree  $r = 20$  with  $\tau = 15$  and  $\theta = 20$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -5$  with  $m_2 = 20$  and  $m_3 = 4$ .
- ( $2^0$ )  $G$  is a strongly regular graph of order  $n = (6k - 1)^2$  and degree  $r = 2k(3k - 1)$  with  $\tau = k^2 + 3k - 1$  and  $\theta = k(k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = 5k - 1$  and  $\lambda_3 = -k$  with  $m_2 = 2k(3k - 1)$  and  $m_3 = 10k(3k - 1)$ .
- ( $\bar{2}^0$ )  $G$  is a strongly regular graph of order  $n = (6k - 1)^2$  and degree  $r = 10k(3k - 1)$  with  $\tau = 25k^2 - 9k - 1$  and  $\theta = 5k(5k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = k - 1$  and  $\lambda_3 = -5k$  with  $m_2 = 10k(3k - 1)$  and  $m_3 = 2k(3k - 1)$ .
- ( $3^0$ )  $G$  is a strongly regular graph of order  $n = (6k + 1)^2$  and degree  $r = 2k(3k + 1)$  with  $\tau = k^2 - 3k - 1$  and  $\theta = k(k + 1)$ , where  $k \geq 4$ . Its eigenvalues are  $\lambda_2 = k$  and  $\lambda_3 = -(5k + 1)$  with  $m_2 = 10k(3k + 1)$  and  $m_3 = 2k(3k + 1)$ .
- ( $\bar{3}^0$ )  $G$  is a strongly regular graph of order  $n = (6k + 1)^2$  and degree  $r = 10k(3k + 1)$  with  $\tau = 25k^2 + 9k - 1$  and  $\theta = 5k(5k + 1)$ , where  $k \geq 4$ . Its eigenvalues are  $\lambda_2 = 5k$  and  $\lambda_3 = -(k + 1)$  with  $m_2 = 2k(3k + 1)$  and  $m_3 = 10k(3k + 1)$ .

**Theorem 2.5** ([5]). *Let  $G$  be a connected strongly regular graph of order  $n$  and degree  $r$  with  $m_2 = 6m_3$  or  $m_3 = 6m_2$ . Then,  $G$  is one of the following strongly regular graphs.*

- ( $1^0$ )  $G$  is the complete bipartite graph  $K_{4,4}$  of order  $n = 8$  and degree  $r = 4$  with  $\tau = 0$  and  $\theta = 4$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -4$  with  $m_2 = 6$  and  $m_3 = 1$ .
- ( $2^0$ )  $G$  is the strongly regular graph  $\overline{3K_5}$  of order  $n = 15$  and degree  $r = 10$  with  $\tau = 5$  and  $\theta = 10$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -5$  with  $m_2 = 12$  and  $m_3 = 2$ .

- (3<sup>0</sup>)  $G$  is the strongly regular graph  $\overline{6K_6}$  of order  $n = 36$  and degree  $r = 30$  with  $\tau = 24$  and  $\theta = 30$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -6$  with  $m_2 = 30$  and  $m_3 = 5$ .
- (4<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (7k - 1)^2$  and degree  $r = k(7k - 2)$  with  $\tau = k^2 + 4k - 1$  and  $\theta = k(k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = 6k - 1$  and  $\lambda_3 = -k$  with  $m_2 = k(7k - 2)$  and  $m_3 = 6k(7k - 2)$ .
- (4<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (7k - 1)^2$  and degree  $r = 6k(7k - 2)$  with  $\tau = 36k^2 - 11k - 1$  and  $\theta = 6k(6k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = k - 1$  and  $\lambda_3 = -6k$  with  $m_2 = 6k(7k - 2)$  and  $m_3 = k(7k - 2)$ .
- (5<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (7k + 1)^2$  and degree  $r = k(7k + 2)$  with  $\tau = k^2 - 4k - 1$  and  $\theta = k(k + 1)$ , where  $k \geq 5$ . Its eigenvalues are  $\lambda_2 = k$  and  $\lambda_3 = -(6k + 1)$  with  $m_2 = 6k(7k + 2)$  and  $m_3 = k(7k + 2)$ .
- (5<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (7k + 1)^2$  and degree  $r = 6k(7k + 2)$  with  $\tau = 36k^2 + 11k - 1$  and  $\theta = 6k(6k + 1)$ , where  $k \geq 5$ . Its eigenvalues are  $\lambda_2 = 6k$  and  $\lambda_3 = -(k + 1)$  with  $m_2 = k(7k + 2)$  and  $m_3 = 6k(7k + 2)$ .
- (6<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 2(7k - 2)^2$  and degree  $r = 3(14k^2 - 8k + 1)$  with  $\tau = 18k^2 - 16k + 2$  and  $\theta = 6k(3k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 2k - 1$  and  $\lambda_3 = -(12k - 3)$  with  $m_2 = 6(14k^2 - 8k + 1)$  and  $m_3 = 14k^2 - 8k + 1$ .
- (6<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 2(7k - 2)^2$  and degree  $r = 4(14k^2 - 8k + 1)$  with  $\tau = 2k(16k - 7)$  and  $\theta = 4(2k - 1)(4k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 12k - 4$  and  $\lambda_3 = -2k$  with  $m_2 = 14k^2 - 8k + 1$  and  $m_3 = 6(14k^2 - 8k + 1)$ .
- (7<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 2(7k + 2)^2$  and degree  $r = 3(14k^2 + 8k + 1)$  with  $\tau = 18k^2 + 16k + 2$  and  $\theta = 6k(3k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 12k + 3$  and  $\lambda_3 = -(2k + 1)$  with  $m_2 = 14k^2 + 8k + 1$  and  $m_3 = 6(14k^2 + 8k + 1)$ .
- (7<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 2(7k + 2)^2$  and degree  $r = 4(14k^2 + 8k + 1)$  with  $\tau = 2k(16k + 7)$  and  $\theta = 4(2k + 1)(4k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 2k$  and  $\lambda_3 = -(12k + 4)$  with  $m_2 = 6(14k^2 + 8k + 1)$  and  $m_3 = 14k^2 + 8k + 1$ .
- (8<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 15(7k - 1)^2$  and degree  $r = 2(105k^2 - 30k + 2)$  with  $\tau = 60k^2 - 35k + 3$  and  $\theta = 10k(6k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 5k - 1$  and  $\lambda_3 = -(30k - 4)$  with  $m_2 = 6(105k^2 - 30k + 2)$  and  $m_3 = 105k^2 - 30k + 2$ .
- (8<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 15(7k - 1)^2$  and degree  $r = 5(105k^2 - 30k + 2)$  with  $\tau = 5(5k - 1)(15k - 1)$  and  $\theta = 5(5k - 1)(15k - 2)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 30k - 5$  and  $\lambda_3 = -5k$  with  $m_2 = 105k^2 - 30k + 2$  and  $m_3 = 6(105k^2 - 30k + 2)$ .
- (9<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 15(7k + 1)^2$  and degree  $r = 2(105k^2 + 30k + 2)$  with  $\tau = 60k^2 + 35k + 3$  and  $\theta = 10k(6k + 1)$ , where  $k \in \mathbb{N}$ . Its

eigenvalues are  $\lambda_2 = 30k + 4$  and  $\lambda_3 = -(5k + 1)$  with  $m_2 = 105k^2 + 30k + 2$  and  $m_3 = 6(105k^2 + 30k + 2)$ .

- ( $\bar{9}^0$ )  $G$  is a strongly regular graph of order  $n = 15(7k + 1)^2$  and degree  $r = 5(105k^2 + 30k + 2)$  with  $\tau = 5(5k + 1)(15k + 1)$  and  $\theta = 5(5k + 1)(15 + 2)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 5k$  and  $\lambda_3 = -(30k + 5)$  with  $m_2 = 6(105k^2 + 30k + 2)$  and  $m_3 = 105k^2 + 30k + 2$ .

**Theorem 2.6** ([5]). *Let  $G$  be a connected strongly regular graph of order  $n$  and degree  $r$  with  $m_2 = 7m_3$  or  $m_3 = 7m_2$ . Then,  $G$  is one of the following strongly regular graphs.*

- ( $1^0$ )  $G$  is the strongly regular graph  $\overline{7K_7}$  of order  $n = 49$  and degree  $r = 42$  with  $\tau = 35$  and  $\theta = 42$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -7$  with  $m_2 = 42$  and  $m_3 = 6$ .
- ( $2^0$ )  $G$  is a strongly regular graph of order  $n = (8k - 1)^2$  and degree  $r = 2k(4k - 1)$  with  $\tau = k^2 + 5k - 1$  and  $\theta = k(k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = 7k - 1$  and  $\lambda_3 = -k$  with  $m_2 = 2k(4k - 1)$  and  $m_3 = 14k(4k - 1)$ .
- ( $\bar{2}^0$ )  $G$  is a strongly regular graph of order  $n = (8k - 1)^2$  and degree  $r = 14k(4k - 1)$  with  $\tau = 49k^2 - 13k - 1$  and  $\theta = 7k(7k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = k - 1$  and  $\lambda_3 = -7k$  with  $m_2 = 14k(4k - 1)$  and  $m_3 = 2k(4k - 1)$ .
- ( $3^0$ )  $G$  is a strongly regular graph of order  $n = (8k + 1)^2$  and degree  $r = 2k(4k + 1)$  with  $\tau = k^2 - 5k - 1$  and  $\theta = k(k + 1)$ , where  $k \geq 6$ . Its eigenvalues are  $\lambda_2 = k$  and  $\lambda_3 = -(7k + 1)$  with  $m_2 = 14k(4k + 1)$  and  $m_3 = 2k(4k + 1)$ .
- ( $\bar{3}^0$ )  $G$  is a strongly regular graph of order  $n = (8k + 1)^2$  and degree  $r = 14k(4k + 1)$  with  $\tau = 49k^2 + 13k - 1$  and  $\theta = 7k(7k + 1)$ , where  $k \geq 6$ . Its eigenvalues are  $\lambda_2 = 7k$  and  $\lambda_3 = -(k + 1)$  with  $m_2 = 2k(4k + 1)$  and  $m_3 = 14k(4k + 1)$ .
- ( $4^0$ )  $G$  is a strongly regular graph of order  $n = 105(8k - 3)^2$  and degree  $r = 3(840k^2 - 630k + 118)$  with  $\tau = 945k^2 - 765k + 153$  and  $\theta = 15(3k - 1)(21k - 8)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 15k - 6$  and  $\lambda_3 = -(105k - 39)$  with  $m_2 = 7(840k^2 - 630k + 118)$  and  $m_3 = 840k^2 - 630k + 118$ .
- ( $\bar{4}^0$ )  $G$  is a strongly regular graph of order  $n = 105(8k - 3)^2$  and degree  $r = 5(840k^2 - 630k + 118)$  with  $\tau = 5(525k^2 - 387k + 71)$  and  $\theta = 15(5k - 2)(35k - 13)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 105k - 40$  and  $\lambda_3 = -(15k - 5)$  with  $m_2 = 840k^2 - 630k + 118$  and  $m_3 = 7(840k^2 - 630k + 118)$ .
- ( $5^0$ )  $G$  is a strongly regular graph of order  $n = 105(8k + 3)^2$  and degree  $r = 3(840k^2 + 630k + 118)$  with  $\tau = 945k^2 + 765k + 153$  and  $\theta = 15(3k + 1)(21k + 8)$ , where  $k \geq 0$ . Its eigenvalues are  $\lambda_2 = 105k + 39$  and  $\lambda_3 = -(15k + 6)$  with  $m_2 = 840k^2 + 630k + 118$  and  $m_3 = 7(840k^2 + 630k + 118)$ .
- ( $\bar{5}^0$ )  $G$  is a strongly regular graph of order  $n = 105(8k + 3)^2$  and degree  $r = 5(840k^2 + 630k + 118)$  with  $\tau = 5(525k^2 + 387k + 71)$  and  $\theta = 15(5k + 2)(35k + 13)$ , where  $k \geq 0$ . Its eigenvalues are  $\lambda_2 = 15k + 5$  and  $\lambda_3 = -(105k + 40)$  with  $m_2 = 7(840k^2 + 630k + 118)$  and  $m_3 = 840k^2 + 630k + 118$ .

**Theorem 2.7** ([5]). *Let  $G$  be a connected strongly regular graph of order  $n$  and degree  $r$  with  $m_2 = 8m_3$  or  $m_3 = 8m_2$ . Then,  $G$  is one of the following strongly regular graphs.*

- (1<sup>0</sup>)  $G$  is the complete bipartite graph  $K_{5,5}$  of order  $n = 10$  and degree  $r = 5$  with  $\tau = 0$  and  $\theta = 5$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -5$  with  $m_2 = 8$  and  $m_3 = 1$ .
- (2<sup>0</sup>)  $G$  is the strongly regular graph  $\overline{4K_7}$  of order  $n = 28$  and degree  $r = 21$  with  $\tau = 14$  and  $\theta = 21$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -7$  with  $m_2 = 24$  and  $m_3 = 3$ .
- (3<sup>0</sup>)  $G$  is the strongly regular graph  $\overline{8K_8}$  of order  $n = 64$  and degree  $r = 56$  with  $\tau = 48$  and  $\theta = 56$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -8$  with  $m_2 = 56$  and  $m_3 = 7$ .
- (4<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (9k - 1)^2$  and degree  $r = k(9k - 2)$  with  $\tau = k^2 + 6k - 1$  and  $\theta = k(k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = 8k - 1$  and  $\lambda_3 = -k$  with  $m_2 = k(9k - 2)$  and  $m_3 = 8k(9k - 2)$ .
- ( $\overline{4}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (9k - 1)^2$  and degree  $r = 8k(9k - 2)$  with  $\tau = 64k^2 - 15k - 1$  and  $\theta = 8k(8k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = k - 1$  and  $\lambda_3 = -8k$  with  $m_2 = 8k(9k - 2)$  and  $m_3 = k(9k - 2)$ .
- (5<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (9k + 1)^2$  and degree  $r = k(9k + 2)$  with  $\tau = k^2 - 6k - 1$  and  $\theta = k(k + 1)$ , where  $k \geq 7$ . Its eigenvalues are  $\lambda_2 = k$  and  $\lambda_3 = -(8k + 1)$  with  $m_2 = 8k(9k + 2)$  and  $m_3 = k(9k + 2)$ .
- ( $\overline{5}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (9k + 1)^2$  and degree  $r = 8k(9k + 2)$  with  $\tau = 64k^2 + 15k - 1$  and  $\theta = 8k(8k + 1)$ , where  $k \geq 7$ . Its eigenvalues are  $\lambda_2 = 8k$  and  $\lambda_3 = -(k + 1)$  with  $m_2 = k(9k + 2)$  and  $m_3 = 8k(9k + 2)$ .
- (6<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 4(9k - 4)^2$  and degree  $r = 3(2k - 1)(18k - 7)$  with  $\tau = 18k(2k - 1)$  and  $\theta = (3k - 2)(12k - 5)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 24k - 11$  and  $\lambda_3 = -(3k - 1)$  with  $m_2 = (2k - 1)(18k - 7)$  and  $m_3 = 8(2k - 1)(18k - 7)$ .
- ( $\overline{6}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 4(9k - 4)^2$  and degree  $r = 6(2k - 1)(18k - 7)$  with  $\tau = 3(48k^2 - 45k + 10)$  and  $\theta = 2(3k - 1)(24k - 11)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 3k - 2$  and  $\lambda_3 = -(24k - 10)$  with  $m_2 = 8(2k - 1)(18k - 7)$  and  $m_3 = (2k - 1)(18k - 7)$ .
- (7<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 4(9k + 4)^2$  and degree  $r = 3(2k + 1)(18k + 7)$  with  $\tau = 18k(2k + 1)$  and  $\theta = (3k + 2)(12k + 5)$ , where  $k \geq 0$ . Its eigenvalues are  $\lambda_2 = 3k + 1$  and  $\lambda_3 = -(24k + 11)$  with  $m_2 = 8(2k + 1)(18k + 7)$  and  $m_3 = (2k + 1)(18k + 7)$ .
- ( $\overline{7}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 4(9k + 4)^2$  and degree  $r = 6(2k + 1)(18k + 7)$  with  $\tau = 3(48k^2 + 45k + 10)$  and  $\theta = 2(3k + 1)(24k + 11)$ , where  $k \geq 0$ . Its eigenvalues are  $\lambda_2 = 24k + 10$  and  $\lambda_3 = -(3k + 2)$  with  $m_2 = (2k + 1)(18k + 7)$  and  $m_3 = 8(2k + 1)(18k + 7)$ .

- (8<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 10(9k - 1)^2$  and degree  $r = 4(90k^2 - 20k + 1)$  with  $\tau = 160k^2 - 55k + 3$  and  $\theta = 20k(8k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 5k - 1$  and  $\lambda_3 = -(40k - 4)$  with  $m_2 = 8(90k^2 - 20k + 1)$  and  $m_3 = 90k^2 - 20k + 1$ .
- (8<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 10(9k - 1)^2$  and degree  $r = 5(90k^2 - 20k + 1)$  with  $\tau = 10k(25k - 4)$  and  $\theta = 5(5k - 1)(10k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 40k - 5$  and  $\lambda_3 = -5k$  with  $m_2 = 90k^2 - 20k + 1$  and  $m_3 = 8(90k^2 - 20k + 1)$ .
- (9<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 10(9k + 1)^2$  and degree  $r = 4(90k^2 + 20k + 1)$  with  $\tau = 160k^2 + 55k + 3$  and  $\theta = 20k(8k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 40k + 4$  and  $\lambda_3 = -(5k + 1)$  with  $m_2 = 90k^2 + 20k + 1$  and  $m_3 = 8(90k^2 + 20k + 1)$ .
- (9<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 10(9k + 1)^2$  and degree  $r = 5(90k^2 + 20k + 1)$  with  $\tau = 10k(25k + 4)$  and  $\theta = 5(5k + 1)(10k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 5k$  and  $\lambda_3 = -(40k + 5)$  with  $m_2 = 8(90k^2 + 20k + 1)$  and  $m_3 = 90k^2 + 20k + 1$ .
- (10<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 28(9k - 1)^2$  and degree  $r = 2(252k^2 - 56k + 3)$  with  $\tau = 112k^2 - 63k + 5$  and  $\theta = 14k(8k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 7k - 1$  and  $\lambda_3 = -(56k - 6)$  with  $m_2 = 8(252k^2 - 56k + 3)$  and  $m_3 = 252k^2 - 56k + 3$ .
- (10<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 28(9k - 1)^2$  and degree  $r = 7(252k^2 - 56k + 3)$  with  $\tau = 14(7k - 1)(14k - 1)$  and  $\theta = 7(7k - 1)(28k - 3)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 56k - 7$  and  $\lambda_3 = -7k$  with  $m_2 = 252k^2 - 56k + 3$  and  $m_3 = 8(252k^2 - 56k + 3)$ .
- (11<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 28(9k + 1)^2$  and degree  $r = 2(252k^2 + 56k + 3)$  with  $\tau = 112k^2 + 63k + 5$  and  $\theta = 14k(8k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 56k + 6$  and  $\lambda_3 = -(7k + 1)$  with  $m_2 = 252k^2 + 56k + 3$  and  $m_3 = 8(252k^2 + 56k + 3)$ .
- (11<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 28(9k + 1)^2$  and degree  $r = 7(252k^2 + 56k + 3)$  with  $\tau = 14(7k + 1)(14k + 1)$  and  $\theta = 7(7k + 1)(28k + 3)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 7k$  and  $\lambda_3 = -(56k + 7)$  with  $m_2 = 8(252k^2 + 56k + 3)$  and  $m_3 = 252k^2 + 56k + 3$ .

**Theorem 2.8** ([6]). *Let  $G$  be a connected strongly regular graph of order  $n$  and degree  $r$  with  $m_2 = 9m_3$  or  $m_3 = 9m_2$ . Then,  $G$  is one of the following strongly regular graphs.*

- (1<sup>0</sup>)  $G$  is the strongly regular graph  $\overline{3K_7}$  of order  $n = 21$  and degree  $r = 14$  with  $\tau = 7$  and  $\theta = 14$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -7$  with  $m_2 = 18$  and  $m_3 = 2$ .
- (2<sup>0</sup>)  $G$  is the strongly regular graph  $\overline{9K_9}$  of order  $n = 81$  and degree  $r = 72$  with  $\tau = 63$  and  $\theta = 72$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -9$  with  $m_2 = 72$  and  $m_3 = 8$ .

- (3<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (10k - 1)^2$  and degree  $r = 2k(5k - 1)$  with  $\tau = k^2 + 7k - 1$  and  $\theta = k(k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = 9k - 1$  and  $\lambda_3 = -k$  with  $m_2 = 2k(5k - 1)$  and  $m_3 = 18k(5k - 1)$ .
- (3<sup>̄0</sup>)  $G$  is a strongly regular graph of order  $n = (10k - 1)^2$  and degree  $r = 18k(5k - 1)$  with  $\tau = 81k^2 - 17k - 1$  and  $\theta = 9k(9k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = k - 1$  and  $\lambda_3 = -9k$  with  $m_2 = 18k(5k - 1)$  and  $m_3 = 2k(5k - 1)$ .
- (4<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (10k + 1)^2$  and degree  $r = 2k(5k + 1)$  with  $\tau = k^2 - 7k - 1$  and  $\theta = k(k + 1)$ , where  $k \geq 8$ . Its eigenvalues are  $\lambda_2 = k$  and  $\lambda_3 = -(9k + 1)$  with  $m_2 = 18k(5k + 1)$  and  $m_3 = 2k(5k + 1)$ .
- (4<sup>̄0</sup>)  $G$  is a strongly regular graph of order  $n = (10k + 1)^2$  and degree  $r = 18k(5k + 1)$  with  $\tau = 81k^2 + 17k - 1$  and  $\theta = 9k(9k + 1)$ , where  $k \geq 8$ . Its eigenvalues are  $\lambda_2 = 9k$  and  $\lambda_3 = -(k + 1)$  with  $m_2 = 2k(5k + 1)$  and  $m_3 = 18k(5k + 1)$ .
- (5<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 9(10k - 3)^2$  and degree  $r = 2(90k^2 - 54k + 8)$  with  $\tau = 36k^2 + 4k - 5$  and  $\theta = (2k - 1)(18k - 5)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 36k - 11$  and  $\lambda_3 = -(4k - 1)$  with  $m_2 = 90k^2 - 54k + 8$  and  $m_3 = 9(90k^2 - 54k + 8)$ .
- (5<sup>̄0</sup>)  $G$  is a strongly regular graph of order  $n = 9(10k - 3)^2$  and degree  $r = 8(90k^2 - 54k + 8)$  with  $\tau = 4(4k - 1)(36k - 13)$  and  $\theta = 4(4k - 1)(36k - 11)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 4k - 2$  and  $\lambda_3 = -(36k - 10)$  with  $m_2 = 9(90k^2 - 54k + 8)$  and  $m_3 = 90k^2 - 54k + 8$ .
- (6<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 9(10k - 3)^2$  and degree  $r = 5(90k^2 - 54k + 8)$  with  $\tau = 225k^2 - 155k + 25$  and  $\theta = (5k - 1)(45k - 14)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 5k - 2$  and  $\lambda_3 = -(45k - 13)$  with  $m_2 = 9(90k^2 - 54k + 8)$  and  $m_3 = 90k^2 - 54k + 8$ .
- (6<sup>̄0</sup>)  $G$  is a strongly regular graph of order  $n = 9(10k - 3)^2$  and degree  $r = 5(90k^2 - 54k + 8)$  with  $\tau = 225k^2 - 115k + 13$  and  $\theta = (5k - 2)(45k - 13)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 45k - 14$  and  $\lambda_3 = -(5k - 1)$  with  $m_2 = 90k^2 - 54k + 8$  and  $m_3 = 9(90k^2 - 54k + 8)$ .
- (7<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 9(10k + 3)^2$  and degree  $r = 2(90k^2 + 54k + 8)$  with  $\tau = 36k^2 - 4k - 5$  and  $\theta = (2k + 1)(18k + 5)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 4k + 1$  and  $\lambda_3 = -(36k + 11)$  with  $m_2 = 9(90k^2 + 54k + 8)$  and  $m_3 = 90k^2 + 54k + 8$ .
- (7<sup>̄0</sup>)  $G$  is a strongly regular graph of order  $n = 9(10k + 3)^2$  and degree  $r = 8(90k^2 + 54k + 8)$  with  $\tau = 4(4k + 1)(36k + 13)$  and  $\theta = 4(4k + 1)(36k + 11)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 36k + 10$  and  $\lambda_3 = -(4k + 2)$  with  $m_2 = 90k^2 + 54k + 8$  and  $m_3 = 9(90k^2 + 54k + 8)$ .
- (8<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 9(10k + 3)^2$  and degree  $r = 5(90k^2 + 54k + 8)$  with  $\tau = 225k^2 + 115k + 13$  and  $\theta = (5k + 2)(45k + 13)$ , where  $k \geq 0$ . Its eigenvalues are  $\lambda_2 = 5k + 1$  and  $\lambda_3 = -(45k + 14)$  with  $m_2 = 9(90k^2 + 54k + 8)$  and  $m_3 = 90k^2 + 54k + 8$ .
- (8<sup>̄0</sup>)  $G$  is a strongly regular graph of order  $n = 9(10k + 3)^2$  and degree  $r = 5(90k^2 + 54k + 8)$  with  $\tau = 225k^2 + 155k + 25$  and  $\theta = (5k + 1)(45k + 14)$ , where  $k \geq 0$ .

- Its eigenvalues are  $\lambda_2 = 45k + 13$  and  $\lambda_3 = -(5k + 2)$  with  $m_2 = 90k^2 + 54k + 8$  and  $m_3 = 9(90k^2 + 54k + 8)$ .
- (9<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 21(10k - 1)^2$  and degree  $r = 3(210k^2 - 42k + 2)$  with  $\tau = 189k^2 - 77k + 5$  and  $\theta = 21k(9k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 7k - 1$  and  $\lambda_3 = -(63k - 6)$  with  $m_2 = 9(210k^2 - 42k + 2)$  and  $m_3 = 210k^2 - 42k + 2$ .
- ( $\bar{9}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 21(10k - 1)^2$  and degree  $r = 7(210k^2 - 42k + 2)$  with  $\tau = 7(147k^2 - 27k + 1)$  and  $\theta = 7(7k - 1)(21k - 2)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 63k - 7$  and  $\lambda_3 = -7k$  with  $m_2 = 210k^2 - 42k + 2$  and  $m_3 = 9(210k^2 - 42k + 2)$ .
- (10<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 21(10k + 1)^2$  and degree  $r = 3(210k^2 + 42k + 2)$  with  $\tau = 189k^2 + 77k + 5$  and  $\theta = 21k(9k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 63k + 6$  and  $\lambda_3 = -(7k + 1)$  with  $m_2 = 210k^2 + 42k + 2$  and  $m_3 = 9(210k^2 + 42k + 2)$ .
- ( $\bar{10}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 21(10k + 1)^2$  and degree  $r = 7(210k^2 + 42k + 2)$  with  $\tau = 7(147k^2 + 27k + 1)$  and  $\theta = 7(7k + 1)(21k + 2)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 7k$  and  $\lambda_3 = -(63k + 7)$  with  $m_2 = 9(210k^2 + 42k + 2)$  and  $m_3 = 210k^2 + 42k + 2$ .

**Theorem 2.9** ([6]). *Let  $G$  be a connected strongly regular graph of order  $n$  and degree  $r$  with  $m_2 = 10m_3$  or  $m_3 = 10m_2$ . Then,  $G$  is one of the following strongly regular graphs.*

- (1<sup>0</sup>)  $G$  is the complete bipartite graph  $K_{6,6}$  of order  $n = 12$  and degree  $r = 6$  with  $\tau = 0$  and  $\theta = 6$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -6$  with  $m_2 = 10$  and  $m_3 = 1$ .
- (2<sup>0</sup>)  $G$  is the strongly regular graph  $\overline{5K_9}$  of order  $n = 45$  and degree  $r = 36$  with  $\tau = 27$  and  $\theta = 36$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -9$  with  $m_2 = 40$  and  $m_3 = 4$ .
- (3<sup>0</sup>)  $G$  is the strongly regular graph  $\overline{10K_{10}}$  of order  $n = 100$  and degree  $r = 90$  with  $\tau = 80$  and  $\theta = 90$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -10$  with  $m_2 = 90$  and  $m_3 = 9$ .
- (4<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (11k - 1)^2$  and degree  $r = k(11k - 2)$  with  $\tau = k^2 + 8k - 1$  and  $\theta = k(k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 10k - 1$  and  $\lambda_3 = -k$  with  $m_2 = k(11k - 2)$  and  $m_3 = 10k(11k - 2)$ .
- ( $\bar{4}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (11k - 1)^2$  and degree  $r = 10k(11k - 2)$  with  $\tau = 100k^2 - 19k - 1$  and  $\theta = 10k(10k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = k - 1$  and  $\lambda_3 = -10k$  with  $m_2 = 10k(11k - 2)$  and  $m_3 = k(11k - 2)$ .
- (5<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (11k + 1)^2$  and degree  $r = k(11k + 2)$  with  $\tau = k^2 - 8k - 1$  and  $\theta = k(k + 1)$ , where  $k \geq 9$ . Its eigenvalues are  $\lambda_2 = k$  and  $\lambda_3 = -(10k + 1)$  with  $m_2 = 10k(11k + 2)$  and  $m_3 = k(11k + 2)$ .

- ( $\bar{5}^0$ )  $G$  is a strongly regular graph of order  $n = (11k+1)^2$  and degree  $r = 10k(11k+2)$  with  $\tau = 100k^2 + 19k - 1$  and  $\theta = 10k(10k + 1)$ , where  $k \geq 9$ . Its eigenvalues are  $\lambda_2 = 10k$  and  $\lambda_3 = -(k + 1)$  with  $m_2 = k(11k + 2)$  and  $m_3 = 10k(11k + 2)$ .
- ( $6^0$ )  $G$  is a strongly regular graph of order  $n = 3(11k - 2)^2$  and degree  $r = 5(33k^2 - 12k + 1)$  with  $\tau = 75k^2 - 42k + 4$  and  $\theta = 15k(5k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 3k - 1$  and  $\lambda_3 = -(30k - 5)$  with  $m_2 = 10(33k^2 - 12k + 1)$  and  $m_3 = 33k^2 - 12k + 1$ .
- ( $\bar{6}^0$ )  $G$  is a strongly regular graph of order  $n = 3(11k - 2)^2$  and degree  $r = 6(33k^2 - 12k + 1)$  with  $\tau = 27k(4k - 1)$  and  $\theta = 6(3k - 1)(6k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 30k - 6$  and  $\lambda_3 = -3k$  with  $m_2 = 33k^2 - 12k + 1$  and  $m_3 = 10(33k^2 - 12k + 1)$ .
- ( $7^0$ )  $G$  is a strongly regular graph of order  $n = 3(11k + 2)^2$  and degree  $r = 5(33k^2 + 12k + 1)$  with  $\tau = 75k^2 + 42k + 4$  and  $\theta = 15k(5k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 30k + 5$  and  $\lambda_3 = -(3k + 1)$  with  $m_2 = 33k^2 + 12k + 1$  and  $m_3 = 10(33k^2 + 12k + 1)$ .
- ( $\bar{7}^0$ )  $G$  is a strongly regular graph of order  $n = 3(11k + 2)^2$  and degree  $r = 6(33k^2 + 12k + 1)$  with  $\tau = 27k(4k + 1)$  and  $\theta = 6(3k + 1)(6k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 3k$  and  $\lambda_3 = -(30k + 6)$  with  $m_2 = 10(33k^2 + 12k + 1)$  and  $m_3 = 33k^2 + 12k + 1$ .
- ( $8^0$ )  $G$  is a strongly regular graph of order  $n = 5(11k - 3)^2$  and degree  $r = 2(55k^2 - 30k + 4)$  with  $\tau = 20k^2 - 33k + 7$  and  $\theta = 2k(10k - 3)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = 3k - 1$  and  $\lambda_3 = -(30k - 8)$  with  $m_2 = 10(55k^2 - 30k + 4)$  and  $m_3 = 55k^2 - 30k + 4$ .
- ( $\bar{8}^0$ )  $G$  is a strongly regular graph of order  $n = 5(11k - 3)^2$  and degree  $r = 9(55k^2 - 30k + 4)$  with  $\tau = 27(3k - 1)(5k - 1)$  and  $\theta = 9(3k - 1)(15k - 4)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = 30k - 9$  and  $\lambda_3 = -3k$  with  $m_2 = 55k^2 - 30k + 4$  and  $m_3 = 10(55k^2 - 30k + 4)$ .
- ( $9^0$ )  $G$  is a strongly regular graph of order  $n = 5(11k + 3)^2$  and degree  $r = 2(55k^2 + 30k + 4)$  with  $\tau = 20k^2 + 33k + 7$  and  $\theta = 2k(10k + 3)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 30k + 8$  and  $\lambda_3 = -(3k + 1)$  with  $m_2 = 55k^2 + 30k + 4$  and  $m_3 = 10(55k^2 + 30k + 4)$ .
- ( $\bar{9}^0$ )  $G$  is a strongly regular graph of order  $n = 5(11k + 3)^2$  and degree  $r = 9(55k^2 + 30k + 4)$  with  $\tau = 27(3k + 1)(5k + 1)$  and  $\theta = 9(3k + 1)(15k + 4)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 3k$  and  $\lambda_3 = -(30k + 9)$  with  $m_2 = 10(55k^2 + 30k + 4)$  and  $m_3 = 55k^2 + 30k + 4$ .
- ( $10^0$ )  $G$  is a strongly regular graph of order  $n = 15(11k - 5)^2$  and degree  $r = 3(165k^2 - 150k + 34)$  with  $\tau = 3(45k^2 - 54k + 15)$  and  $\theta = 3(3k - 1)(15k - 7)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 6k - 3$  and  $\lambda_3 = -(60k - 27)$  with  $m_2 = 10(165k^2 - 150k + 34)$  and  $m_3 = 165k^2 - 150k + 34$ .
- ( $\bar{10}^0$ )  $G$  is a strongly regular graph of order  $n = 15(11k - 5)^2$  and degree  $r = 8(165k^2 - 150k + 34)$  with  $\tau = 2(480k^2 - 429k + 95)$  and  $\theta = 24(2k - 1)(20k - 9)$ ,

- where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 60k - 28$  and  $\lambda_3 = -(6k - 2)$  with  $m_2 = 165k^2 - 150k + 34$  and  $m_3 = 10(165k^2 - 150k + 34)$ .
- (11<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 15(11k + 5)^2$  and degree  $r = 3(165k^2 + 150k + 34)$  with  $\tau = 3(45k^2 + 54k + 15)$  and  $\theta = 3(3k + 1)(15k + 7)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 60k + 27$  and  $\lambda_3 = -(6k + 3)$  with  $m_2 = 165k^2 + 150k + 34$  and  $m_3 = 10(165k^2 + 150k + 34)$ .
- ( $\overline{11}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 15(11k + 5)^2$  and degree  $r = 8(165k^2 + 150k + 34)$  with  $\tau = 2(480k^2 + 429k + 95)$  and  $\theta = 24(2k + 1)(20k + 9)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 6k + 2$  and  $\lambda_3 = -(60k + 28)$  with  $m_2 = 10(165k^2 + 150k + 34)$  and  $m_3 = 165k^2 + 150k + 34$ .
- (12<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 70(11k - 5)^2$  and degree  $r = 4(770k^2 - 700k + 159)$  with  $\tau = 2(560k^2 - 469k + 97)$  and  $\theta = 28(2k - 1)(20k - 9)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 140k - 64$  and  $\lambda_3 = -(14k - 6)$  with  $m_2 = 770k^2 - 700k + 159$  and  $m_3 = 10(770k^2 - 700k + 159)$ .
- ( $\overline{12}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 70(11k - 5)^2$  and degree  $r = 7(770k^2 - 700k + 159)$  with  $\tau = 14(245k^2 - 226k + 52)$  and  $\theta = 14(7k - 3)(35k - 16)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 14k - 7$  and  $\lambda_3 = -(140k - 63)$  with  $m_2 = 10(770k^2 - 700k + 159)$  and  $m_3 = 770k^2 - 700k + 159$ .
- (13<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 70(11k + 5)^2$  and degree  $r = 4(770k^2 + 700k + 159)$  with  $\tau = 2(560k^2 + 469k + 97)$  and  $\theta = 28(2k + 1)(20k + 9)$ , where  $k \geq 0$ . Its eigenvalues are  $\lambda_2 = 14k + 6$  and  $\lambda_3 = -(140k + 64)$  with  $m_2 = 10(770k^2 + 700k + 159)$  and  $m_3 = 770k^2 + 700k + 159$ .
- ( $\overline{13}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 70(11k + 5)^2$  and degree  $r = 7(770k^2 + 700k + 159)$  with  $\tau = 14(245k^2 + 226k + 52)$  and  $\theta = 14(7k + 3)(35k + 16)$ , where  $k \geq 0$ . Its eigenvalues are  $\lambda_2 = 140k + 63$  and  $\lambda_3 = -(14k + 7)$  with  $m_2 = 770k^2 + 700k + 159$  and  $m_3 = 10(770k^2 + 700k + 159)$ .

**Theorem 2.10** ([7]). *Let  $G$  be a connected strongly regular graph of order  $n$  and degree  $r$  with  $m_2 = 11m_3$  or  $m_3 = 11m_2$ . Then,  $G$  is one of the following strongly regular graphs.*

- (1<sup>0</sup>)  $G$  is the strongly regular graph  $\overline{11K_{11}}$  of order  $n = 121$  and degree  $r = 110$  with  $\tau = 99$  and  $\theta = 110$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -11$  with  $m_2 = 110$  and  $m_3 = 10$ .
- (2<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (12k - 1)^2$  and degree  $r = 2k(6k - 1)$  with  $\tau = k^2 + 9k - 1$  and  $\theta = k(k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = 11k - 1$  and  $\lambda_3 = -k$  with  $m_2 = 2k(6k - 1)$  and  $m_3 = 22k(6k - 1)$ .
- ( $\overline{2}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (12k - 1)^2$  and degree  $r = 22k(6k - 1)$  with  $\tau = 121k^2 - 21k - 1$  and  $\theta = 11k(11k - 1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = k - 1$  and  $\lambda_3 = -11k$  with  $m_2 = 22k(6k - 1)$  and  $m_3 = 2k(6k - 1)$ .
- (3<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = (12k + 1)^2$  and degree  $r = 2k(6k + 1)$  with  $\tau = k^2 - 9k - 1$  and  $\theta = k(k + 1)$ , where  $k \geq 10$ . Its eigenvalues are  $\lambda_2 = k$  and  $\lambda_3 = -(11k + 1)$  with  $m_2 = 22k(6k + 1)$  and  $m_3 = 2k(6k + 1)$ .

- ( $\bar{3}^0$ )  $G$  is a strongly regular graph of order  $n = (12k+1)^2$  and degree  $r = 22k(6k+1)$  with  $\tau = 121k^2 + 21k - 1$  and  $\theta = 11k(11k+1)$ , where  $k \geq 10$ . Its eigenvalues are  $\lambda_2 = 11k$  and  $\lambda_3 = -(k+1)$  with  $m_2 = 2k(6k+1)$  and  $m_3 = 22k(6k+1)$ .
- ( $4^0$ )  $G$  is a strongly regular graph of order  $n = 385(12k-5)^2$  and degree  $r = 5(4620k^2 - 3850k + 802)$  with  $\tau = 5(1925k^2 - 1645k + 351)$  and  $\theta = 35(5k-2)(55k-23)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 35k-15$  and  $\lambda_3 = -(385k-160)$  with  $m_2 = 11(4620k^2 - 3850k + 802)$  and  $m_3 = 4620k^2 - 3850k + 802$ .
- ( $\bar{4}^0$ )  $G$  is a strongly regular graph of order  $n = 385(12k-5)^2$  and degree  $r = 7(4620k^2 - 3850k + 802)$  with  $\tau = 7(2695k^2 - 2225k + 459)$  and  $\theta = 35(7k-3)(77k-32)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 385k - 161$  and  $\lambda_3 = -(35k-14)$  with  $m_2 = 4620k^2 - 3850k + 802$  and  $m_3 = 11(4620k^2 - 3850k + 802)$ .
- ( $5^0$ )  $G$  is a strongly regular graph of order  $n = 385(12k+5)^2$  and degree  $r = 5(4620k^2 + 3850k + 802)$  with  $\tau = 5(1925k^2 + 1645k + 351)$  and  $\theta = 35(5k+2)(55k+23)$ , where  $k \geq 0$ . Its eigenvalues are  $\lambda_2 = 385k + 160$  and  $\lambda_3 = -(35k+15)$  with  $m_2 = 4620k^2 + 3850k + 802$  and  $m_3 = 11(4620k^2 + 3850k + 802)$ .
- ( $\bar{5}^0$ )  $G$  is a strongly regular graph of order  $n = 385(12k+5)^2$  and degree  $r = 7(4620k^2 + 3850k + 802)$  with  $\tau = 7(2695k^2 + 2225k + 459)$  and  $\theta = 35(7k+3)(77k+32)$ , where  $k \geq 0$ . Its eigenvalues are  $\lambda_2 = 35k+14$  and  $\lambda_3 = -(385k+161)$  with  $m_2 = 11(4620k^2 + 3850k + 802)$  and  $m_3 = 4620k^2 + 3850k + 802$ .

**Theorem 2.11** ([7]). *Let  $G$  be a connected strongly regular graph of order  $n$  and degree  $r$  with  $m_2 = 12m_3$  or  $m_3 = 12m_2$ . Then,  $G$  is one of the following strongly regular graphs.*

- ( $1^0$ )  $G$  is the complete bipartite graph  $K_{7,7}$  of order  $n = 14$  and degree  $r = 7$  with  $\tau = 0$  and  $\theta = 7$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -7$  with  $m_2 = 12$  and  $m_3 = 1$ .
- ( $2^0$ )  $G$  is the strongly regular graph  $\overline{3K_9}$  of order  $n = 27$  and degree  $r = 18$  with  $\tau = 9$  and  $\theta = 18$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -9$  with  $m_2 = 24$  and  $m_3 = 2$ .
- ( $3^0$ )  $G$  is the strongly regular graph  $\overline{4K_{10}}$  of order  $n = 40$  and degree  $r = 30$  with  $\tau = 20$  and  $\theta = 30$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -10$  with  $m_2 = 36$  and  $m_3 = 3$ .
- ( $4^0$ )  $G$  is the strongly regular graph  $\overline{6K_{11}}$  of order  $n = 66$  and degree  $r = 55$  with  $\tau = 44$  and  $\theta = 55$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -11$  with  $m_2 = 60$  and  $m_3 = 5$ .
- ( $5^0$ )  $G$  is the strongly regular graph  $\overline{12K_{12}}$  of order  $n = 144$  and degree  $r = 132$  with  $\tau = 120$  and  $\theta = 132$ . Its eigenvalues are  $\lambda_2 = 0$  and  $\lambda_3 = -12$  with  $m_2 = 132$  and  $m_3 = 11$ .
- ( $6^0$ )  $G$  is a strongly regular graph of order  $n = (13k-1)^2$  and degree  $r = k(13k-2)$  with  $\tau = k^2 + 10k - 1$  and  $\theta = k(k-1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = 12k - 1$  and  $\lambda_3 = -k$  with  $m_2 = k(13k-2)$  and  $m_3 = 12k(13k-2)$ .

- ( $\bar{6}^0$ )  $G$  is a strongly regular graph of order  $n = (13k-1)^2$  and degree  $r = 12k(13k-2)$  with  $\tau = 144k^2 - 23k - 1$  and  $\theta = 12k(12k-1)$ , where  $k \geq 2$ . Its eigenvalues are  $\lambda_2 = k-1$  and  $\lambda_3 = -12k$  with  $m_2 = 12k(13k-2)$  and  $m_3 = k(13k-2)$ .
- ( $7^0$ )  $G$  is a strongly regular graph of order  $n = (13k+1)^2$  and degree  $r = k(13k+2)$  with  $\tau = k^2 - 10k - 1$  and  $\theta = k(k+1)$ , where  $k \geq 11$ . Its eigenvalues are  $\lambda_2 = k$  and  $\lambda_3 = -(12k+1)$  with  $m_2 = 12k(13k+2)$  and  $m_3 = k(13k+2)$ .
- ( $\bar{7}^0$ )  $G$  is a strongly regular graph of order  $n = (13k+1)^2$  and degree  $r = 12k(13k+2)$  with  $\tau = 144k^2 + 23k - 1$  and  $\theta = 12k(12k+1)$ , where  $k \geq 11$ . Its eigenvalues are  $\lambda_2 = 12k$  and  $\lambda_3 = -(k+1)$  with  $m_2 = k(13k+2)$  and  $m_3 = 12k(13k+2)$ .
- ( $8^0$ )  $G$  is a strongly regular graph of order  $n = 3(13k-3)^2$  and degree  $r = 4(39k^2 - 18k + 2)$  with  $\tau = 48k^2 - 45k + 7$  and  $\theta = 12k(4k-1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 3k-1$  and  $\lambda_3 = -(36k-8)$  with  $m_2 = 12(39k^2 - 18k + 2)$  and  $m_3 = 39k^2 - 18k + 2$ .
- ( $\bar{8}^0$ )  $G$  is a strongly regular graph of order  $n = 3(13k-3)^2$  and degree  $r = 9(39k^2 - 18k + 2)$  with  $\tau = 3(81k^2 - 34k + 3)$  and  $\theta = 9(3k-1)(9k-2)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 36k-9$  and  $\lambda_3 = -3k$  with  $m_2 = 39k^2 - 18k + 2$  and  $m_3 = 12(39k^2 - 18k + 2)$ .
- ( $9^0$ )  $G$  is a strongly regular graph of order  $n = 3(13k+3)^2$  and degree  $r = 4(39k^2 + 18k + 2)$  with  $\tau = 48k^2 + 45k + 7$  and  $\theta = 12k(4k+1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 36k+8$  and  $\lambda_3 = -(3k+1)$  with  $m_2 = 39k^2 + 18k + 2$  and  $m_3 = 12(39k^2 + 18k + 2)$ .
- ( $\bar{9}^0$ )  $G$  is a strongly regular graph of order  $n = 3(13k+3)^2$  and degree  $r = 9(39k^2 + 18k + 2)$  with  $\tau = 3(81k^2 + 34k + 3)$  and  $\theta = 9(3k+1)(9k+2)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 3k$  and  $\lambda_3 = -(36k+9)$  with  $m_2 = 12(39k^2 + 18k + 2)$  and  $m_3 = 39k^2 + 18k + 2$ .
- ( $10^0$ )  $G$  is a strongly regular graph of order  $n = 10(13k-2)^2$  and degree  $r = 3(130k^2 - 40k + 3)$  with  $\tau = 90k^2 - 70k + 8$  and  $\theta = 15k(6k-1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 5k-1$  and  $\lambda_3 = -(60k-9)$  with  $m_2 = 12(130k^2 - 40k + 3)$  and  $m_3 = 130k^2 - 40k + 3$ .
- ( $\bar{10}^0$ )  $G$  is a strongly regular graph of order  $n = 10(13k-2)^2$  and degree  $r = 10(130k^2 - 40k + 3)$  with  $\tau = 5(200k^2 - 59k + 4)$  and  $\theta = 10(5k-1)(20k-3)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 60k-10$  and  $\lambda_3 = -5k$  with  $m_2 = 130k^2 - 40k + 3$  and  $m_3 = 12(130k^2 - 40k + 3)$ .
- ( $11^0$ )  $G$  is a strongly regular graph of order  $n = 10(13k+2)^2$  and degree  $r = 3(130k^2 + 40k + 3)$  with  $\tau = 90k^2 + 70k + 8$  and  $\theta = 15k(6k+1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 60k+9$  and  $\lambda_3 = -(5k+1)$  with  $m_2 = 130k^2 + 40k + 3$  and  $m_3 = 12(130k^2 + 40k + 3)$ .
- ( $\bar{11}^0$ )  $G$  is a strongly regular graph of order  $n = 10(13k+2)^2$  and degree  $r = 10(130k^2 + 40k + 3)$  with  $\tau = 5(200k^2 + 59k + 4)$  and  $\theta = 10(5k+1)(20k+3)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 5k$  and  $\lambda_3 = -(60k+10)$  with  $m_2 = 12(130k^2 + 40k + 3)$  and  $m_3 = 130k^2 + 40k + 3$ .

- (12<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 14(13k - 1)^2$  and degree  $r = 6(182k^2 - 28k + 1)$  with  $\tau = 504k^2 - 119k + 5$  and  $\theta = 42k(12k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 7k - 1$  and  $\lambda_3 = -(84k - 6)$  with  $m_2 = 12(182k^2 - 28k + 1)$  and  $m_3 = 182k^2 - 28k + 1$ .
- ( $\overline{12}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 14(13k - 1)^2$  and degree  $r = 7(182k^2 - 28k + 1)$  with  $\tau = 14k(49k - 5)$  and  $\theta = 7(7k - 1)(14k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 84k - 7$  and  $\lambda_3 = -7k$  with  $m_2 = 182k^2 - 28k + 1$  and  $m_3 = 12(182k^2 - 28k + 1)$ .
- (13<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 14(13k + 1)^2$  and degree  $r = 6(182k^2 + 28k + 1)$  with  $\tau = 504k^2 + 119k + 5$  and  $\theta = 42k(12k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 84k + 6$  and  $\lambda_3 = -(7k + 1)$  with  $m_2 = 182k^2 + 28k + 1$  and  $m_3 = 12(182k^2 + 28k + 1)$ .
- ( $\overline{13}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 14(13k + 1)^2$  and degree  $r = 7(182k^2 + 28k + 1)$  with  $\tau = 14k(49k + 5)$  and  $\theta = 7(7k + 1)(14k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 7k$  and  $\lambda_3 = -(84k + 7)$  with  $m_2 = 12(182k^2 + 28k + 1)$  and  $m_3 = 182k^2 + 28k + 1$ .
- (14<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 30(13k - 6)^2$  and degree  $r = 5(390k^2 - 360k + 83)$  with  $\tau = 10(75k^2 - 76k + 19)$  and  $\theta = 10(5k - 2)(15k - 7)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 10k - 5$  and  $\lambda_3 = -(120k - 55)$  with  $m_2 = 12(390k^2 - 360k + 83)$  and  $m_3 = 390k^2 - 360k + 83$ .
- ( $\overline{14}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 30(13k - 6)^2$  and degree  $r = 8(390k^2 - 360k + 83)$  with  $\tau = 2(960k^2 - 865k + 194)$  and  $\theta = 40(2k - 1)(24k - 11)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 120k - 56$  and  $\lambda_3 = -(10k - 4)$  with  $m_2 = 390k^2 - 360k + 83$  and  $m_3 = 12(390k^2 - 360k + 83)$ .
- (15<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 30(13k + 6)^2$  and degree  $r = 5(390k^2 + 360k + 83)$  with  $\tau = 10(75k^2 + 76k + 19)$  and  $\theta = 10(5k + 2)(15k + 7)$ , where  $k \geq 0$ . Its eigenvalues are  $\lambda_2 = 120k + 55$  and  $\lambda_3 = -(10k + 5)$  with  $m_2 = 390k^2 + 360k + 83$  and  $m_3 = 12(390k^2 + 360k + 83)$ .
- ( $\overline{15}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 30(13k + 6)^2$  and degree  $r = 8(390k^2 + 360k + 83)$  with  $\tau = 2(960k^2 + 865k + 194)$  and  $\theta = 40(2k + 1)(24k + 11)$ , where  $k \geq 0$ . Its eigenvalues are  $\lambda_2 = 10k + 4$  and  $\lambda_3 = -(120k + 56)$  with  $m_2 = 12(390k^2 + 360k + 83)$  and  $m_3 = 390k^2 + 360k + 83$ .
- (16<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 66(13k - 1)^2$  and degree  $r = 2(858k^2 - 132k + 5)$  with  $\tau = 264k^2 - 143k + 9$  and  $\theta = 22k(12k - 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 11k - 1$  and  $\lambda_3 = -(132k - 10)$  with  $m_2 = 12(858k^2 - 132k + 5)$  and  $m_3 = 858k^2 - 132k + 5$ .
- ( $\overline{16}$ <sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 66(13k - 1)^2$  and degree  $r = 11(858k^2 - 132k + 5)$  with  $\tau = 22(11k - 1)(33k - 2)$  and  $\theta = 11(11k - 1)(66k - 5)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 132k - 11$  and  $\lambda_3 = -11k$  with  $m_2 = 858k^2 - 132k + 5$  and  $m_3 = 12(858k^2 - 132k + 5)$ .

- (17<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 66(13k + 1)^2$  and degree  $r = 2(858k^2 + 132k + 5)$  with  $\tau = 264k^2 + 143k + 9$  and  $\theta = 22k(12k + 1)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 132k + 10$  and  $\lambda_3 = -(11k + 1)$  with  $m_2 = 858k^2 + 132k + 5$  and  $m_3 = 12(858k^2 + 132k + 5)$ .
- (17<sup>0</sup>)  $G$  is a strongly regular graph of order  $n = 66(13k + 1)^2$  and degree  $r = 11(858k^2 + 132k + 5)$  with  $\tau = 22(11k + 1)(33k + 2)$  and  $\theta = 11(11k + 1)(66k + 5)$ , where  $k \in \mathbb{N}$ . Its eigenvalues are  $\lambda_2 = 11k$  and  $\lambda_3 = -(132k + 11)$  with  $m_2 = 12(858k^2 + 132k + 5)$  and  $m_3 = 858k^2 + 132k + 5$ .

### 3. CONCLUDING REMARKS

Using equations (1.3) and (1.4), and applying the same procedure as in articles [4–7], we can establish the parameters  $n$ ,  $r$ ,  $\tau$  and  $\theta$  for strongly regular graphs with  $m_2 = qm_3$  and  $m_3 = qm_2$  for any fixed value  $q \in \mathbb{N}$ , by using only one parameter  $k$ . All results presented in this work has been verified using a computer program `srgpar.exe`, which was written by the author in the programming language `Borland C++ Builder 5.5` using only one parameter  $k$ .

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