UNIVERSITY OF KRAGUJEVAC
FACULTY OF SCIENCE

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SERBIAN MATHEMATICAL CONGRESS

BOOK OF ABSTRACTS

UNIVERSITY OF KRAGUJEVAC
FACULTY OF SCIENCE

XIV SERBIAN MATHEMATICAL CONGRESS
(14SMAK 2018)

BOOK OF ABSTRACTS

May 16–19, 2018
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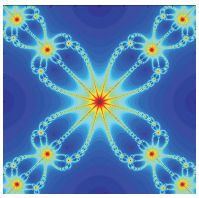

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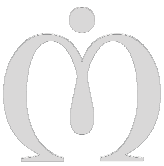

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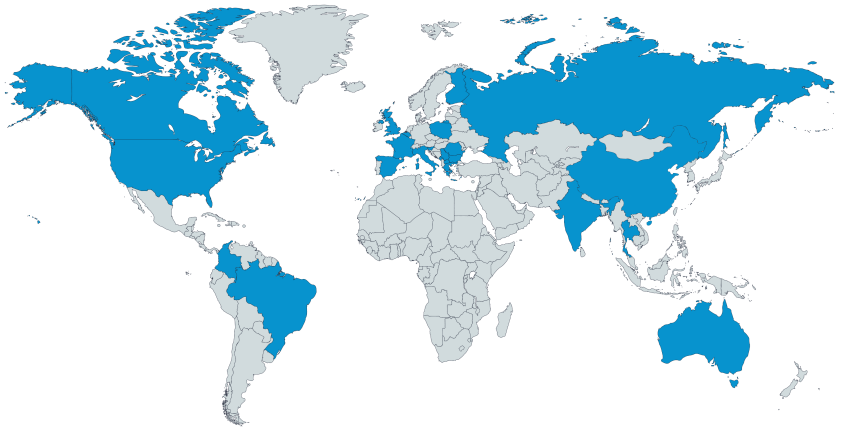

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HOST CITY – KRAGUJEVAC

When you come to Kragujevac for the first time, the first your hosts will say is the following: "Kragujevac is the heart of Šumadija and Serbia". "Kragujevac has always been first in everything!" However, you should not think that it is a matter of host who can boast! Here are some interesting facts about the city you will be welcome.

Kragujevac is situated in the central part of Serbia on the banks of the Lepenica River, between the flanks of mountains Rudnik, Crni vrh and Gledićke planine. The area covers 835 square kilometres at an altitude of 173-200 metres. The geographic centre of Serbia is 8 kilometres northwest of the city centre. Kragujevac is the fifth largest city in Serbia (after Belgrade, Novi Sad, Niš and Priština) with around 190,000 inhabitants and is recognised as the driver of the industrial, economic, cultural and educational development of Šumadija and Pomoravlje. In the area of Kragujevac, there are numerous material remains from prehistoric period. Illyrians, Romans, and later Slavs significantly affected the development of this region. The town was first recorded in the Ottoman population census (defter) in 1476 as Kragujfoča (although some other sources included different names Kragujevdž and Karagovindža). The name of Kragujevac is connected with the bird "kraguj" – a species of the hawk used for hunting.

On May 6, 1818, at the Assembly of National Elections in the Monastery Vraćevšnica, Kragujevac was elected the capital. Before the relocation of the capital to Belgrade, Kragujevac evolved as the first cultural and educational centre. Thus, in 1807 was established the elementary school which was attended by male children; the Gymnasium followed in 1833; Licej (the forerunner of Belgrade University) started in 1838; the General-craft School opened in 1845; the Military-craft School began its work in 1845 and the Teachers' Training School was established in 1870.

The College for females was founded in 1891, and in 1903 the Teachers Training School for females was opened.

For the first time in the history of the Serbian state, a whole set of institutions were established in Kragujevac. The first judicial institution, the Court of Kragujevac, was established in 1820. The first newspaper, "Novine srbske", were moved from Belgrade to Kragujevac in 1834. During this period, a Slavo-Serb writer Joakim Vujić established the theatre in Kragujevac – Knjaževsko-srpski teatar – in 1835, and then, as the musical program accompanying the main event, the choir – Knjaževsko-srbska banda – appeared under the leadership of the chapel-master, Josip Schlesinger.

During the First World War, Kragujevac was once again the capital city where the Regent Aleksandar Karađorđević resided as well as the Supreme Command of the Serbian Army led by the general Radomir Putnik. As an important strategic centre, Kragujevac was frequently demolished in both wars. In the Second World War, in the mass shooting of the civilian population on October 21, 1941, the Germans killed about 7,000 people residing in Kragujevac, including 300 pupils and fifteen children aged 8-12 years. The city was freed on October 21, 1944. The commemorative event "Great School Lesson" – Veliki školski čas – was held for the first time in the Memorial Park of Šumarice in 1957. The Memorial Museum "21st October" – 21. oktobar – dedicated to the victims of Kragujevac was built in 1976.

In the post-war period, Kragujevac has been rapidly evolving. The licence-based agreement with Fiat was signed on August 12, 1954, and the first car – "Fića" – was assembled in the car factory "Zastava" in 1955 when the industrial development of the city commenced, along with the rapid increase in the number of inhabitants, the establishment of the University, and the development of other important institutions.

People in Kragujevac are hard-working and creative. We host numerous international events and festivals: Domestic-script based festival of the best Serbian theatre performances – Joakim Fest; International Small-scene Theatre Festival – Joakim Interfest; International Music Festival – OKTOH, International Jazz Festival – OF, International Festival of Antiwar Caricature; International Chamber Choir Festival; International Puppet Festival – Zlatna iskra; International Art Photography Festival – Fotorama; music festival – Arsenal Fest; Theater Encounters of Serbian Gymnasiums...

FACULTY OF SCIENCE
UNIVERSITY OF KRAGUJEVAC

The most important event for the development of higher education in Serbia was the establishment of the Great School, or so-called Ustanička velika škola and later Dositej's Great School in Belgrade in 1808, which lasted under the rule of Karađorđe until 1813.

As Prince Miloš Obrenović (1780-1860) decided, the Great School moved from Belgrade to Kragujevac in 1833 and there continued to operate as the first secondary school in the Principality of Serbia.

At that time, Serbia was in a great need for highly educated people. At the suggestion of the then Minister of Education, as Prince Miloš decided on July 1, 1838, the Lyceum was established in Kragujevac as the first college in Serbia.

At that time the Lyceum represented the highest-ranking educational institution in Serbia. The first rector of the Lyceum in Kragujevac was Atanasije Nikolić (1803-1882), the professor of enormous creative spirit and energy who greatly contributed to the realization of the idea of higher education in Serbia and its further development as a precursor to the first University. At first, only the Department of Philosophy was founded at the Lyceum. In 1853 the Department of Natural and Technical Sciences was founded at the Lyceum.

Prince Miloš abdicated in 1839, and the state administration moved from Kragujevac to Belgrade. In the same year, the Lyceum was separated from the Gymnasium and continued to operate in Kragujevac until 1841, when it moved to Belgrade – the new capital.

A major reform of the Lyceum took place in 1863, when the Great School was established in Belgrade. The former departments were transformed into the faculties of the Great School: Faculty of Philosophy, Faculty of Law, and Faculty of Engineering. Passing the University Act in 1905, the Great School became the University of Belgrade.

In 1871 the first Teacher Training School in Serbia was established. In 1877, Teacher Training School was moved to Belgrade, and in 1903 again returned to Kragujevac as Women Teacher Training School. But, the development of pedagogical thinking and professional and pedagogical training of teachers in Kragujevac was not over.

Teachers' Training School in Kragujevac ceased to exist in 1971, and the Teachers Training College came to an end in 1975. The activities of the Teachers' Training College in the field of natural sciences and mathematics were the foundations for establishing the corresponding departments of the newly established Faculty of Science in Kragujevac.

In the early sixties of the last century, Kragujevac welcomed the establishment of the Faculty of Economics and the Faculty of Mechanical Engineering as departments of the University of Belgrade. Two additional departments, the Faculty of Law and the Faculty of Science, were established in 1972. Faculty of Science became independent in 1976. In the same year, 138 years after the establishment of the Lyceum, the then University Svetozar Marković and today the University of Kragujevac was founded. Today the University has 12 faculties and over 18, 000 students.

Faculty of Science offers 18 study programmes at all levels of study. The Faculty provides that learning outcomes are based on the qualifications for all three levels of education in the scientific fields of biology, ecology, physics, chemistry, mathematics and computer science in accordance with the descriptors defining levels in the European Qualifications Framework.

A necessary condition for successful education at the Faculty of Science is closely connected with continuous scientific research and professional training of academic staff. Besides education, the academic staff is engaged in a number of national and international projects in the fields of chemistry, biochemistry, biology, ecology, physics, mathematics, informatics and environmental protection. Research projects are conducted at the Faculty, and, sometimes, in collaboration with other research institutions at home and abroad.

Faculty of Science in Kragujevac is the publisher of an international journal – MATCH Communications in Mathematical and in Computer Chemistry (having the impact factor 3.139 in 2017). In addition, the Faculty publishes two national scientific journals: Kragujevac Journal of Mathematics and Kragujevac Journal of Science.

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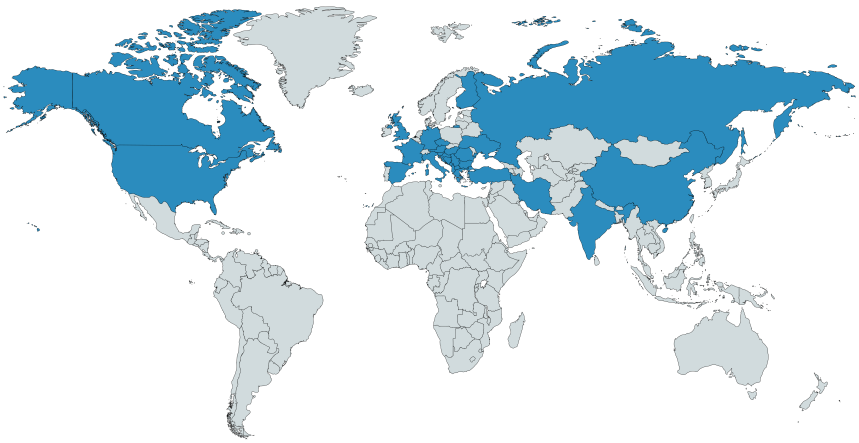

PLENARY LECTURES – ABSTRACTS

Billiards within quadrics, Riemann surfaces, isomonodromy deformations, and extremal polynomials

Vladimir Dragović[1,2]

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[2]Mathematical Institute SANU, Belgrade, Serbia

We will review recent and new results on billiards within confocal quadrics and their dynamical, geometric, and arithmetic properties. By connecting these questions with the analysis on Riemann surfaces, in particular elliptic and hyperelliptic curves, we construct solutions to the Painlevé VI and Schlesinger equations on isomonodromy deformations. We map the billiard dynamics within confocal conics to rectangular billiard dynamics, which leads to a novel concept in the ergodic theory, "the genericity along the curves" (Fraczek, Shi, Ulcigrai). By developing a bridge toward the theory of extremal polynomials and Pell's equations, we derive fundamental properties of the billiard dynamics, winding numbers and frequency map. As an application, we provide a detailed description of periodic trajectories in an arbitrary dimension d with small periods T, $d \leq T \leq 2d$, emphasizing the cases $d = 3$, $d = 4$. In part, the results are joint with Milena Radnović and in part with Vasilisa Shramchenko. The results are obtained as parts of grants 174020 MPNTR and NSF 1444147.

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Constructive theory of orthogonal polynomials and new applications

Gradimir V. Milovanović[1]

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Constructive theory of orthogonal polynomials was developed in eighties in a series of papers by Walter Gautschi. It opened the door for extensive computational work on orthogonal polynomials and their applications not only in mathematics, but in other computational and applied sciences. Beside the basic procedures for numerical generation of coefficients in the three-term recurrence relation for orthogonal polynomials for arbitrary measures, in this lecture we present some details on the stability analysis of such algorithms, Christoffel modifications of the measure and corresponding algorithms, as well as available software. This theory enables the construction of many new classes of strongly non-classical orthogonal polynomials (very often with certain exotic weights), development of other types of orthogonality (s and σ-orthogonality, orthogonality on radial rays, Sobolev type of orthogonality, multiple orthogonality, etc.), applications in diverse areas of applied and numerical analysis (numerical integration, interpolation, integral equations, ...), approximation theory (moment-preserving spline approximation, ...), integration of fast oscillating functions, summation of slowly convergent series, etc. Particular attention will be paid to some of these issues.

Weyl asymptotic formulas for infinite order ΨDOs and Sobolev type spaces

Stevan Pilipović[1]

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I will present results on the Weyl asymptotic formulae for the operators that are not of power-log-type as in the finite order (distributional) setting, but of log-type, which in turn yields that the eigenvalues of infinite order ΨDO, with appropriate assumptions, are "very sparse". The heat kernel analysis needed for the proofs of the Weyl asymptotic formulae for the class of operators is based on the complex powers of hypo-elliptic type ΨDO of infinite order. In this way, we obtain the semigroup $T(t)f = \sum_{j=0}^{\infty} e^{-t\lambda_j}(f,\varphi_j)\varphi_j$, $f \in L^2(\mathbb{R}^d)$, $t \geq 0$, with the infinitesimal generator $-\overline{A}$ (the closure of $-a^w$ in $L^2(\mathbb{R}^d)$) where λ_j and φ_j are the eigenvalues and eigenfunctions of $\overline{A}$; a^w is the Wayl operator for the symbol a.

Infinite order Sobolev type spaces $H^*_{A_p,\rho}(f)$, where the order is given by a functions f belonging to a certain class of "admissible" functions of sub-exponential (i.e. ultrapolynomial) growth. $H^*_{A_p,\rho}(f)$ satisfies most of the familiar results for the classical, finite order, Sobolev spaces. Moreover, I will present the Fredholm properties of infinite order ΨDOs having hypoelliptic symbols satisfying elliptic bounds with respect to an admissible function f.

The talk is based on collaborative works with Bojan Prangoski and Jasson Vindas.

Finite element approximation of implicitly constituted fluid flow models

Endre Süli[1]

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Classical models describing the motion of Newtonian fluids, such as water, rely on the assumption that the Cauchy stress is a linear function of the symmetric part of the velocity gradient of the fluid. This assumption leads to the Navier-Stokes equations. It is known however that the framework of classical continuum mechanics, built upon an explicit constitutive equation for the Cauchy stress, is too narrow to describe inelastic behavior of solid-like materials or viscoelastic properties of materials. Our starting point in this work is therefore a generalization of the classical framework of continuum mechanics, called the *implicit constitutive theory*, which was proposed recently in a series of papers by Rajagopal. The underlying principle of the implicit constitutive theory in the context of viscous flows is the following: instead of demanding that the Cauchy stress is an explicit (and, in particular, linear) function of the symmetric part of the velocity gradient, one may allow a nonlinear, implicit and not necessarily continuous relationship between these quantities. The resulting general theory therefore admits non-Newtonian fluid flow models with implicit and possibly discontinuous power-law-like rheology.

We develop the analysis of finite element approximations of implicit power-law-like models for viscous incompressible fluids. The Cauchy stress and the symmetric part of the velocity gradient in the class of models under consideration are related by a, possibly multi-valued, maximal monotone graph. Using a variety of weak compactness techniques, including Chacon's biting lemma, we show that a subsequence of the sequence of finite element solutions converges to a weak solution of the problem as the discretization parameter, measuring the granularity of the finite element triangulation, tends to zero. A key new technical tool in our analysis is a finite element counterpart of the Acerbi-Fusco Lipschitz truncation of Sobolev functions.

The talk is based on a series of recent papers with Lars Diening (Bielefeld) and Christian Kreuzer (Dortmund), and ongoing research with Tabea Tscherpel (Oxford).

Towards a structure theory of Maharam algebras

Boban Veličković[1]

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Maharam algebras are complete Boolean algebras that admit a strictly positive continuous submeasure. The famous Control Measure Problem (formulated by D. Maharam in the 1940s) asks if every Maharam problem is in fact a measure algebras. This problem was resolved 2005 by Talagrand who produced a counterexample. We survey some old and some new results on the structure of Maharam algebras. In particular, we discuss a construction (joint with Z. Perovic) of Maharam algebras of arbitrary high countable exhaustivity rank.

On applications of Algebraic topology (Chessboard complexes)

Siniša Vrećica[1]

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The results of Algebraic topology were successfully applied in establishing many very important results in different areas of Mathematics, such as Fundamental theorem of algebra, Brouwer fixed point theorem (which was called Fundamental theorem of analysis when appeared), the ham sandwich theorem, the proof of Kneser conjecture etc.

We illustrate the applicability of topological methods and results by presenting an important configuration space - chessboard complex, and by showing how its properties could be used in solving the problems in other areas of Mathematics. Chessboard complex appears in different ways: as a coset complex of the symmetric group by some of its subgroups (stabilizing some elements), as a matching complex of a complete bipartite graph, as a complex of partial injective functions from one finite set to the other, as a deleted join of a finite set.

Actually, we define several versions of this complex and show how each of them is motivated by some mathematical question. For example, we show how a cycle-free chessboard complex appears in establishing the symmetric analogue of the cyclic homology of algebras, and how generalized and symmetrized versions appear in establishing the generalizations of van Kampen-Flores theorem and Tverberg-type theorems.

Our dominant interest is in the connectivity properties of a chessboard complex (which reduces to determining its homology groups), but we consider some other properties as well.

The talk is based on joint papers with R. Živaljević, and some with D. Jojić.

WORKSHOP LECTURERS – ABSTRACTS

WORKSHOP: TIME-FREQUENCY ANALYSIS

Space of tempered distributions on positive orthant

Smiljana Jakšić[1]

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We start this talk by the Laguerre characterization of the space of tempered distributions on positive orthant. We apply the characterization to prove boundedness of Weyl pseudo-differential operators with radial symbols on the space of tempered distributions.

Translation-modulation invariant Banach spaces of distributions and ultradistributions

Stevan Pilipović[1]

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We introduce and study a new class of translation-modulation invariant Banach spaces of distributions and ultradistributions. These spaces show stability under Fourier transform and tensor products; furthermore, they have a natural Banach convolution module structure over a certain associated Beurling algebra, as well as a Banach multiplication module structure over an associated Wiener-Beurling algebra. We also investigate a new class of modulation spaces, the Banach spaces of distributions $\mathcal{M}^F$ on $\mathbb{R}^d$, associated to translation-modulation invariant Banach spaces of distributions and ultradistributions F on $\mathbb{R}^{2d}$.

Joint work with P. Dimovski, B. Prangoski and J. Vindas.

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Fréchet frames for spaces of distributions and ultradistributions

Diana T. Stoeva[1]

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In this talk we focus on certain localized frames and show that they form Fréchet frames for appropriately chosen Fréchet spaces, in particular, for the Schwartz space of rapidly decreasing functions and for spaces of sub-exponentially decreasing functions. Furthermore, we consider series expansions and characterizations of the above mentioned spaces and their duals, based on the Hermite orthonormal bases, and extend them to the frame-setting. Finally, we introduce a more general "localization"-concept and present corresponding results.

The talk is based on a joint work with Stevan Pilipović.

Wave front sets and time-frequency analysis

Nenad Teofanov[1]

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The concept of wave front set is the main notion in microlocal analysis. We present different situations where this concept turned out to be useful. More precisely, we illustrate its use in the geometric separation problem, quantum field theory and partial differential equations. Then we give a short overview of different definitions of wave front sets, adjusted to particular aims. For example, we consider wave front sets in the context of Gevrey and extended Gevrey regularity, cf. [4, 5, 6].

The second part of the lecture is devoted time-frequency analysis of wave front sets. In particular, we take a look at micro-local analysis of modulation spaces [2, 3] and discuss the relation between the continuum and the discrete counterparts [1].

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Time-frequency analysis on compact groups

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Phase-space analysis or time-frequency analysis is a subfield of Fourier analysis. It is traditionally exercised in Euclidean spaces, presenting signals simultaneously both in time and in frequency. We introduce a natural family of time-frequency transforms for signals on any compact group. We also study the properties of the related pseudo-differential operators for signal processing.

WORKSHOP: ALGEBRAIC TOPOLOGY – METHODS, APPLICATIONS AND COMPUTATION

The multi-cover persistence of Euclidean balls

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Given a locally finite set X in $\mathbb{R}^d$ and a positive radius, the k-fold cover of X and r consists of all points that have k or more points of X within distance r. The order-k Voronoi diagram decomposes the k-fold cover into convex regions, and we use the dual of this decomposition to compute homology and persistence in scale and in depth.

The persistence in depth is interesting from a geometric as well as algorithmic viewpoint. The main tool in understanding its structure is a rhomboid tiling in $\mathbb{R}^{d+1}$ that combines the duals for all values of k into one. We mention a straightforward consequence, namely that the cells in the dual are generically not simplicial, unless $k = 1$ or $d = 1, 2$.

LS-category of moment-angle manifolds and Massey products

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We give various bounds for the Lusternik-Schnirelmann category of moment-angle complexes $\mathcal{Z}_K$ and show how this relates to vanishing of Massey products in $H^*(\mathcal{Z}_K)$. In particular, we characterise the Lusternik-Schnirelmann category of moment-angle manifolds $\mathcal{Z}_K$ over triangulated d-spheres K for $d \leq 2$, as well as higher dimension spheres built up via connected sum, join, and vertex doubling operations. This characterisation is given in terms of the combinatorics of K, the cup product length of $H^*(\mathcal{Z}_K)$, as well as a certain Massey products. Some of the applications include calculations of the Lusternik-Schnirelmann category and the description of conditions for vanishing of Massey products for moment-angle complexes over fullerenes and k-neighbourly complexes.

Chessboard complex and its generalizations

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The *chessboard complex* $\Delta_{n,m}$ can be defined as the simplicial complex whose faces are all non-taking rook placements (no two rooks on the same row or column) on a $m \times n$ "chessboard". This simplicial complex appears in many interesting combinatorial situations, and some of its topological properties (connectivity, the structure of an orientable pseudomanifold) played the fundamental role in the proof of some interesting non-trivial results (colored Tverberg Theorem). There are some natural generalizations of a chessboard complex:

- Chessboard complex on a triangular board $\Psi_{a_n,\ldots,a_1}$ (a left justified board with a_i rows of length i);
- Multiple chessboard complex $\Delta_{m,n}^{k_1,\ldots,k_n;l_1,\ldots,l_m}$ (at most k_i rooks in the i-th row and at most l_j in the j-th column);
- Symmetric multiple chessboard complex

$$\Sigma_{m,n}^{k_1,\ldots,k_n;\mathbf{1}} := \bigcup_{\pi\in G} \Delta_{m,n}^{k_{\pi(1)},\ldots,k_{\pi(n)};1,1,\ldots,1}.$$

We use standard combinatorial tools (shellability and discrete Morse theory) to investigate some topological properties of these complexes. These complexes naturally appear as appropriate configuration spaces for problems of Tverberg type, and improved estimates of their connectivity often leads to new results. Also, we will show that an "optimal multiple chessboard complex" can be naturally interpreted as a relative and a generalization of Bier spheres.

This talk is based on the joint work with S. Vrećica and R. Živaljević.

Groups of automorphisms and integrability in finite terms

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Elementary functions are easy to differentiate but hard to integrate: an indefinite integral of an elementary function is usually not an elementary function [1].

Theorem 1 (Liouville's theorem)**.** *An integral y of $f \in K$ where K is a differential field belongs to an elementary extension of K if and only if y is representable in the form*

$$y(x) = \int_{x_0}^{x} f(t)\,dt = A_0(x) + \sum_{i=1}^{n} \lambda_i \ln A_i(x),$$

where A_i are functions in the field K for $i = 0, \ldots, n$.

For large classes of functions algorithms based on this theorem make it possible to either evaluate an integral or to prove that the integral cannot be "evaluated in finite terms".

In the talk I will discuss a proof of the Liouville's theorem. I will show that it can be proved by the Galois theory arguments. Liouvill's theorem is based on two statements. The first one suggested by Abel deals with a finite group of automorphisms. The second statement deals with an n-dimensional commutative Lie group of automorphisms.

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On Calabi-Yau representatives in the SU-bordism ring

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To find nice geometric representatives of bordism classes and bordism ring generators for various bordism theories has been a classical problem in algebraic and differential topology since 1960s. From that time till nowadays much was done regarding this problem, starting with the influential works by Conner, Floyd, Milnor, Novikov, Stong, and others. In 1958 F. Hirzebruch stated a problem, which remains open until now: to find a nonsingular (connected) complex algebraic variety in a given unitary bordism class. It was proved in 1960s that Milnor hypersurfaces generate the unitary bordism ring over integers, which is a polynomial ring due to a classical result of J. Milnor and S. P. Novikov, and similar generators also exist for unoriented and oriented bordism rings.

In 1962 S. P. Novikov proved that the special unitary bordism ring over integers with 2 reversed is isomorphic to a polynomial ring with one generator in each even real dimension greater than two. Z. Lü and T. E. Panov (2014) constructed a quasitoric representative for each multiplicative generator of this ring, starting with real dimension 10; quasitoric manifolds represent zero in dimensions 4, 6, and 8.

In this talk we are going to discuss Hirzebruch problem for SU-bordism. J.Mosley (2016) proved that a nonsingular complex algebraic variety may not exist in a given SU-bordism class already in dimension 4. However, we show [1] that for each multiplicative generator in the SU-bordism ring such a representative (disconnected in general) can be found using V. V. Batyrev's construction (1993) of Calabi-Yau hypersurfaces in toric Fano varieties over reflexive polyhedra.

This is a joint work with Zhi Lü (Fudan University) and Taras E. Panov (Moscow State University).

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Faithful topological quantum field theories

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It is evident that one aspect of topological quantum field theories (TQFTs) concerns with the corresponding invariants of manifolds. However, the completeness of these invariants is seldom investigated in the literature. This talk is about faithful one and two dimensional TQFTs and its aim is to foreshadow some possible results concerning higher dimensions.

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WORKSHOP: MODERN CHEMICAL GRAPH THEORY

Aspects of the Randic entropy and some related measures

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In this talk, we introduce the Randic entropy and some related measures based on Shannons entropy. Also we discuss some aspects thereof like the uniqueness and also of related measures.

How to find reaction mechanisms?

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A plethora of theoretical and computational methods on different levels of abstraction are used to interpret experimental findings in Chemistry. While this traditional role of Theoretical and Computational Chemistry is very important, e.g. to choose between equally plausible hypothesis, the potential of this discipline as a true discovery tool for fundamentally novel chemical behavior has not yet been fully taped. The specification and identification of recurrent reaction patterns and high-order reaction behavior in reactive systems or the discovery of possible elementary energetic constraints that shape the structure of reaction networks on the large scale, requires an adequate formalism allowing to first express and second study these type of phenomena. Over the past years, we have developed such a formalism, which is rooted in category theory, and models chemical transformation on an atomic level as algebraic graph rewrite. A key feature of our formalism is the possibility to construct arbitrary chemical reaction spaces in a unified manner. This characteristics paves the way for a rigorous static and dynamic analysis of reactive systems as well as to attack questions connected to the temporal ordering of reaction steps or causality. I will present deterministic and stochastic applications of our graph-grammar formalism ranging from enzyme mechanisms to polyketide biosynthesis to the design of metabolic pathways.

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Partition distance in graphs

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If G is a graph and $\mathcal{P}$ is a partition of $V(G)$, then the partition distance of G is the sum of the distance between all pairs of vertices that lie in the same part of $\mathcal{P}$. This concept generalizes several metric concepts (such as the (terminal) Wiener index). It will be demonstrated that the partition distance of a graph can be obtained from the Wiener index of weighted quotient graphs induced by the transitive closure of the Djoković-Winkler relation as well as by any partition coarser than it. Many earlier results follow from the obtained theorems. Applying the main results, upper bounds on the partition distance of trees with prescribed order and radius will also be shown and corresponding extremal trees presented.

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Oriented graphs with extremal skew energy

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An oriented graph has a skew symmetric adjacency matrix. The skew energy of an oriented graph is the sum of the norms of the eigenvalues of its skew symmetric adjacency matrix. In this talk we will survey the results about oriented graphs with maximum or minimum skew energy. Some open questions are also presented.

On two conjectures regarding the set of values of Wiener index

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The Wiener index $W(G)$ of a simple connected graph G is defined as the sum of distances over all pairs of vertices in a graph. Since it was one of the first indices introduced in literature, it was extensively studied. One of the proposed problems was the inverse Wiener index problem, i.e. for a given value w the problem of finding a graph G for which $W(G) = w$, where G can be a graph on any number of vertices. That problem was first solved computationally for w up to 10^8 and then it was fully solved in 2006 when two papers were independently published proving that there are only 49 integers that are not the value of Wiener index for any graph.

Recently, a related Wiened index interval problem was introduced in [1] where the question is what values can Wiener index have on a class of graphs with a given number of vertices and what is the largest interval of consecutive integers among those values. In the same paper some strong results are given on the class $\mathcal{G}_n$ of all simple graphs on n vertices and the following two conjectures are made for the class $\mathcal{T}_n$ of all trees on n vertices ($W[\mathcal{T}_n]$ denotes the set of all values of Wiener index for a tree from $\mathcal{T}_n$, while $W^{int}[\mathcal{T}_n]$ denotes the largest interval of consecutive integers in $W[\mathcal{T}_n]$).

Theorem 2. *The cardinality of* $W[\mathcal{T}_n]$ *equals* $\frac{1}{6}n^3 + \Theta(n^2)$.

Theorem 3. *The cardinality of* $W^{int}[\mathcal{T}_n]$ *equals* $\Theta(n^3)$.

We present the proof of these conjectures which is the strongest possible in terms of the highest power of n.

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Chemistry as a graph rewriting system: generative exploration of chemical space

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Undirected, labeled graph have been recognized as appropriate models of molecules, and indeed underly the established notation in chemistry. Chemical reactions thus are naturally viewed as transformations of (usually not connected) graph. This simple idea is rigorously formalized by graph grammars and thus amenable to practical implementation. Grounded soundly in category theory, the framework can be extended to accomodate also geometric aspects beyond pure topology, i.e., stereochemistry. An important aspect of the mathematical framework is its capability to describe composite rules, setting the stage for computational approaches to disentangle "overall" chemical reactions into elementary steps. The rule based framework, furthermore, provides explicit atom-atom maps.

Starting molecules and reaction rules naturally span chemical spaces, in which molecular types are points and chemical reactions form directed hyperedges. These spaces may be finite or infinite, as in the case of reaction rules that support polymerization. These structure of these spaces fall outside the realm of standard topology, but retains a rich structure that naturally generalizes certain types of proximity or - equivalently - separation spaces. These structures in turn are equivalent to abstract notions of connectivity and reachability.

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Mathematical aspects of Balaban index

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Balaban index is defined as

$$J(G) = \frac{m}{m-n+2} \sum \frac{1}{\sqrt{w(u) \cdot w(v)}},$$

where the sum is taken over all edges of a connected graph G, n and m are the cardinalities of the vertex and the edge set of G, respectively, and $w(u)$ (resp. $w(v)$) denotes the sum of distances from u (resp. v) to all the other vertices of G.

In the talk, I will present the following results regarding this index:

- an upper bound for the Balaban index of regular graphs, and also an improved bound for fullerene graphs;
- graphs of prescirbed order with minimum Balaban index;
- accumulation points of Balaban index;
- Balaban index of nanotubes, etc.

On the Randić's Matrix: old and new

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Inspired by Randić's idea, Wang et al. redefined and renamed the D_{MAX}-matrix of a graph as the *eccentricity matrix*, which is constructed from the distance matrix so that in each row and each column it only retains the eccentricities, while other elements of the distance matrix are set to be zero. In this report, we first review some applications of this matrix in the chemical graph theory, and then partially answer an open problem by its algebraic properties.

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WORKSHOP: DATA SCIENCE

Utilizing combined power of R and Tableau for optimization processes in financial sector

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Inside of financial sector there has been a lot of changes driven by 4th industrial revolution and development of Data Science. Now, more than ever, financial sector is in need for a fast and verifiable complex data structures, detailed data engineering processes that support predictive analytics and all of that wrap up in data storytelling art through mastery of data visualization. All of this and much more is needed to make a good business calls on daily level that can influence life and jobs of thousands of people.

In this talk you shall get an overview how does combine power of R and Tableau can be a game changer in financial sector. Keywords here are portable, supportable and scalable. Through real case scenario in multinational corporation, which NCR corporation is, you shall see how utilizing R and Tableau can help to optimize processes and predict product failures using expert systems method. The NCR corporation is a worlds largest supplier of multivendor ATM software and applications, with a more than 130 years history of doing business in financial sector.

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Anomaly detection at scale and the role of Bayesian ensembles for detector selection

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Anomaly detection services could prove very useful in securing SLA (service-level agreement) requirements, like latency or high-availability, for applications deployed in the cloud. However, a good anomaly detection service (ADS) is itself a non-trivial requirement. Difficulties come from the fact that hundreds of collected infrastructure metrics represent unlabeled, unbalanced, multivariate time series that are usually autocorrelated or otherwise non-stationary in nature and can exhibit complex contextual or collective anomalies and high false positive rate of point anomalies. Furthermore, ADS itself is usually required to serve several other functions besides obvious diagnostics and real-time detection of anomalous events, for example: predictive maintenance, root-cause analysis, alerting, accountability, reporting. First goal of this research project is to scope and properly define data science, data engineering and business-related requirements for ADS solution as cloud-based analytics platform, with synthetic data, that can serve as a blueprint for real-life implementation.

From data science perspective, anomaly detection is predominantly done in unsupervised fashion. There are many approaches to this problem: machine learning detectors like one-class SVM or robust PCA, forecasting methods like ARIMAX or Holt-Winters or deep learning methods for anomaly detection with GANs [1], LSTMs or robust autoencoders [2]. There is a general consensus that combination of multiple detectors into ensembles could be beneficial to overall accuracy of detection, although ensembles for unsupervised anomaly detection are more recent and emerging area of research [3]. Second goal of this research project is to investigate various bayesian ensemble learning models with emphasis on usage of bayesian approach for detector selection. Work presented is a continuation of previous efforts [4], with implementation in pyMC3 python library [5], on Yahoo Webscope S5 dataset [6].

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Feature evaluation and selection: beyond classification

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In data science, feature selection algorithms aim to reduce the dimensionality of the data and increase the performance of prediction models. Several successful feature evaluation approaches exist, mostly focusing on classification. We will present ideas of classical supervised approaches (filter, wrapper, and embedded methods), both heuristic and o ptimization based. We will focus our presentation on recent results in extensions to supervised learning (multi-task, multi-view, and multilabel), as well as unsupervised and semi-supervised learning. We will present important issues in feature subset selection such as stability, redundancy, and higher order interactions.

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Data science in enterprise world competencies - models and technologies

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Data science and machine learning has enormous potential for changing companies, our everyday life and world around us. Some companies, historians and futurists see it as a holy grail of a modern world. Implementing such projects require very special set of skills, technologies and mindset. Data science is on the edge of both scientific and business worlds and a successful data scientists know how to bridge the gap between them in order to bring benefits from the first to latter and vice versa.

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Sentiment analysis of social media posts

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In today's digital world, people regularly post public comments, pictures and videos on various on-line social media and social networking platforms. Such a large volume of data is a valuable source of real-time information about public interests, opinions and attitudes in relation to various topics. In this presentation we will explore automatic analysis of sentiment expressed in social media text posts. We will provide an overview of sentiment analysis approaches with a particular focus on the machine learning one, and provide examples of several practical applications.

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Approximate recursive Bayesian estimation of recurrent neural networks: on-line learning of synaptic weights, neuron activities and network structure

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We have derived algorithms for on-line training of the recurrent neural networks using approximate recursive Bayesian estimation of unknown probabilty density function of its state, represented as concatenated vector of synaptic weights and neuron activities. Joint estimation of synaptic weights and neuron activities generalizes the heuristic known as teacher forcing, which enables filtering out the noise from date during the training. Optimal solution of the recursive Bayesian estimation for recurrent neural natworks is intractable, due to the nonlinearty of the network dynamics, therfore approximate solutons have to be considered. We have derived a class of derivative free algorithms for on-line training of recurrent networks, using Stirling's interpolation formula and the Unscented transformation. For the case when the non-Gaussian (multi modal or heavy tailed) noise is present on training data, we have derived learning algorithms using Gaussian mixture as the approximation of probability density function of the RNN state. Finally, we have used statistics, recursively updated during sequential Bayesian estimation, to derive criteria for growing and pruning of synaptic connections and hidden neurons in recurrent neural networks. The performance of the proposed learning algorithms is demonstrated on problems of chaotic and nonstationary time series prediction, blind signal separation and deconvolution and dynamic system identification.

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CONTRIBUTED TALKS – ABSTRACTS

FIELD 1 – REAL AND FUNCTIONAL ANALYSIS, PROBABILITY AND STATISTICS, DIFFERENTIAL EQUATIONS

The localization of a frame for weighted shift-invariant spaces

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We investigate the concept of the localization of frames and the properties of the dual frame in the weighted shift-invariant spaces

$$V_\mu^p(\Phi) = \left\{ \sum_{i=1}^{r} \sum_{j\in\mathbb{Z}} c_i(j)\phi_i(\cdot - j) \mid \{c_i(j)\}_{j\in\mathbb{Z}} \in \ell_\mu^p,\ i = 1, \dots, r \right\},$$

$p \in [1, \infty]$, with specially chosen functions ϕ_i, $i = 1, \dots, r$. We determine whether the frame operator preserves this localization and the dual frame possesses the same localization properties as the original frame.

On tauberian theorems for statistical weighted Norlund-Cesaro-Euler mean method of summability

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In this paper we establish some new Tauberian theorems for the statistical weighted Norlund-Cesaro-Euler mean method of summability via the weighted Norlund-Cesaro-Euler general control modulo of the oscillatory behavior of nonnegative integer order of a real sequence. The main results improve the well-known classical Tauberian theorems which are given for weighted Norlund-Cesaro-Euler mean method of summability and statistical convergence.

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A further generalization of Kakutani's fixed point theorm in KKM spaces

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In 1941 S. Kakutani proved an important fixed point theorem (any upper semi-continuous nonempty closed convex-valued multifunction $F : K \to K$ has at least one fixed point, where $K \subseteq R^n$ is nonempty, compact and convex set of R^n). In this paper we present new extension of Kakutani's theorem. Our result generalize the fixed point theorems obtained by S. Eilenberg and D. Montgomery, F. H. Bohnenblust and S. Karlin, I. L. Glicksberg, K. Fan, C. J. Himmelberg, E. Tarafdar [5] and S. Park [2, 3, 3, 4].

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Basic properties of an eigenparameter-dependent q-boundary value problem

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This paper is devoted to study a boundary value problem consisting a differential equation of second order with q-Jackson derivative and eigenparameter dependent boundary conditions. We introduce a modified inner product in a suitable direct sum space $L_q^2[0,\pi] \oplus \mathbb{C}^2$ and define a symmetric linear operator in this space in such a way that the considered problem can be interpreted as an eigenvalue problem of this operator. We investigate the eigenvalue and eigenfunction properties of this boundary value problem and we construct Green's function.

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Computing asymptotic formulas for eigenvalues and eigenfunctions of a boundary value problem with retarded argument

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The paper is devoted to study the asymptotic behavior of eigenvalues and eigenfunctions of a boundary value problem which is generated by a differential equation with a retarded argument. The literature of the boundary value problems with retarded argument begins with the works of [1, 2, 3, 4, 5, 6]. Differential equations with retarded argument have many applications in the theory of automatic control, in the theory of self-oscillatory systems, in the study of problems connected with combustion in rocket engines, in a number of problems in economics, biophysics, and many other fields. Several physical applications of such problems can be found in [5].

Our problem differs from S. B. Norkin's [5] with the discontinuous coefficient in the differential equation and the discontinuities inside the interval in which we investigate the boundary value problem. These differences affect the expression of the equivalent integral expression for the solution of the boundary value problem which yields another difference in the expression of the characteristic equation. The characteristic equation of the boundary value problem plays a very important role while examining the properties of the eigenvalues and eigenfunctions.

Studies about the boundary value problems which is generated with a differential equation with retarded argument can not only be restricted to the investigation of the characteristics for the eigenvalues and eigenfunctions. For instance, in [7, 8], inverse Sturm-Liouville problems with a delay on finite interval are examined.

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Transmission eigenvalues, discreetness and existence

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The interior transmission problem, which arises in inverse inverse scattering theory, is a boundary value problem compounded of two partial differential equations of second order defined in a bounded domain that corresponds to the scatterer. The boundary value problem is not elliptic in the sense of Agmon-Douglas-Nirenberg so the classical theory of PDE does not provide a direct answer for its solvability. Its homogeneous version is referred to as the transmission eigenvalue problem, which is nonlinear and non self-adjoint eigenvalue problem. In this paper the focus is to prove descretness and existence of real transmission eigenvalues of the following problem

$$\begin{aligned} \Delta w + k^2 n w &= 0 && \text{in} && \Omega, \\ \Delta v + k^2 v &= 0 && \text{in} && \Omega, \\ w - v &= -\eta \frac{\partial v}{\partial \nu} && \text{on} && \partial\Omega, \\ \frac{\partial w}{\partial \nu} &= \frac{\partial v}{\partial \nu} && \text{on} && \partial\Omega. \end{aligned}$$

This transmission eigenvalue problem, which appears in the analysis of inverse scattering problem for an inhomogeneous media with thin coating, has not been studied in the existing literature. It presents additional difficulties due to more complicated boundary conditions. Establishing the discreetness of transmission eigenvalues is important in order to prove the solvability of the interior transmission problem, since the latter satisfies the Fredholm Alternative. On the other hand the existence of real transmission eigenvalues, which is much harder question due to non-selfadjointness, is important for solving the inverse scattering problem since they provide information on refractive index n of the scattering media

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L^2-type exponentiality tests based on V-empirical Laplace transform and Puri-Rubin characterization

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In this paper we propose new goodness-of-fit tests that employ the equidistribution characterization of the exponential distribution due to Puri and Rubin. Based on V-empirical Laplace transforms of equidistributed statistics, test statistics of L^2-type are constructed. They are degenerate V-statistics with estimated parameters. Their asymptotic properties are derived. To assess their quality, the approximate Bahadur efficiency is used. For small sample sizes, a simulated power study is performed. The tests are shown to be very efficient and powerful in comparison to many other exponentiality tests.

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Causality with finite horizon of the past

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Motivated by Markov process of order p (p-markovianity) in the discrete time case and recent studies of stochastic systems with memory, we suggest the new concept of causality for continuous time stochastic processes which deal with finite horizon of the past. Also, we present results which show connections between given concept of causality and marginalization of continuous time Markov processes.

Mutual position of two subspaces and two operators

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The first part of this talk will be mainly expository, orbiting around the problem of closedness of the linear sum of two closed subspaces in a Hilbert space. We will present some well-known results regarding this problem, give a few examples from different contexts and some interpretations. The second part of the talk will be devoted to the following problem: when do two bounded operators, defined on different subspaces of the same Hilbert space, have an equal continuous (or closed) extension to the whole space? In solving such a problem we will recall some old results of Ando on Lebesgue-type decomposition of operators, and Izumino on Quotient operators, but we will also invoke new results regarding unbounded pseudoinverses and orthoprojections. An application of these results will be given in the end of the talk.

Some generalizations of β-duals of sequence spaces

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We will start with the set $M(X,Y)$, multiplier space, defined by:

$$M(X,Y) = \{a = (a_k) \in \omega \mid ax \in Y, \text{ for all } x \in X\},$$

where ω denote the space of all complex-valued sequences and X and Y are sequence spaces. Specially, putting $Y = cs$, where cs is the set of convergent series, the multiplier space becomes β-dual of X. In this talk, we will present some generalized results related to X^β and extend some of existing. Finally, we will illustrate these generalizations with some examples and applications.

New estimators for the parameters of the $SDLINAR(1)$ model

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In this paper we present a new estimators for the parameters of the $INAR$ model with skewed discrete Laplace marginal distribution. These estimators are based on the sample cross-covariances. We discuss the asymptotic behavior of the estimators.

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Rational contraction in multiplicative metric spaces

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The purpose of this paper is to prove that instead of a rational contraction shown in the paper Afrah A. N. Abdou, *Fixed point theorems for generalized contraction mappings in multiplicative metric spaces,* J. Nonlinear Sci. Appl. **9** (2016), 2347–2363, a more general contractive condition can be obtained in multiplicative metric spaces, which is equivalent to a contractive condition in metric spaces.

Probabilistic properties of Colombeau stochastic processes

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Colombeau stochastic processes (CSPs) are defined as Colombeau functions with values in the space of random variables with finite pth moments and with values in the space of real valued random variables endowed with almost sure convergence.

In [1], the notion of point values of CSPs in compactly supported generalized points is established and relaying on this results we prove the measurability of the corresponding random variables with values in Colombeau algebra of compactly supported generalized constants endowed with the topology generated by sharp open balls. We have studied the structure of generalized correlation functions and representation of generalized characteristic functions of CSPs.

In [2], CSPs with independent values are defined and we give a characterization of such processes via their generalized correlation function in the classical Colombeau algebra of generalized numbers. Also, we have studied the properties of stationary CSPs, distinguishing between strict stationarity and weak stationarity.

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Applications of Lyapunov functions to Caputo fractional differential equations

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One approach to study various stability properties of solutions of nonlinear Caputo fractional differential equations is based on using Lyapunov like functions. A basic question which arises is the definition of the derivative of the Lyapunov like function along the given fractional equation. In this paper several definitions known in the literature for the derivative of Lyapunov functions among Caputo fractional differential equations are given. Applications and properties are discussed. The purpose of this paper is to refine the fundamental theorems and to discuss and illustrate some of these results and to present some new ones. A Caputo fractional Dini derivative of a Lyapunov function among nonlinear Caputo fractional differential equations is presented. Comparison results using this definition and scalar fractional differential equations are presented and several sufficient conditions for stability and asymptotic stability with respect to part of the variables are given. Several examples are given to illustrate the theory.

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Remarks on optional sequences and the incomplete samples

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For the given probability space (Ω, β, P) and the given sequence of reals $\{a_n\}_{n\in N}$ the corresponding optional sequence is defined. The existence theorem is proved showing that if Ω is the infinite set and P is atomless, then there exists some decomposition of Ω into disjoint subsets $\mathcal{D}_{\mathcal{M}}$ such that $\cup\mathcal{D}_{\mathcal{M}} = \Omega$ and $P(\Gamma) < \epsilon$, for every $\Gamma \in \mathcal{D}_{\mathcal{M}}$ and every real number $\epsilon > 0$. Also, if the above decomposition is at most countable, it is proved that for every real number $\epsilon > 0$ there exists some optional sequence, such that the probability of appearance of any member of the sequence is less then ϵ. The missing data problem has also been perceived and related to the optional sequences. Some illustrative examples and properties of optional sequences of full measure are presented supporting the proofs.

Hardy type inequalities on function spaces

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We shall discuss various weighted norm inequalities of Hardy type covering classical Hardy inequality as well as the latest trend. These inequalities will be discussed in the framework of Lebesgue spaces as well as grand Lebesgue spaces.

Estimation of stress-strength parameter for a new Weibull distribution

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This paper deals with the estimation of $R = P\{X < Y\}$, where X and Y are independent random variables from Peng-Yan Extended Weibull distribution. The MLE of R, its asymptotic distribution and confidence interval based on it, as well as exact confidence interval are obtained. The procedure for deriving bootstrap-p confidence interval is presented. The UMVUE of R and UMVUE of its variance are derived. The Bayes estimator of R is obtained. A simulation study is performed in order to compare these estimators.

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Hilbert matrix on mixed norm spaces

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We prove that the Hilbert matrix operator H is bounded on the mixed norm space $H^{p,q,\alpha}_{\nu}$ if and only if $0 < \kappa_{p,\alpha,\nu} < 1$, where $\kappa_{p,\alpha,\nu} = \nu - \alpha - \frac{1}{p} + 1$. In particular, Hilbert matrix operator H is bounded on the weighted Bergman space $A^{p,\alpha}$ if and only if $1 < \alpha + 2 < p$ and it is bounded on the Dirichlet space $\mathcal{D}^p_\alpha$ if and only if $\max\{-1, p-2\} < \alpha < 2p-2$. Also, it is well known that the norm of the Hilbert matrix operator H on the Bergman space A^p is equal to $\frac{\pi}{\sin\frac{2\pi}{p}}$, when $4 \leq p < \infty$, and it was also conjectured that

$$\|H\|_{A^p \to A^p} = \frac{\pi}{\sin\frac{2\pi}{p}},$$

when $2 < p < 4$. Following [1] we prove this conjecture.

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Cauchy-Schwartz inequality revisited

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The Cauchy-Schwartz inequality for Hilbert C^*-mosules: $|\langle x, y\rangle| \leq ||x||\ \langle y, y\rangle^{1/2}$ can be applied to derive many operator inequalities.

On a theorem of Brian Fisher in the framework of w-distance

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In 1980, Fisher proved very interesting fixed point result for the pair of maps. In 1996, Kada, Suzuki and Takahashi introduced and studied the concept of w-distance in fixed point theory. In this lecture, we generalize Fisher's result for pair of mappings on metric space to complete metric space with w-distance. The obtained results do not require the continuity of maps, but more relaxing condition (C;k). As a corollary we obtain a result of Chatterjea.

Goodness-of-fit tests for the exponential distribution based on U-empirical kernel density estimators

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Characterizations based on the equidistribution of two statistics have become very popular for the construction of goodness-of-fit tests. Various approaches that use different U and V-empirical functions have been proposed. In this paper, we suggest a new method based on U-empirical kernel density estimators. We propose a class of exponentiality tests based on recent characterizations from [1] and [2]. We examine their properties using simulated powers. Their isotones will also be presented for some choice of alternative distributions.

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Stochastic analysis of the predator–prey model with Allee effect on prey

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This paper presents the analysis of stochastic Rosenzweig–MacArthur predator–prey model with Allee effect on prey population of form

$$dx(t)=x(t)\left[\frac{bx(t)}{A_1+x(t)}-d_1-\alpha x(t)-\frac{sy(t)}{1+sh_1x(t)}\right]dt-\sigma_1x(t)dw_1(t),$$
$$dy(t)=y(t)\left[\frac{c_1sx(t)}{1+sh_1x(t)}-d_2\right]dt-\sigma_2y(t)dw_2(t),$$

with initial value $x(0)=x_0$, $y(0)=y_0$. For this model, we first prove the existence and uniqueness of global positive solution by using the comparison theorem for stochastic differential equations. Then, we consider extinction of predator and prey population and conditions under which the extinction occurs. We also find the conditions for parameters of the model under which the solution of the system is globally attractive in mean. Finally, the numerical illustration with real life example is carried out to confirm our theoretical results. More precisely, we consider interaction between wolf and moose populations on Isle Royale to confirm our theoretical results.

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Inequalities involving polar derivative of a polynomial with restricted zeros

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It is well known that if $P(z)$ is a polynomial of degree n, then $\max_{|z|=1} |P'(z)| \leq n \max_{|z|=1} |P(z)|$. This inequality is known as Bernstein's inequality. The subject of inequalities for polynomials and related classes of functions plays an important and crucial role in obtaining inverse theorems in Approximation Theory. Many a times results related to inverse theorems have depended upon first obtaining the corresponding analogue or generalization of Markov's and Bernstein's inequalities. These inequalities have motivated the researchers for significant new literature in Mathematics. Bernstein's inequality and its generalizations concerning the growth of polynomials have entered into different domains, in different norms. Here we obtain some results concerning the inequalities involving polar derivative of a complex polynomial with restricted zeros. The results presented over here improve upon the earlier results.

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Norm inequalities for a class of elementary operators

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Let $\sum_{n=1}^{\infty}(\|A_nh\|^2+\|A_n^*h\|^2+\|B_nh\|^2+\|B_n^*h\|^2)<+\infty$ for all h in a Hilbert space $\mathcal{H}$, for some families $\{A_n\}_{n=1}^{\infty}$ and $\{B_n\}_{n=1}^{\infty}$ of bounded operators on $\mathcal{H}$, where at least one of them consists of mutually commuting normal operators. For a symmetrically normed (s.n.) function Φ and $p\geqslant 2$, let $\Phi^{(p)^*}$ denote a s.n. function adjoint to p-modification $\Phi^{(p)}$ of Φ, then for all $X\in\mathcal{C}_{\Phi^{(p)^*}}(\mathcal{H})$

$$\left\|\sum_{n=1}^{\infty}A_nXB_n\right\|_{\Phi^{(p)^*}}\leqslant\left\|\left(\sum_{n=1}^{\infty}A_n^*A_n\right)^{1/2}X\left(\sum_{n=1}^{\infty}B_nB_n^*\right)^{1/2}\right\|_{\Phi^{(p)^*}}.$$

Amongst other applications, this new Cauchy-Schwarz type norm inequality was used to explore a class of elementary operators induced by an analytic functions with non-negative Taylor coefficients to prove that

$$\left\|f\left(\sum_{n=1}^{\infty}A_n\otimes B_n\right)X\right\|_{\Phi^{(p)^*}}\leqslant\left\|\sqrt{f\left(\sum_{n=1}^{\infty}A_n^*\otimes A_n\right)(I)}\times X\sqrt{f\left(\sum_{n=1}^{\infty}B_n\otimes B_n^*\right)(I)}\right\|_{\Phi^{(p)^*}},$$

where $A_n\otimes B_n$ stands for the bilateral multipliers $A_n\otimes B_n\colon\mathcal{B}(\mathcal{H})\to\mathcal{B}(\mathcal{H})\colon X\mapsto A_nXB_n$. Different applications and examples for the obtained norm inequalities are also provided.

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Compact and "compact" operator over Hilbert C^*-module

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We construct a topology on the standard Hilbert module $H_{\mathcal{A}}$ over a unital C^*-algebra and topology on $H_{\mathcal{A}}^{\#}$ (the extension of the module $H_{\mathcal{A}}$ by the algebra $\mathcal{A}^{**}$) such that any "compact" operator, (i.e., any operator in the norm closure of the linear span of the operators of the form $z \mapsto x\langle y, z\rangle$, $x, y \in H_{\mathcal{A}}$ (i.e., $z \mapsto x\langle y, z\rangle$, $x, y \in H_{\mathcal{A}}^{\#}$)) maps bounded sets into totally bounded sets.

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Goodness-of-fit tests in conditional duration models

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We propose specification tests for the innovation distribution in conditional duration models. The new tests are based either on the cumulative distribution function, or on exponential transforms such as the Laplace transform and the characteristic function, or on characterizations of the innovation-distribution under test. We study the finite-sample performance of the proposed procedures in comparison with alternative tests which employ nonparametric density estimates as well as with tests based on entropy. A bootstrap version of the tests is utilized in order to study the small sample behavior of the procedures. A real-data example illustrates the applicability of our method and confirms conclusions drawn by earlier authors.

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Backward Euler and forward-backward Euler methods for pantograph stochastic differential equations under nonlinear growth conditions

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The main object of consideration is the implicit backward Euler method for a class of pantograph stochastic differential equations with coefficient which satisfy the generalized Khasminskii-type conditions. The one-sided Lipschitz condition on the drift coefficient is required in order to guarantee the existence and uniqueness of the backward Euler solution. In order to overcome some measurability difficulties, the forward-backward Euler method is employed. Under the conditions which are introduced, the convergence in probability on finite time intervals is established for the discrete and continuous forward-backward Euler solutions, as well as for discrete backward Euler solution. Moreover, under certain more restrictive nonlinear growth conditions it is proved that both discrete backward and forward-backward Euler solutions are globally a.s. asymptotically polynomially stable. The stability result is based on the application of the semimartingale convergence theorem. Numerical examples are provided to support the theoretical results.

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Extensions of two minimax theorems of S. Park

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In this paper we prove two general minimax theorems which generalize famous classical saddle point theorems of M. Sion [4] and J. von Neumann. Our theorems also include some results of S. Park [1, 2, 3]. Results of this type have many applications in the Game theory, because they gives existence of solution of zero sums games.

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Conditional least squares estimation of the parameters of Random environment INAR models of higher order

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Two different random environment INAR models of higher order, $\mathrm{RrNGINAR}_1(p)$ and $\mathrm{RrNGINARmax}(p)$, are presented as the newer approach in modeling non-stationary nonnegative integer-valued autoregressive processes. Their interpretation is given, in order to better understand the circumstances of these models applications on random environment counting processes. The estimation statistics defined using conditional least squares method are newly introduced and their properties are tested on the replicated simulated data obtained by the RrNGINAR models constructed with different parameter values. Obtained CLS estimates are presented and discussed.

Markov inequalities in L_2-norms with the Laguerre and Gegenbauer weights

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In this talk I will present some tight upper and lower bounds for the best constants in the Markov inequality for the first derivative of algebraic polynomials in the L_2-norms induced by the Laguerre and the Gegenbauer weight functions. The technique employed for their derivation is sharp estimation of certain norms of related matrices.

This research is partially supported by the Bulgarian National Research Fund through Contract DN 02/14.

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Comparison of efficiencies of some symmetry tests around an unknown center

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In this paper, some recent and classical tests of symmetry are modified for the case of an unknown center. The unknown center is estimated with its α-trimmed mean estimator. The asymptotic behavior of the new tests is explored. The local approximate Bahadur efficiency is used to compare the tests to each other as well as to some other tests.

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Some numerical radius and norm inequalities in Hilbert space operators

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Let $\mathcal{B}(\mathcal{H})$ denote the C^*-algebra of all bounded linear operator on a complex Hilbert space $\mathcal{H}$ with inner product $\langle\cdot,\cdot\rangle$. For $A \in \mathcal{B}(\mathcal{H})$ let $\omega(A) = \sup\{|\langle x, Ax\rangle| : \|x\| = 1\}$, $\|A\| = \sup\{\|Ax\| : \|x\| = 1\}$ and $|A| = (A^*A)^{1/2}$ denote the numerical radius, the usual operator norm of A and the absolute value of A, respectively. It is well know that $\omega(\cdot)$ is a norm on $\mathcal{B}(\mathcal{H})$, and that for all $A \in \mathcal{B}(\mathcal{H})$,

$$\frac{1}{2}\|A\| \leq \omega(A) \leq \|A\|.$$

It is shown that, if $A \in \mathcal{B}(\mathcal{H})$ is a hyponormal operator. Then,

$$\omega(A) \leq \frac{1}{2\left(1 + \frac{\xi^2_{|A|}}{8}\right)} \left\| |A| + |A^*| \right\|,$$

where $\xi_{|A|} = \inf_{\|x\|=1} \left\{ \frac{\langle (|A|-|A^*|)x, x\rangle}{\langle (|A|+|A^*|)x, x\rangle} \right\}$.

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A note on Meir-Keeler theorem in the context of b-metric spaces

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We consider the famous Meir-Keeler's theorem in the context of b-metric spaces. Our result generalizes several known results in existing literature.

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Compactness of some bounded linear operators from *cs*-space

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The space of sequences of bounded variation is a β-dual for the sequence space of convergent series. Both of those spaces can be represented as matrix domains of triangles, the space cs is an AK space and the space bv can be obtained from an AK space. Now, if we combine these properties with the known theory of matrix domains, we can give the representation for certain classes of bounded linear operators and get the conditions for their compactness.

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Inverse problems for Sturm-Liouville operators with a delay less than half the length of the interval and Robin boundary conditions

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This paper deals with an inverse problem for non-self-adjoint second-order differential operators with a constant delay less than $\pi/2$ and a potential from $L_2[\tau, \pi]$ under Robin boundary conditions. We study the inverse spectral problem of recovering operators from their spectral characteristics. Two boundary value problems are considered and we prove that a delay and a potential are uniquely determined from their spectra.

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Some systems of Sylvester-type equations

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Motivated by the work of Baksalary and Kala [1], who studied matrix equations

$$AXB + CYD = E$$

and

$$AX - YB = C,$$

and some recent results on generalized Sylvester matrix equations, we investigate systems of one-sided and two-sided Sylvester-type equations for matrices and bounded linear operators between Hilbert spaces. We establish pure algebraic solvability conditions and give expression for the general solution in terms of generalized inverses.

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Best approximation questions for new types of contractions

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In this article, we study existence of best proximity points for new types of contractions defined on a complete metric space. Our results in this article improve and generalize some recent results in the literature. Some examples are given to demonstrate the generality of our results.

Generalized Hopf bifurcation in coupled excitable systems

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Bifurcations of equilibria of two coupled FitzHugh-Nagumo excitable systems are analyzed. It is shown that there are domains for values of coupling and excitability parameters for all three types of Hopf bifurcation. The point of codimension 2 generalized Hopf bifurcation depends on time-scale ratio of the two variables.

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Fixed point theorems for G-pata operators on metric spaces endowed with a graph

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The purpose of of this paper is to introduce the pata type contraction in metric space endowed with a graph and to prove some fixed point results for G-pata operators in such spaces.

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Admissible frames and the Paulsen problem

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Paulsen problem, its generalizations and related topics in the field of admissible frames have been of interests in recent years. In this talk we will have a new approach to this topic and we will introduce a new algorithm for increasing the degree of tightness of any finite admissible frame.At the continue, we will show that this algorithm provides an answer for the Paulsen problem.

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Improved estimates for the best constant in a Markov L_2-inequality

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Here we present new two-sided estimates for the best (i.e., the smallest possible) constant $c_n(\alpha)$ in the Markov inequality

$$\|p_n'\|_{w_\alpha} \le c_n(\alpha)\|p_n\|_{w_\alpha}, \quad p_n \in \mathcal{P}_n\,,$$

where $\mathcal{P}_n$ is the set of algebraic polynomials of degree at most n, $w_\alpha(x) := x^\alpha\, e^{-x}$, $\alpha > -1$, is the Laguerre weight function, and $\|\cdot\|_{w_\alpha}$ is the associated L_2-norm,

$$\|f\|_{w_\alpha} = \left(\int_0^\infty w_\alpha(x)|f(x)|^2\,dx\right)^{1/2}.$$

Our approach is based on the fact that $c_n^{-2}(\alpha)$ equals to the smallest zero of the n^{th} degree polynomial Q_n in a sequence of polynomials orthogonal with respect to a measure supported on $[0,\infty)$ and defined by an explicit three-term recurrence relation. We employ computer algebra to evaluate the seven lowest degree coefficients of Q_n and to obtain thereby bounds for $c_n(\alpha)$. This work is a continuation of a recent investigations [1], where estimates for $c_n(\alpha)$ were proven on the basis of the four lowest degree coefficients of Q_n.

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Statistical causality and martingale problems

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In this paper we consider a statistical concept of causality in continuous time, between filtered probability spaces, which is based on Granger's definition ([1]) of causality. On the other side, concept of martingale problem was first introduced by Stroock and Varadhan in [2]. Martingale problems should be considered as a stochastic counterpart of ordinary differential equations. It is usually applied for modelling dynamical phenomena in physics, finance, etc. In this paper we show the equivalence between some models of causality and extremal solution of the martingale problem. Also, we prove that the given causality concept is closely connected to the concept of extremality of measures for the solutions of the stopped martingale problem and of the local martingale problem. We consider the connection of the stopped martingale problem to the original martingale problem, as well as the connection between the local and stopped martingale problem.

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On MLEs of the parameters of a new extended Weibull distribution based on record values

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In this paper, we proved that the MLEs of the parameters of a new extended Weibull lifetime model with possible upside-down bathtub shape and monotonic hazard rate function, proposed by Peng and Yan in [1], exist and are unique based on upper kth record values.

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FIELD 2 – LOGIC, ALGEBRA, SET THEORY, DISCRETE MATHEMATICS, NUMBER THEORY

Some bounds on the energy of signed complete bipartite graphs

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A *signed graph* G^σ is a pair (G, σ), where G is a graph, and $\sigma : E(G) \longrightarrow \{-1, +1\}$ is a function. Assume that $m \leq n$ are two positive integers. Let

$$A = \left[\begin{array}{c|c} 0 & B \\ \hline B^t & 0 \end{array}\right]$$

is the adjacency matrix of $K^\sigma_{m,n}$. In this talk we show that for every sign function σ, $2\sqrt{mn} \leq E(K^\sigma_{m,n}) \leq 2m\sqrt{n}$, where $E(K^\sigma_{m,n})$ is the energy of $K^\sigma_{m,n}$. Also it is proved that the equality holds for the upper bound if there exists a Hadamard matrix of order n for which B is an m by n submatrix of H. Also if the equality holds, then every two distinct rows of B are orthogonal. We prove that for the lower bound the equality holds if and only if $K^\sigma_{m,n}$ is switching equivalent to $K_{m,n}$.

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On Kurepa's left factorial conjecture

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Kurepa's conjecture states that there is no odd prime p that divides $!p = 0!+1!+\cdots+(p-1)!$. We introduce new optimization techniques and perform the computation of $!p$ modulo p for all $p < 2^{40}$. Additionally, we consider the generalized Kurepa's left factorial given by $!^k n = (0!)^k + (1!)^k + \cdots + ((n-1)!)^k$, and show that for all integers $1 < k < 100$ there exists an odd prime p such that $p \mid !^k p$. We also investigate the existence of primes $p > 5$ for which the residues of $2!$, $3!$, $\ldots$, $(p-1)!$ modulo p are all distinct. We describe the connection between this problem and Kurepa's left factorial function, and report that there are no such primes less than 2^{40}.

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Some inequalities for elementary symmetric polynomials in the complex domain

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We are investigating problem of finding the upper bound of the modulus of elementary symmetric polynomials $e_k(z_1, \ldots, z_n)$, where variables $z_1, \ldots, z_n$ are subject to conditions $z_1 + \cdots + z_n = 0$ and $|z_j| \leq R$ for all $j = 1, \ldots, n$. We give a sharp upper bound in the case $k = n - 1$. It turns out that the same estimate is valid for the real variables case, in fact, the complex case is reduced to the real one. We also give an estimate in the case $k = n - 2$, which is sharp for even n. These estimates are then applied to give results on location of zeros of polynomials.

Irregularity of graphs

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Let G be a graph with vertex set $V(G)$. The Total irregularity of G is defined as $\mathrm{irr}_t(G) = \sum_{\{u,v\} \subseteq V(G)} |\deg_G(u) - \deg_G(v)|$, where $\deg_G(v)$ is the degree of the vertex v of G. This graph parameter was introduced by Abdo and Dimitrov in 2014. In this talk, we report our recent results on values of this graph parameter on some classes of graphs.

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Bounds of nilpotency class of powerful p-groups

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Powerful p-groups were introduced by Lubotzky and Mann in 1987 in [1]. A finite p-group G is called *powerful* if either p is odd and $[G, G] \subseteq G^p$ or $p = 2$ and $[G, G] \subseteq G^4$. We will discuss results that bound the nilpotency class of a powerful p-group in terms of the exponent of a quotient by a normal abelian subgroup.

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Study of (σ, τ)-generalized derivations with their composition of semiprime rings

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The main purpose of this paper study and investigate some results concerning (σ, τ)-generalized derivations D associated with derivation d of semiprime ring and prime ring R, where σ and acts as two automorphism mappings of R. During this work, we suppose let R be an associative ring with the center of R which is denoted by $Z(R)$ and let σ, τ be automorphism mappings on R. We depend on the commutator $[x, y]_{\sigma,\tau} = x\sigma(y) - \tau(y)x$ (resp. $(xoy)_{\sigma,\tau} = x\sigma(y) + \tau(y)x$) for all $x, y \in R$. Moreover, let $D : R \to R$ is an additive map and $d : R \to R$ is a derivation.

If $D(xy) = D(x)\sigma(y) + \tau(x)d(y)$ holds for all $x, y \in R$; then D is called a (σ, τ)-generalized derivation associated with d. We divided this paper into sections the preliminaries with some results contained in the first section while the second section we emphasis on composition of (σ, τ)-generalized derivations of the Leibniz's formula, where we introduce the general formula to computes the composition of (σ, τ)-generalized derivations and illustrated that by example. We supple some results about that where the σ and τ be two automorphism mappings of R such that their commute with D and d.

In fact, there are some applications of (σ, τ)-derivations which develop an approach to deformations of Lie algebras which have many applications in models of quantum phenomena and in analysis of complex systems. The map has been extensively investigated in pure algebra. Recently, it has been treated for the Banach algebra theory.

Following some results.

Theorem 4. *Let R be a 2-torsion free semiprime ring and σ and τ be two automorphism mappings of R. Suppose that there exists a (σ, τ)-generalized derivation D such that $[D(x), x]_{\sigma,\tau} = 0$ for all $x \in R$, then*

(i) *if the generalized derivation D commuting mapping of R then d is commuting mapping of R;*

(ii) *if the derivation d commuting mapping of R then $D$is 2-commuting mapping of R.*

Theorem 5. *Let n and r be a fixed positive integers. Let R be a 2-torsion free semiprime ring, σ and τ be two automorphism mappings of R such that the mappings σ and τ are commute with D and d, D a (σ, τ)-generalized derivation with an associated derivation d of R such that $[D^n(x), x^n]_{(\sigma,\tau)} = 0$, then*

$$\sum_{r=0}^{n} \binom{n}{r} D^{n-r}(x)d^r(x), x^{2n}] = -[D^n(x), x^{2n}]x \in Z(R), \quad \text{for all } x \in R.$$

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Some lower bounds of the energy of graphs

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Let G be a simple graph with the vertex set $V(G)$ and with the adjacency matrix $A(G)$. The energy $E(G)$ of G is defined to be the sum of the absolute values of all eigenvalues of $A(G)$. Also let n and m be number of edges and vertices of the graph respectively. A regular graph is a graph where each vertex has the same number of neighbours. Given a graph G, its line graph $L(G)$ is a graph such that each vertex of $L(G)$ represents an edge of G; and two vertices of $L(G)$ are adjacent if and only if their corresponding edges share a common endpoint in G. In this paper we show that for every regular graphs and also for every line graphs such that $\delta(G) \geq 3$ we have, $E(G) \geq \frac{2m}{n} + n - 1$. Also it was proved that for any bipartite graph G, $2\mu(G) \leqslant E(G)$ such that $\mu(G)$ is the matching number of G and equality holds if and only if G is the disjoint union of some complete bipartite graphs with perfect matchings and some isolated vertices. We generalize this result by showing that it holds for an arbitrary graph.

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On the number of critical points of a polynomial in a disc

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Let $p(z)$ be n-th degree polynomial and let $z_1, \ldots, z_{n-1}$ be its zeroes. We prove that at least $\left[\frac{n-1}{2}\right]$ of its critical points lie in any circle C that is centered at the arithmetic mean of these zeroes and contains them.

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Ordinances of the vectors of the n-dimensional Boolean cube in accordance with their weights

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The problem "Given a Boolean function f of n variables by its Truth Table vector, denoted by $TT(f)$. Find (if exists) a vector $\alpha \in \{0,1\}^n$ of minimal (or maximal) weight, such that $f(\alpha) = 1$." arises in computing the algebraic degree of Boolean functions or vectorial Boolean functions called S-boxes. The solutions to this problem have useful generalizations and applications (for example, in generating all subsets of a given set in accordance with their cardinalities, or in generating combinations etc.). To find effective solutions we examine the ways of ordering the vectors of the Boolean cube in accordance with their weights. The notion "k-th layer" of the n-dimensional Boolean cube is involved in the definition and examination of the "weight order" relation. It is compared with the known relation "precedes". We enumerate the maximum chains for both relations. An algorithm that generates the vectors of the n-dimensional Boolean cube in accordance with their weights is developed. The lexicographic order is chosen as a second criterion for an ordinance of the vectors of equal weights. The algorithm arranges the vectors in a unique way called a weight-lexicographic order. It is represented by the (serial) numbers of the vectors, instead of the vectors itself. Its time and space complexities are $\Theta(2^n)$, i.e., of linear type with respect to the size of the output. The obtained results are summarized and added as a new sequence (A294648) in the OEIS.

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On Randić energy of a graph

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Let $G = (V, E)$ be a simple graph of order n with vertex set $V = V(G) = \{v_1, v_2, ..., v_n\}$ and edge set $E = E(G)$. Let d_i be the degree of the vertex $v_i \in V(G)$, $i = 1, 2, \ldots, n$. The Randić matrix $\mathbf{R} = \mathbf{R}(G) = ||R_{ij}||_{nxn}$ is defined by

$$R_{ij} = \begin{cases} \frac{1}{\sqrt{d_i d_j}}, & \text{if the vertices } v_i \text{ and } v_j \text{ are adjacent,} \\ 0, & \text{otherwise.} \end{cases}$$

The eigenvalues of matrix $\mathbf{R}$, denoted by $\rho_1, \rho_2, \ldots, \rho_n$, are called the Randić eigenvalues of graph G. The Randić energy of graph G, denoted by RE, is a graph invariant defined by

$$RE = RE(G) = \sum_{i=1}^{n} |\rho_i|.$$

Some upper and lower bounds on Randić energy are presented.

A New Graph Theoretical Invariant in Terms of Degree Sequence

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The first two authors recently defined a new invariant just in terms of a degree sequence that gives a lot of combinatorial and topological information about the realizations of the given degree sequence. Several properties of this invariant determined by all the authors will be presented here.

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Convex heptagons with border trapezoids

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A convex heptagon $A_1A_2A_3A_4A_5A_6A_7$ is given. Let A_i, A_{i+1}, A_{i+2}, A_{i+3} be the four consecutive vertices of this heptagon. The quadrilateral $A_iA_{i+1}A_{i+2}A_{i+3}$ is called *border quadrilateral*. The paper presents the proof of the following claim: if the given heptagon has six border quadrilaterals that are trapezoids, then the seventh border quadrilateral is also a trapezoid. The existence of convex heptagons with border trapezoids in integer lattice was also discussed. Both these problems present continuation in research of analogous problems for convex pentagons and hexagons, which autors solved in previous work.

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Weakly linear equations and inequalities for matrices over an additively idempotent semiring and applications

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Quantales, complete residuated lattices, complete Heiting algebras, and related residuated algebraic structures, represent an excellent basis for studying systems of fuzzy equations and inequalities. In contrast, semirings generally are not residuated structures, but for matrices over an additively idempotent semiring there is a kind of relative residuation which allows us to define and study Boolean residuals of matrices. These residuals will be used to solve weakly linear systems of matrix equations and inequalities. Iterative algorithms for testing the existence and computing the greatest solutions of these systems will be described. Bearing in mind that the behavior of timed automata is described using matrices over an additivly idempotent semiring, the previous methodology will be applied for testing behavioral equivalence between these automata.

The talk reports a joint work with M. Ćirić and J. Ignjatović.

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The reciprocity law for the twisted second moment of Dirichlet L-functions over rational function fields

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We investigate the reciprocity law for the twisted second moments of Dirichlet L-functions over rational function fields, corresponding to two irreducible polynomials in $F_q[t]$, where F_q is a finite field with q elements. This formula is the analogue of the formulas for Dirichlet L-functions over the field of rational numbers obtained by B. Conrey and M. Young.

The class of trees with nonsingular acyclic matrix with most $n-2$ P-vertices

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This paper concerns P-vertices and P-set of nonsingular acyclic matrices. A vertex is P-vertex iff $m_{A(i)} = m_A + 1$ where m_A is multiplicity of eigenvalue 0 in matrix A. It is shown that double star DS_n with n vertices is an example of tree such that for each nonsingular matrix A whose graph is DS_n the number of P-vertices of A is most $n-2$. Also, here is constructed a nonsingular matrix whose graph is double star with exactly $n-2$ P-vertices.

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A short note on the lower bounds for the Kirchhoff index of graphs

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Let G be a simple connected graph with $n \geq 2$ vertices, m edges and Laplacian eigenvalues $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_{n-1} \geq \mu_n = 0$. The Kirchhoff index $Kf(G)$, of a simple connected graph is defined as [2]

$$Kf(G) = \sum_{i<j} r_{ij},$$

where r_{ij} is the effective resistance between the vertices i and j. A more appropriate formula from practical point of view, was put forward in [1] (see [6])

$$Kf(G) = n \sum_{i=1}^{n-1} \frac{1}{\mu_i}.$$

The topological index, later called general Randić index R_{-1}, is defined as [5]

$$R_{-1} = R_{-1}(G) = \sum_{i \sim j} \frac{1}{d_i d_j},$$

where $i \sim j$ denotes that vertices i and j are adjacent, and d_i denotes the degree of the vertex i.
In [3] (see also [4]) the following inequality was proved

$$Kf(G) \geq -1 + 2(n-1)R_{-1}. \tag{1}$$

In this paper we will prove the following inequality

$$Kf(G) \geq \frac{n^2(n-1) - m}{m} - 2(n-1)R_{-1}. \tag{2}$$

A comparison of the inequality (1) and (2), as well as the inequality (2) with other known inequalities for the lower bounds of $Kf(G)$ are considered.

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On the normal edge-transitive Cayley graphs

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For graph Γ, let X be a subgroup of $Aut(\Gamma)$, Γ is called X-vertex-transitive or X-edge-transitive, if X is transitive on the set of vertices or the set of edges, respectively. The Cayley graph $X = \mathrm{Cay}(G, S)$ is is normal edge-transitive if and only if $\mathrm{Aut}(G, S)$ is either transitive on S or has two orbits in S in the form of T and T^{-1}, where T is a non-empty subset of S and $S = T \cup T^{-1}$. In this paper, we study some properties of normal edge-transitive Cayley graphs.

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On graded 2-nil-good rings

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Recently, in [M. S. Abdolyousefi, N. Ashrafi, H. Chen, On 2-nil-good rings, J. Algebra Appl. DOI 10.1142/S0219498818501104], 2-nil-good rings are introduced as rings in which every element can be written as a sum of two units and a nilpotent. We study group graded rings in which every homogeneous element can be written as a sum of two homogeneous units and a homogeneous nilpotent. We name such rings *graded* 2*-nil-good rings*. After establishing basic properties of such rings, we focus on their extensions. In particular, we discuss (graded) 2-nil-good property for (graded) group rings and also deal with the question of how the graded 2-nil-good property of a group graded ring depends on the 2-nil-good property of the component which corresponds to the neutral element of the grading group.

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On certain sums involving the Riemann zeta-function $\zeta(s)$

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A discussion involving the evaluation of the sum

$$\sum_{T<\gamma\le T+H} |\zeta(\tfrac{1}{2}+i\gamma)|^2$$

and some related integrals is presented, where γ denotes imaginary parts of complex zeros of the Riemann zeta-function $\zeta(s)$. It is shown unconditionally that the above sum is $\ll H\log^2 T\log\log T$ for $T^{2/3}\log^4 T \ll H \le T$. Under these conditions it is also shown that

$$\int_T^{T+H} |\zeta(\tfrac{1}{2}+it)|^2 S(t)\,\mathrm{d}t \ll H\log T\log\log T,$$
$$\int_T^{T+H} |\zeta(\tfrac{1}{2}+it)|^2 S^2(t)\,\mathrm{d}t \ll H\log T(\log\log T)^2,$$

where $S(T)=\frac{1}{\pi}\arg\zeta\left(\frac{1}{2}+iT\right)$. This generalizes the results of [1].

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A breaf survey on rigid rings

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In this paper we consider rigid rings end their generalizations. The notation R is used for an associative ring with identity, σ will denote an endomorphism of a ring R and $R[x;\sigma]$ is a skew polynomial ring with multiplication subject to the relation $xr = \sigma(r)x$ for all $r \in R$. A ring R is σ-rigid if $a\sigma(a) = 0$ implies $a = 0$ for all $a \in R$. We say that a ring R is weak $\sigma-$ rigid if $a\sigma(a) \in \mathrm{nil}(R)$ if and only if $a \in \mathrm{nil}(R)$. We deal with preserving rigid property under constructions of product, limits and various extensions over rigid rings.

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On matrix and polynomial extensions over generalizations of Armendariz rings

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This paper deals with generalizations of Armendariz rings, so called weak Armendariz rings. A ring R is called Armendariz if $f(x)g(x) = 0$ implies $a_ib_j = 0$, for all polynomials $f(x) = \sum_{i=0}^{n} a_ix^i$ and $g(x) = \sum_{j=0}^{m} b_jx^j$ from $R[x]$. A ring is called a weak-Armendariz if $f(x)g(x) = 0$ implies $a_ib_j \in \mathrm{nil}(R)$. Recall that generalization of Armendariz and rigid ring is σ-skew Armendariz ring. Ring R is called σ-skew Armendariz if $f(x)g(x) = 0$ implies $a_i\sigma^i(b_j) = 0$, for all $f(x) = \sum_{i=0}^{n} a_ix^i$ and $g(x) = \sum_{j=0}^{m} b_jx^j$ from $R[x;\sigma]$. As a generalization of σ-skew Armendariz rings, there is a notion of weak σ-skew Armendariz ring R as a ring in which $f(x)g(x) = 0$ implies $a_i\sigma^i(b_j)$ is the nilpotent element of R for all $f(x) = \sum_{i=0}^{n} a_ix^i$ and $g(x) = \sum_{j=0}^{m} b_jx^j$ from $R[x;\sigma]$. We construct weak Armendariz structure which is preserved under ring isomorphism. Our main result is that Armendariz property can be transformed from the ring to its matrix or polynomial extension. In this paper we generalize some results which are related to σ-skew Armendariz rings, to the weak σ-skew Armendariz case. Central Armendariz rings are also considered.

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Structural examinations of graphs with smallest least eigenvalue

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Connected graphs of fixed order and size that minimize the least eigenvalue appear in several references published in a recent past. In all of them, the structure of such graphs is examined in details. For example, it is known that such a graph must be either bipartite or a join of two nested split graphs (not both totally disconnected). Moreover, if the graph is bipartite then it has a special structure, that is it must be a so-called double nested graph.

Our contribution to this problem refers to the non-bipartite case, where we identify the unique graph of fixed order n and size m that minimizes the least eigenvalue whenever its size satisfies $m = \lceil \frac{n}{2} \rceil \lfloor \frac{n}{2} \rfloor + a$, where a is a fixed constant in $\left[1, \lceil \frac{n}{2} \rceil - 1\right]$.

In the lecture, we present the entire background along with our results.

Double Roman domination number on cardinal product of graphs

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Roman domination was named after the way the Roman emperor Constantine (274-337 AD) distributed his legions to defend borders of the Empire against barbarian attacks. Some border areas had 1 or 2 legions, and some had no legions, but they had at least 1 neighbouring area with 2 legions.

Double Roman domination is a stronger version of Roman domination that doubles the protection by ensuring that any attack can be defended by at least two legions.

A function $f : V \to \{0, 1, 2, 3\}$ is a *double Roman dominating function* (DRDF) on a graph G if it satisfies the following conditions. Let V_i denote the set of vertices assigned i by function f.

(i) If $f(v) = 0$, then vertex v has at least two neighbors in V_2 or one neighbor in V_3.

(ii) If $f(v) = 1$, then vertex v has at least one neighbor in $V_2 \cup V_3$.

The *double Roman domination number* $\gamma_{dR}(G)$ equals the minimum weight of a double Roman dominating function on G, and a double Roman dominating function of G with weight $\gamma_{dR}(G)$ is called a γ_{dR}-*function* of G.

In this paper we determine some upper and lower bounds for double Roman domination numbers on cardinal product of any two graphs and some exact values for the cardinal product of paths and cycles.

Local fusion graph of certain finite groups

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Suppose G is a finite group and X is a conjugacy class of involutions in G. The local fusion graph $F(G, X)$ has X as its vertex set, with distinct vertices x and y joined by an edge if, and only if, xy has odd order. The aim of this talk is to present our latest results on computing the local fusion graph of some finite groups.

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The free calculus and non-isomorphism of finitely presented groups

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It is well-known that in general case determining whether or not two presentations $\mathbf{(x:r)}$ and $\mathbf{(x':r')}$ define isomorphic groups is undecidable problem. The main goal of this talk is to demonstrate how technique named free calculus can resolve this problem for some group presentations. Using free calculus we can attach to each finitely presented group a chain of elementary ideals. Any two finite presentations of the same group have same chain of elementary ideals, thus considering these ideals we can distinguish some groups presented by generators and finitely many relations. One concrete example related to an open question will also be given.

A class of models of bounded arithmetic and continuous logic

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We investigate a class of Boolean-valued models based on random variables using continuous first-order logic. Such models provide a rich framework for studying bound arithmetic and propositional proof complexity [1], and they can be naturally considered as continuous structures in the sense of [2].

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H-coloring revisited

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In this paper we give a new, shortened proof of NP-completeness of CSP problem for undirected, non bipartite graphs, of interest for generalization to QCSP problem. We also give some illustrative examples.

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The large sum graph related to comultiplication modules

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Let R be a commutative ring and M be an R-module. We define the large sum graph, denoted by $\acute{G}(M)$, as a graph with the vertex set of non-large submodules of M and two distinct vertices are adjacent if and only if $N + K$ is a non-large submodule of M. In this article, we investigate the connection between the graph-theoretic properties of $\acute{G}(M)$ and algebraic properties of M when M is a comultiplication R-module. This is a part of my jointly paper which is accepted for publication in the journal "Le Matematiche".

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The graph $\Gamma_S(L)$, where S is a filter

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In this paper, we study some graph-theoretical properties of $\Gamma_S(L)$, a graph which the vertex set is all elements of a finite lattice L and two distinct vertices a and b are adjacent if and only if $a \vee b \in S$, where S is a $\wedge$-closed subset of L. As a consequence of our work, some results in [2] are extended to the case that S is a filter.

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On some classes of graphs whose second largest eigenvalue does not exceed $\frac{\sqrt{5}-1}{2}$

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Let λ_2 be the second largest eigenvalue of the adjacency matrix of a graph. We determine all trees and all bicyclic graphs for which λ_2 does not exceed $\frac{\sqrt{5}-1}{2}$. In description of these classes we use mappings that preserve sgn $\left(\lambda_2 - \frac{\sqrt{5}-1}{2}\right)$.

On the geometric-arithmetic index

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Abstract. Let $G(k,n)$ be the set of connected simple n-vertex graphs with minimum vertex degree k. The geometric-arithmetic index $GA(G)$ of a graph G is defined by $GA(G) = \sum_{uv} \frac{2\sqrt{d_u d_v}}{d_u+d_v}$, where $d(u)$ is the degree of vertex u and the summation extends over all edges uv of G. In this paper we characterized graphs on which GA index attains minimum value, when number of vertices of degree k is $n-1$ and $n-2$. We also gave a conjecture about the extremal graphs on which this index attains its minimum value and lower bound for this index for graphs with given minimum degree k, where $k \leq \lfloor k_0 \rfloor$, $k_0 = q_0(n-1)$, $q_0 \approx 0.0874$ is the unique positive root of equation $q\sqrt{q} + q + 3\sqrt{q} - 1 = 0$.

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Metric dimension of the multiple antiprism graphs

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Imran et al. in [1] prove that metric dimension of some classes of convex polytopes, especially of double antiprism A_n (noted also with Q_n in other papers) is constant, and equal to three. Authors in [2] prove that infinite classes of convex polytopes generated by wheel related graphs have unbounded metric dimension, and ask for the characterization of graphs with unbounded metric dimension.

In order to answer the asked questions we will investigate metric dimension of the multiple antiprism graphs.

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On the notions of structure species in the senses of Tsalenko, Sonner, and Blanchard

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Blanchard [1] introduced the concept of structure species on a category very close to the concept of structure species in the sense of Bourbaki [2], and proved that the concept introduced by him is equivalent to the concept of structure species in the sense of Sonner [3]. According to Sonner, a structure species on a category $\mathcal{X}$ is an univalent functor $F\colon \mathcal{Y} \to \mathcal{X}$ such that for any object Y of the category $\mathcal{Y}$ and any isomorphism $j\colon F(Y) \to ??$ into $\mathcal{X}$ there exists an isomorphism $i\colon Y \to ?$ in $\mathcal{Y}$ such that $j = F(i)$. Finally, Tsalenko [4] introduced the notion of structured category over the category $\mathcal{X}$, meaning by this the ordered pair $(\mathcal{Y},\ F)$, formed by the category $\mathcal{Y}$ and univalent functor F from $\mathcal{Y}$ to $\mathcal{X}$. It is clear that if $F\colon \mathcal{Y} \to \mathcal{X}$ is the structure species in Sonner sense, then $(\mathcal{Y},\ F)$ is a structured category over $\mathcal{X}$ in the sense of Tsalenko, and that the converse, generally speaking, is incorrect. Nevertheless, the following theorem holds.

Theorem 6. *Let $F\colon \mathcal{Y} \to \mathcal{X}$ be an univalent functor. Then there exist the category $\hat{\mathcal{Y}}$, the structure species (in the sense of Blanchard) $\hat{F}\colon \hat{\mathcal{Y}} \to \mathcal{X}$ and the equivalence $(U,\ V)$ between the categories $\mathcal{Y}$ and $\hat{\mathcal{Y}}$ such that $\hat{F} \circ U = F$ and $V \circ U = 1_{\mathcal{X}}$.*

Corollary 1. *Let $\mathcal{Y}$ be a category with a generator (respectively, with a cogenerator). Then there is the structure species Σ on the category of sets such that the category $\mathcal{Y}$ (respectively, $\mathcal{Y}^{\mathrm{op}}$, the dual to $\mathcal{Y}$) is equivalent to the category of all Σ-objects and Σ-morphisms.*

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On the compressed essential graph of a commutative ring

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Let R be a commutative ring. In this paper, we introduce and study the compressed essential graph of R. The compressed essential graph of R is the graph $EG_E(R)$, whose vertices are equivalence classes of zero-divisors of R and two distinct vertices $[x]$ and $[y]$ are adjacent if and only if $\operatorname{ann}(xy)$ is an essential ideal of R. It is shown that for a reduced ring R, $EG_E(R) = \Gamma_E(R)$, $\Gamma_E(R)$ denotes the compressed zero-divisor graph of R, and $V(EG_E(R)) = 2^n - 2$ also it has $c(n,k)$ vertices of degree $2^k - 1$, for all $1 \leq k \leq n-1$, where R is Noetherian and $|\operatorname{Ass}(R)| = n > 2$. Furthermore, it shown that for a non-reduced ring R with $3 < |EG_E(R)| < \infty$, $EG_E(R) = \Gamma_E(R)$ if and only if

(i) $\operatorname{Nil}(R) = \operatorname{ann}_R(Z(R))$;

(ii) if $\operatorname{ann}_R(a)$ is an essential ideal of R, then $a \in \operatorname{Nil}(R)$;

(iii) every non-zero element of $\operatorname{Nil}(R)$ is irreducible in $Z(R)$.

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Face enumeration on matroid base polytope

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To a generalized permutohedron Q is associated the quasisymmetric function $F(Q)$, defined in [1], which enumerates positive integer lattice points lying in maximal cones of the normal fan of Q. In [2] is defined its weighted refinements $F_q(Q)$

$$F_q(Q) := \sum_{\omega \in \mathbf{Z}_+^n} q^{\mathrm{rk}_Q(\mathcal{F}_\omega)} x_{\omega_1} x_{\omega_2} \cdots x_{\omega_n},$$

where $\mathrm{rk}_Q(\mathcal{F}_\omega)$ is a rank function on the face poset of the standard permutohedron determined by Q. Particularly, the enumerator $F_q(Q)$ contains the information about the f-vector of the generalized permutohedron Q. We study the special case of matroid base polytopes $Q = P_M$ and calculate their f-vectors. We also show that the corresponding weighted quassisymmetric enumerator $F_q(P_M)$ coincides with a universal morphism of combinatorial Hopf algebras of matroids to quasisymmetric functions.

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Elements in a ring which can be represented as a sum of idempotents and one nilpotent element

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The conditions that allow an element of an associative, unital, not necessarily commutative ring R, to be represented as a sum of (commuting) idempotents and one nilpotent element are analyzed. An element a of a ring R is s-nil-clean if it can be written in the following form:

$$a = e_1 + \cdots + e_s + n,$$

where elements $e_1, \ldots, e_s$ are idempotents and n is nilpotent. If an element a can be written in this form so that elements in this sum are pairwise commutative, we say that this element is strongly s-nil-clean. If every element in R is (strongly) s-nil clean, we say that R is a (strongly) s-nil-clean ring. We examine some interesting properties of s-nil clean rings.

On the spectrum of derangement graphs

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The spectrum of a graph is the multiset $\{\lambda_1, \ldots, \lambda_n\}$, where λ_i's are the roots of the characteristic polynomial of the adjacency matrix of given graph. Let G be a permutation group, a derangement graph is a graph with vertex set G and two vertices are adjacent if and only if they do not intersect. In this paper, we compute the spectrum of derangement graphs of well-known groups.

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The maximal modulus of a reciprocal algebraic integer

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Let α be an algebraic integer of degree d, which is reciprocal. The house of α is the largest modulus of its conjugates. We compute the minimum of the houses of all reciprocal algebraic integers of degree d which are not roots of unity, say $m_R(d)$, for d at most 34. We proved lemmas useful to avoid unnecessary calculations. The computations suggest several conjectures.

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Propositional logics with metric operators

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We introduce and investigate a formal language that is an extension of classical propositional language obtained by adding new binary operators of the form $D_{\leqslant s}$ and $D_{\geqslant s}$, $s \in \mathbb{Q}_0^+$. Our language allows making formulas such as $D_{\leqslant s}(\alpha, \beta)$ with the intended meaning "distance between formulas α and β is less than or equal to s". The semantics of the proposed language consists of possible worlds with a distance function defined between sets of worlds.

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Applications of some analytic inequalities in obtaining bounds for the resolvent energy of graphs

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Let M be a square matrix of order n. The resolvent matrix $\mathcal{R}_M(z)$, of matrix M is defined as $\mathcal{R}_M(z) = (zI_n - M)^{-1}$, where I_n is the unit matrix of order n and z is a complex variable. Let G be a simple graph, and let A, L, and Q be its adjacency, Laplacian, and signless Laplacian matrix, respectively . Eigenvalues of matrices A, L and Q we denote by $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$, $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_n$ and $q_1 \geq q_2 \geq \cdots \geq q_n$, respectively. We consider resolvent matrices $\mathcal{R}_A(n)$, $\mathcal{R}_L(n+1)$ and $\mathcal{R}_Q(2n-1)$. The resolvent, Laplacian resolvent and signless Laplacian resolvent energy of a graph G are defined as

$$ER(G) = \sum_{i=1}^{n} \frac{1}{n - \lambda_i}, \quad RL(G) = \sum_{i=1}^{n} \frac{1}{(n+1) - \mu_i},$$

$$RQ(G) = \sum_{i=1}^{n} \frac{1}{2n - 1 - q_i},$$

respectively. Using analytic inequalities, some lower and upper bounds for these graph invariants are obtained.

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FIELD 3 – GEOMETRY, TOPOLOGY, ALGEBRAIC GEOMETRY AND TOPOLOGY

A class of four dimensional CR submanifolds of the nearly Kähler six sphere

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A submanifold M of the nearly Kähler sphere $S^6(1)$ is called a CR submanifold if there exists a C^∞-differential almost complex distribution $U : x \to U_x \subset T_xM$, i.e., $JU = U$ on M, such that its orthogonal complement $U^\perp$ in TM is totally real distribution, i.e., $JU^\perp \subset T^\perp M$, where $T^\perp M$ is the normal bundle over M in $S^6(1)$. Since the four dimensional CR submanifolds of $S^6(1)$ can not be totally geodesic, we investigate four dimensional CR submanifolds that admit the distribution $D(p) = \{X \in TpM \mid h(X,Y) = 0, \text{ for all } Y \in TpM\}$, of the maximal possible dimension which is two and classify them using sphere curves and vector fields along those curves.

Mapping degrees among 4-dimensional quasitoric manifolds

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We study the set $D(M, N)$ of all possible mapping degrees from M to N when M and N are quasitoric 4-manifolds. In some of the cases, we completely describe this set. Our results rely on Theorems proved by Duan and Wang and the sets of integers obtained are interesting from the number theoretical point of view, for example those representable as the sum of two squares $D(\mathbb{C}P^2\sharp\mathbb{C}P^2, \mathbb{C}P^2)$ or the sum of three squares $D(\mathbb{C}P^2\sharp\mathbb{C}P^2\sharp\mathbb{C}P^2, \mathbb{C}P^2)$. In addition to the general results about the mapping degrees between quasitoric 4-manifolds, we establish connections between Duan and Wangs approach, quadratic forms, number theory and lattices.

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Geodesically equivalent metrics on homogenous spaces

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Two metrics on a manifold are geodesically equivalent if sets of their unparameterized geodesics coincide. In this paper we show that if two left G-invariant metrics of arbitrary signature on homogenous space G/H are geodesically equivalent, they are affinely equivalent, i.e. they have the same Levi-Civita connection. We also prove that existence of non-proportional, geodesically equivalent, G-invariant metrics on homogenous space G/H implies that their holonomy algebra cannot be full. We give an algorithm for finding all left invariant metrics geodesically equivalent to a given left invariant metric on a Lie group. Using that algorithm we prove that no two left invariant metric, of any signature, on sphere S^3 are geodesically equivalent. However, we present examples of Lie groups that admit geodesically equivalent, non-proportional, left-invariant metrics.

This is joint work with T. Šukilović and S. Vukmirović.

On generalized Bishop frame of null Cartan curve in Minkowski 3-space

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We define generalized Bishop frame of a null Cartan curve in Minkowski 3-space by using its Bishop's frame vector fields. We obtain the Cartan equations according to the generalized Bishop frame and give the relations between the generalized Bishop curvatures and Bishop curvatures. In particular, we also show that among all null Cartan curves in $\mathbb{E}_1^3$, only the null Cartan cubic has two generalized Bishop frames, one of which coincides with its Bishop frame. We also show that there exists a null Cartan curve whose generalized Bishop curvatures and Bishop curvatures are equal, but whose generalized Bishop frame and Bishop frame do not coincide. As an application, we characterize a k-type null Cartan slant helices for $k \in \{0, 1, 2\}$ according to the generalized Bishop frame, in terms of their generalized Bishop curvatures.

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Algebraic geometrical description of Neumann systems on Stiefel varieties

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We study geometric and algebraic geometric properties of the continuous and discrete Neumann systems on cotangent bundles of Stiefel varieties $V_{n,r}$. The systems are integrable in the non-commutative sense, and by applying a $2r \times 2r$–Lax representation, we show that generic complex invariant manifolds are open subsets of affine (non-compact) Prym varieties on which the complex flow is linear. The characteristics of the varieties and the direction of the flow are calculated explicitly. Next, we construct a family of (multi-valued) integrable discretizations of the Neumann systems and describe them as translations on the Prym varieties, which are written explicitly in terms of divisors of points on the spectral curve. It appears that the systems inherit or naturally generalize the basic properties of the classical Neumann system on S^{n-1} and, therefore, of the Jacobi–Mumford systems: the structure of the Lax matrices, the spectral curve, the equations of motion, linearization on Abelian varieties, and, in the discrete case, the formula for the translation on them. The results are obtained in a collaboration with Yuri Fedorov.

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New insight into Mihailo Petrovic's dissertation

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Mihailo Petrovic's dissertation was submitted to the Ecole Normale Superieure and approved in 1894, by the commission consisting of Hermite, Picard and Painleve. Many articles have been written since about the dissertation and its results and on Petrovic's influence on mathematics in Serbia. However, there has been no substantial analysis of Petrovic's methods. The present work is trying to change this attitude. Petrovic's methods were based primarily on the so called Newton polygon. First described in Newton's letter to Oldenburg, dated October 26th, 1676, the method has been forgotten and much later widely used in the work of Puiseux around 1850's, and then forgotten again for a long time. It has revived in the work of the school of V. I. Arnold. The main idea of the method is to treat qualitative behavior of polynomials (in Petrovic's case polynomials in two variables $y, y\prime$ with functional coefficients $\phi(x)$) by the combinatorial geometry of the convex hull of the points representing its exponents. The second part of Petrovic's dissertation is devoted to the generalisation of his plane results (first order ODE) to higher order ODE. He tried to obtain results by the new planar version of the Newton polygon. The work of Arnold and his school, and also of the author, has showed how to use higher dimensional polyhedra instead of polygons. In this case, it would be the right way to generalise Petrovic's results for the first order ODE. This work is developing this idea further.

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Geodesic mappings and their generalizations

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Our aim is to study geodesic mappings and their generalizations. The generalizations we mean holomofphically-projective and F-planar mappings. The fundamental terms and facts it is possible to find in monography [6].

In our study we find new form of a fundamental equations of above mentioned mappings. Those equations are appropriate for (pseudo-) Riemannian spaces of second order approach. We also refined fundamental equations of F-planar mappings, see [3, 4].

Geodesic and holomorphically projective mappings of spaces with equiaffine connection onto (pseudo-) Riemannian and Kählerian spaces were studied in [4, 9], see [6]. Those questions are connected to metrizability of a manifolds with affine connection. It was proved by É. Cartan that manifold with affine connection is projective equivalent to manifold with equiaffine connection as locally as globaly, see [2]. Above mentioned results are acceptable for projective and holomorphically projective metrizability of spaces with affine connection.

Holomorphically projective mappings were studied for parabollical Kähler spaces as well [7]. We also studied geodesic mappings of special spaces, for example semisymmetric projective Euclidean spaces [8].

J. Mikeš [6] studied F-planar mappings of spaces with equiaffine connection onto (pseudo-) Riemannian manifolds. Those questions are connected to metrizability as well. F_2^{ε}-planar mappings was studied in [1].

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Cyclotomic polynomial and fundamental group

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Coefficients of cyclotomic polynomial can be interpreted topologically, as the torsion in the homology of a certain simplicial complex associated with the degree of cyclotomic polynomial. We discuss an open question by studying fundamental group of certain 2-dimensional subcomplex for cyclotomic polynomials whose degree is a product of three arbitrary primes.

Algebraic non-integrability of magnetic billiards on the sphere and hyperbolic plane

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We consider billiard ball motion in a convex domain on a constant curvature surface influenced by the constant magnetic field. We examine the existence of integral of motion which is polynomial in velocities. We prove that if such an integral exists then the boundary curve of the domain determines an algebraic curve in $\mathbf{C}^3$ which must be nonsingular. Using this fact we deduce that for any domain different from round disc for all but finitely many values of the magnitude of the magnetic field billiard motion does not have Polynomial in velocities integral of motion.

Chain connected sets in a topological space

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In the paper Shekutkovski [1] are compared two definitions of connectedness, the standard one and the definition using coverings. The second one seems to be very effective description of quasicomponents.

In our paper instead as a space, we generalized the notion to a set in a topological space called chain connected set.

Definition 1. *A set C in a topological space X is chain connected if for every two elements $x, y \in \mathcal{U}$ and every open covering $\mathcal{U}$ of X in X, there exists a chain in $\mathcal{U}$ which connects x and y.*

Also we introduced a notion of chain separated sets in a space and we study the properties of chain connected sets in a topological space. Moreover, we prove the properties of connected spaces using chain connectivity. Chain connectedness of two points in a topological space is an equivalence relation. Components of a chain connectedness of a set in a topological space are union of quasicomponents of the set, and if the set is equal with the space, components of a chain connectedness matches with a quasicomponents.

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Proximate groups of higher order

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Using the intrinsic definition of shape based on proximate sequences for compact and all topological spaces based on proximate nets indexed by open coverings in the paper Shekutkovski et all. [1] we define proximate fundamental group. In this paper the proximate groups of higher order will be introduced.

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Computer classification of fundamental domains of plane discontinuous groups

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H. Poincaré (1882) attempted to describe a plane crystallographic group in the Bolyai-Lobachevsky hyperbolic plane $\mathbf{H}^2$ by appropriate fundamental polygon. This initiative he extended also to space. B. N. Delone (Delaunay) in 1960's refreshed this very hard topic for Euclidean space groups by the so-called stereohedron problem: to give all fundamental domains for a given space group, with few partial results.

A. M. Macbeath (1967) completed the initiative of H. Poincaré in classifying the 2-orbifolds by giving each with a signature. That is by a base surface with orientable or non-orientable genus, by some singular points on it, as rotational centres with given periods, by some boundary components, in each with given dihedral corners. All these are characterized up to an equivariant isomorphism, also indicated in this talk. There is a nice curvature formula that describes whether the above (good) orbifold, i.e., co-compact plane group (with compact fundamental domain) is realizable either in the sphere $\mathbf{S}^2$, or in the Euclidean plane $\mathbf{E}^2$, or in the hyperbolic plane $\mathbf{H}^2$, respectively.

Our initiative in 1990's was to combine the two above descriptions. Namely, how to give all the combinatorially different fundamental domains for any above plane group. Z. Lučić and E. Molnár completed this by a graph theoretical tree enumeration algorithm. That time N. Vasiljević implemented this algorithm to computer (program COMCLASS), of super-exponential complexity, by certain new ideas as well.

In the time of the Yugoslav war we lost our manuscript, then the new one has been surprisingly rejected (?!). Now we have refreshed our manuscript to submit again and that is to appear as [1]. Here we intend to present a report on it, also with some new problems.

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On k-type null Cartan slant helices according to Darboux frame in Minkowski 3-space

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We define k-type null Cartan slant helices for $k \in \{0, 1, 2\}$ lying on the timelike surface in Minkowski 3-space according to their Darboux frame. We study these helices by using their geodesic curvature, normal curvature and geodesic torsion. Additionally, we determine their axes and consider the special cases when the mentioned helices are geodesic curves and principal curvature lines lying on the timelike surface in E_1^3. We show that null Cartan cubics lying on B-scrolls are 0-type and 2-type null Cartan slant helices and prove that geodesic null Cartan isophotic curves are the null Cartan slant helices. Furthermore, we obtain some interesting relations between 0-type, 1-type and 2-type null Cartan slant helices and provide the related examples.

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On the characteristic rank of vector bundles over oriented Grassmannians

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We study the cohomology algebra of the Grassmann manifold $\widetilde{G}_{k,n}$ of oriented k-dimensional subspaces in $\mathbb{R}^{n+k}$ via the characteristic rank of the canonical vector bundle $\widetilde{\gamma}_{k,n}$ over $\widetilde{G}_{k,n}$ (denoted by charrank$(\widetilde{\gamma}_{k,n})$). Using Gröbner bases for the ideals determining the cohomology algebras of the "unoriented" Grassmannians $G_{k,n}$ we prove that charrank$(\widetilde{\gamma}_{k,n})$ increases with k. In addition to that, we calculate the exact value of charrank$(\widetilde{\gamma}_{4,n})$, and for $k \geq 5$ we improve a general lower bound for charrank$(\widetilde{\gamma}_{k,n})$ obtained by Korbaš. Some corollaries concerning the cup-length of $\widetilde{G}_{4,n}$ will also be given.

Joint work with Branislav Prvulović.

On certain mappings onto Ricci symmetric manifolds

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On mappings onto Ricci symmetric manifolds

Let us recall that manifolds with affine connection and Riemannian manifolds are called *Ricci symmetric* if in them the Ricci tensor is absolutely parallel.

Let $f\colon M_n \to \bar{M}_n$ be a diffeomorphism (possibly a bijection of "sufficiently high" differentiability class) between n-dimensional manifolds

M_n and $\bar{M}_n$. The above considerations allow us to suppose that the manifolds in fact coincide, $\bar{M} \equiv M$.

Hence $A_n = (M, \nabla)$ and $\bar{A}_n = (\bar{M}, \bar{\nabla}) \equiv (\bar{M}, \nabla)$ be manifolds M and $\bar{M}$ with affine connections ∇ and $\bar{\nabla}$, respectively. Then the type (1,2) tensor field

$$P = \bar{\nabla} - \nabla$$

is called the *deformation tensor* of the connections ∇ and $\bar{\nabla}$ with respect to f on M.

We proved that A_n admits mapping f onto Ricci symmetric manifolds $\bar{A}_n$ if and only if it satisfies the following equation

$$\nabla_m \bar{R}_{ij} = P^{\alpha}_{mi} \bar{R}_{\alpha j} + P^{\alpha}_{mj} \bar{R}_{i\alpha},$$

where $\bar{R}_{ij}$ are components of the Ricci tensor on $\bar{A}_n$.

Here we considered certain special mappings onto Ricci symmetric manifolds.

Conformal mappings onto Ricci symmetric manifolds

The following Theorem 7 ([1]) is devoted to conformal mappings onto Ricci symmetric manifolds. These results are practically generalized results which we have obtained for conformal mappings of Riemannian manifolds onto Einstein spaces [3, 5] and geodesic mappings of manifolds with affine connection onto symmetric Riemannian manifolds [4, 6, 7].

Theorem 7. *n-dimensinal (pseudo-) Riemannian manifold V_n admits conformal mapping onto Ricci symmetric (pseudo-) Riemannian manifold $\bar{V}_n$ if and only if on V_n exists a solution of the following closed Cauchy type equations system in covariant derivative respective unknown functions $\psi(x)$, $\psi_i(x)$, $\mu(x)$, and $\bar{R}_{ij}(x)$ $\left(= \bar{R}_{ji}(x)\right)$:*

$$\begin{aligned}
\psi_{,i} &= \psi_i \\
\psi_{i,j} &= \frac{\mu}{n-2}\, g_{ij} + \psi_i \psi_j - \frac{1}{n-2} \left(\bar{R}_{ij} - R_{ij}\right), \\
\bar{R}_{ij,k} &= 2\psi_k \bar{R}_{ij} + \psi_i \bar{R}_{jk} + \psi_j \bar{R}_{ik} - \psi^{\alpha} \bar{R}_{i\alpha} g_{jk} - \psi^{\alpha} \bar{R}_{j\alpha} g_{ik}, \\
(n-1)\mu_{,k} &= g^{\alpha\beta} \Big((n-2)\psi_\gamma R^{\gamma}_{\beta k\alpha} - (n-1)\psi_\beta \bar{R}_{\alpha k} - \psi_\beta R_{\alpha k} \Big) \\
&\quad + \left(R + (n-1)\mu\right) \psi_k - \frac{R_{,k}}{2},
\end{aligned}$$

where comma denotes covariant derivative on V_n.

Here functions $\bar{R}_{ij}(x)$ are components of the Ricci tensor on manifold $\bar{V}_n$.

Geodesic mappings onto Ricci symmetric manifolds

Analogical results were obtained for geodesic mappings [2].

Theorem 8. *n-dimensinal manifold A_n with affine connection admits geodesic mapping onto Ricci symmetric manifold $\bar{A}_n$ with affine connection if and only if on A_n exists a solution of the following closed Cauchy type equations system in covariant derivative respective unknown functions $\psi_i(x)$ and $\bar{R}_{ij}(x)$*

$$\bar{R}_{ij,m} = 2\psi_m \bar{R}_{ij} + \psi_i \bar{R}_{mj} + \psi_j \bar{R}_{im},$$
$$\psi_{i,j} = \frac{1}{n^2-1}\left(n\bar{R}_{ij} + \bar{R}_{ji} - (nR_{ij} + R_{ji})\right) + \psi_i\psi_j,$$

where comma denotes covariant derivative on A_n.

Here functions $\bar{R}_{ij}(x)$ are components of the Ricci tensor on manifold $\bar{A}_n$.

We set a number of principian parameters on which depend the general solutions of the Cauchy type equation systems in Theorems 7 and 8.

It is interesting to note that if Ricci symmetric manifold $\bar{A}_n$ is equiaffine (i.e., the Ricci tensor is symmetric), then integrability condition of equations in Theorem 8 is linear respective unknown functions $\psi_i(x)$ and $\bar{R}_{ij}(x)$.

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p-toroids and their 3-triangulations

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It is known that we can always 3-triangulate (i.e. divide into tetrahedra) convex polyhedra but not always non-convex ones. Polyhedra topologically equivalent to sphere with p handles, shortly p-toroids, could not be convex. So, it is interesting to investigate possibilities and properties of their 3-triangulations. Here, we will study the minimal necessary number of tetrahedra for the triangulation of a 3-triangulable p-toroid. For that purpose we will develop the concepts of piecewise convex polyhedra and graph of connection.

On the symmetries of the 4-dimensional nilpotent Lie groups

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The symmetry of a tensor T is a 1-parameter group of diffeomorphisms of manifold (M, g), leaving T invariant. Therefore, we consider a vector field X satisfying the condition $\mathcal{L}_X T = 0$, where $\mathcal{L}$ denotes the Lie derivative. Examples of symmetries are isometries (for $T = g$ and X being a Killing vector field), but also homotheties, curvature collineations ($T = R$ where R is the curvature tensor), Ricci collineations (where $T = \rho$ is the Ricci tensor), Weyl collineations ($T = W$ being the Weyl conformal curvature tensor), etc.

We investigate symmetries of the four-dimensional nilpotent Lie groups, equipped with various left-invariant metrics of arbitrary signature. First, we give a full classification of left-invariant metrics on Lie groups $H_3 \times \mathbb{R}$ and G_4 and then we consider their geometry.

This is a part of an ongoing project with prof. Wafaa Batat, Ecole Nationale Polytechnique d'Oran, Algeria.

Orthogonal shadows and index of Grassmann manifolds

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In this paper we study the $\mathbb{Z}/2$ action on real Grassmann manifolds $G_n(\mathbb{R}^{2n})$ and $\widetilde{G}_n(\mathbb{R}^{2n})$ given by taking (appropriately oriented) orthogonal complement. We completely evaluate the related $\mathbb{Z}/2$ Fadell–Husseini index utilizing a novel computation of the Stiefel–Whitney classes of the wreath product of a vector bundle. These results are used to establish the following geometric result about the orthogonal shadows of a convex body: For $n = 2^a(2b+1)$, $k = 2^{a+1} - 1$, C a convex body in $\mathbb{R}^{2n}$, and k real valued functions $\alpha_1, \ldots, \alpha_k$ continuous on convex bodies in $\mathbb{R}^{2n}$ with respect to the Hausdorff metric, there exists a subspace $V \subseteq \mathbb{R}^{2n}$ such that projections of C to V and its orthogonal complement $V^{\perp}$ have the same value with respect to each function α_i, which is $\alpha_i(p_V(C)) = \alpha_i(p_{V^{\perp}}(C))$ for all $1 \leq i \leq k$.

Cyclohedron and Kantorovich-Rubinstein polytopes

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We show that the cyclohedron (Bott-Taubes polytope) W_n arises as the polar dual of a Kantorovich-Rubinstein polytope $KR(\rho)$, where ρ is an explicitly described quasi-metric (asymmetric distance function) satisfying strict triangle inequality. From a broader perspective, this phenomenon illustrates the relationship between a nestohedron $\Delta_{\widehat{\mathcal{F}}}$ (associated to a building set $\widehat{\mathcal{F}}$) and its non-simple deformation $\Delta_{\mathcal{F}}$, where $\mathcal{F}$ is an *irredundant* or *tight basis* of $\widehat{\mathcal{F}}$ [2, Definition 21]. Among the consequences are a new proof of a recent result of Gordon and Petrov (Arnold Math. J. **3**(2) (2017), 205–218) about f-vectors of generic Kantorovich-Rubinstein polytopes and an extension of a theorem of Gelfand, Graev, and Postnikov, about triangulations of the type A, positive root polytopes.

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FIELD 4 – APPLIED MATHEMATICS, NUMERICAL ANALYSIS

A Nyström method for approximating the solutions of an integral equation arising from a problem in mathematical biology

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We consider integral equations of the following type

$$f(x)\int_0^1 k(x-y)dy+\int_0^1 k(y-x)f(y)dy=g(x),\quad 0<x<1, \tag{3}$$

where k is a given convolution kernel, g is a known function and f is the unknown.

The above integral equation is of interest because it arises from a problem in mathematical biology [1].

We propose to approximate the solutions of (3) by a Nyström method using the Gauss-Legendre quadrature rule. The stability and the convergence are proved in uniform spaces of continuous functions. Finally, numerical tests showing the effectiveness of the method are presented.

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Rational interpolation and root-finding methods

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Root seeking algorithms constitute indispensable components of problem solving. The aim of this paper is to continue investigation conducted in [1] and [2], regarding fixed point iterative methods for solving nonlinear equations based on particular type rational interpolant. Rational interpolants of different kinds will be discussed and analyzed. A systematization will be conducted with a purpose to assemble efficient multipoint iterative procedures for solving nonlinear equations. This formidable task already commenced in cited papers. Further investigations require access to and feedback from different interlocking theories such as Stability, Interpolation, Interval Mathematics, Complexity and Dynamics. As a result, local and global convergence conditions are derived.

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The applications of generalized logistic map in some mathematical models

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Probably the most famous model with chaotic behaviour is the logistic map. The relative simple form of logistic map made this equation suitable for modelling in many fields, including biology, physics, stock market, cryptography, traffic, tourism models...Lately, as a need to optimize some of those models, some generalizations of logistic maps occurred, see [2, 3]. We present few of these generalizations and their implementation in some mathematical models. Finally we propose one discrete dynamical traffic flow model as a modification of model from [1] based on the flow-density-speed fundamental diagram and Pipes-Munjal model from 1967 and analyse its stability.

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New methods for solving minimization problems with coupled constraints

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In our talk we will consider minimization problems with coupled constraints. We will present gradient-type methods (gradient projection, proximal, extragradient method), Newton-type methods and consensus-based algorithm for solving these problems.

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Numerical solution of parabolic-hyperbolic transmission problem

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Layers with material properties which significantly differ from those of the surrounding medium appear in a variety of applications. The layer may have a structural role (as in the case of glue), a thermal role (as in the case of a thin thermal insulator), an electromagnetic or optical role etc. Mathematical models of energy and mass transfer in domains with layers lead to so called interface or transmission problems. In this paper we consider a class of non-standard parabolic-hyperbolic transmission problem in disjoint domains. As a model example we take an area consisting of two non-adjacent rectangles. In each rectangle an Robin's initial-boundary value problem is given. The interaction between their solutions is described using nonlocal integral conjugation conditions Robin-Dirichlet type on the boundaries of the observed subareas. For the model problem the existence and uniqueness of its weak solution in appropriate Sobolev-like space is proved. A finite difference scheme approximating this problem is proposed and analyzed. An estimate of the convergence rate, compatible with the smoothness of the input data is obtained.

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Construction of the optimal set of quadrature rules for four integrals in the sense of Borges

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Borges in [1] considered a problem that arises in the evaluation of computer graphics illumination models. Starting with that problem, he examined the problem of numerically evaluating a set of $r \in \mathbb{N}$, $r \geq 2$, definite integrals of the form

$$\int_E f(x)\, w_j(x)\mathrm{d}x, \quad j = 1, 2, \ldots, r,$$

where w_j, $j = 1, 2, \ldots, r$, are the weight functions.

We consider the problem of numerically evaluating a set of $r \in \mathbb{N}$, $r \geq 2$, definite integrals with the same integrand and over the same interval of integration, but with different weight functions, related to an arbitrary multi–index. Optimal set of quadrature rules for mentioned problem was studied in [1] and [2]. The stable numerical method for the construction of such optimal set of quadrature rules for two and three weight functions was given in [3].

In this paper we present numerical method for the construction of an optimal set of quadrature rules for four weight functions in the sense of Borges.

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New optimality conditions for the vector continuous-time programming problem

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We consider the vector continuous-time programming problem. Unfortunately, some of the main results from [1], regarding optimality conditions are incorrect, as shown in [2]. In order to obtain optimality conditions, we improve new connection between the vector continuous-time programming problem and the weighting scalar problem. We obtain new optimality conditions for the considered problem.

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Subsampled inexact Newton methods for minimizing large sums of convex functions

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This paper deals with the minimization of large sum of convex functions by Inexact Newton (IN) methods employing subsampled Hessian approximations. The Conjugate Gradient method is used to compute the inexact Newton step and global convergence is enforced by a non-monotone line search procedure. The aim is to obtain methods with affordable costs and fast convergence. Assuming strictly convex functions, a set of rules for the forcing parameters and subsample sizes are derived that ensure local linear/superlinear convergence of the proposed methods. The random choice of the Hessian subsample is also investigated and bounds ensuring a good approximation of the true Hessian with some high probability are provided. For such Hessian approximations and suitable forcing terms convergence in the mean square, both for finite and infinite sums of functions, is proved. Finally, convergence of IN methods is investigated in the case of sum of convex function and strongly convex objective function. Numerical results on well known binary classification problems are also given. Adaptive strategies for selecting forcing terms and Hessian subsample size, streaming out of the theoretical analysis, are employed and the numerical results showed that they yield effective IN methods.

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Error bounds for Kronrod extension of generalizations of Micchelli-Rivlin quadrature formula for analytic functions

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We consider Kronrod extension of generalizations of the well known Micchelli-Rivlin quadrature formula, with the highest algebraic degree of precision, for the Fourier-Chebyshev coefficients. For analytic functions the remainder term of these quadrature formulas can be represented as a contour integral with a complex kernel. We study the kernel, on elliptic contours with foci at the points ∓ 1 and a sum of semi-axes $\rho > 1$, for the quoted quadrature formulas. Starting from the explicit expression of the kernel, we determine the locations on the ellipses where maximum modulus of the kernel is attained. So we derive effective L^∞-error bounds for these quadrature formulas. Complex-variable methods are used to obtain expansions of the error in these quadrature formulas over the interval $[-1, 1]$. Finally, effective L^1-error bounds are also derived for these quadrature formulas. Numerical examples which illustrate the calculation of these error bounds are included.

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Distributed order fractional wave equation

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In this work, the classical wave equation is generalized for the case of viscoelastic materials by the use of distributed order fractional model, and describe wave propagation in infinite viscoelastic media. We consider, analyze and solve the distributed order wave equation given as system:

$$\begin{aligned} \frac{\partial}{\partial x}\sigma(x,t) &= \rho\,\frac{\partial^2}{\partial t^2}u(x,t), \\ \int_0^1 \phi_\sigma(\alpha)\,{}_0D_t^\alpha\sigma(x,t)\,d\alpha, &= E\int_0^1 \phi_\epsilon(\alpha)\,{}_0D_t^\alpha\epsilon(x,t)\,d\alpha \\ \epsilon(x,t) &= \frac{\partial}{\partial x}u(x,t), \end{aligned}$$

where u, σ and ϵ are displacement, stress and strain, x real number and $t > 0$, $\rho = \text{const.}$ is the density of the media, $E = \text{const.}$ is the generalized Young modulus of elasticity, and ϕ_σ and ϕ_ϵ are constitutive functions or distributions, describing material properties. The left, resp. the right hand side in the second equation is a distributed order fractional derivative of σ, resp. ϵ, with ${}_0D_t^\alpha$ being Riemann-Liouville fractional derivative of order α.

The first equation is the equation of motion and it is a consequence of the Second Newton Law. The second equation is the constitutive equation of distributed order fractional type, and the third equation is the strain measure for small local deformations. In fact, the system is derived from the basic equations of elasticity, where the equation of motion and the strain measure are preserved, since they hold true for any type of deformable body, and only the constitutive equation, which is the Hooke law for an elastic body, is changed by distributed order fractional model, and thus adapted for viscoelastic type media.

We study existence and uniqueness of fundamental solutions for the generalized Cauchy problem corresponding to distributed order wave equation. As consequence, we establish existence, uniqueness, and obtain explicit form of the solution to a class of wave equations, corresponding to the linear fractional order constitutive models, and we also study a genuine distributed order wave equation. The wave speed is found to be connected with the material properties at initial time instant, more precisely with the glass modulus.

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Hybrid model of accelerated double step size method

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This paper presents a hybridization of accelerated double step size model introduced in [9]. Based on the hybrid iterations presented in [4], we define a hybrid form of accelerated model with two step length values. The values of the iterative step sizes are calculated using the Armios' line search procedure. In this regard, we estimate the initial value for the Backtracking algorithm and so the improvement, compared to the relevant algorithm used in [9], is achieved. We prove that this hybrid double step size scheme is linearly convergent on the set of uniformly convex functions.

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The Moore-Penrose inverse and a dual method of quadratic optimization

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In the present paper we discuss the primal and the dual solution of a specific convex optimization problem, that is, the constrained minimization of a positive semidefinite quadratic form H, using the Moore Penrose inverse. The difference of a classical approach of convex optimization techiques is that we treat both (primal and dual) problems using only vectors $x \in \mathcal{N}(H)^{\perp}$. We present results about the solutions arising from the dual formulation of the problem. Moreover, we examine the primal and dual solutions with the use of the General Normal Equation in the case when the constraint equation is inconsistent.

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Two-term Hu-Storey method for large-scale nonlinear monotone systems

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We propose a two-term Hu-Storey method for solving large-scale monotone systems, which is based on derivative-free conjugate gradient approach and hyperplane projection technique. The conjugate gradient approach is efficient for large-scale systems due to low memory requirement, while projection strategy is suitable for monotone equations because it enables simply globalization. The derivative-free, function-value-based line search is combined with Hu-Storey search direction and projection procedure, in order to construct a globally convergent method. Numerical experiments indicate great robustness and efficiency of proposed method.

Discrete iterations for computing generalized inverses of time-varying matrix

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We consider discrete-time iterative methods for computing inverse and pseudoinverse of time-varying matrices. These methods are obtained by discretizing corresponding ZNN (Zhang Neural Network) models. The proposed schemes incorporate scaled Hyperpower iterative methods as well as the Newton iteration in certain cases. We apply the general linear Multi-step method to obtain the general discretization rule. It comprises all previously proposed discretization schemes, including Euler and the Taylor-type difference rules. One particular rule, based on the 4th order Adams-Bashforth method, is proposed and numerically compared with other known iterative schemes. In addition, we propose the extension of the ZNN model for pseudoinverse computation of singular or rectangular matrices. Convergence properties of the continuous-time ZNN model in the case of the Moore-Penrose inverse and its discretization are also considered.

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Anti-Gaussian quadrature rule for trigonometric polynomials

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An anti-Gaussian quadrature formula is an $(n+1)$-point formula with algebraic degree of exactness $2n+1$. Its error is equal in magnitude but of opposite sign to that of the n-point Gaussian formula. In this paper, we investigate an anti-Gaussian quadrature rule with maximal trigonometric degree of exactness with respect to an even weight function on $[-\pi, \pi)$. Also, we give the method for its construction based on relations between nodes and weights of the quadrature rule for trigonometric polynomials and those of the quadrature rule for algebraic polynomials which were given in [1].

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On some iteration schemes for numerical computation of fixed points

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The talk refers to some new iteration schemes for reckoning fixed points on the setting of Banach spaces, satisfying various contractive conditions. Their rate of convergence is studied by comparison with other processes, including the classical ones of Mann, Ishikawa and Agarwal et al. Numerical examples are given to support the results presented. Some remarks with respect to different numerical iteration procedures are also in view. Some polynomiographs connected to this research are also presented.

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Gauss quadrature and incurable breakdown in the Lanczos algorithm

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The main message of the mismatch theorem [2, Theorem 4.1] is that incurable breakdown in the Lanczos algorithm occurs only when a minimal realization of the transfer function given by the input of the algorithm has been found. In this talk we present how this result can be proved by means of the Gauss quadrature, using results from [1, Chapter 5].

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On generalized Newton type iterative methods with high efficiency

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In this talk, the aim is to discuss various Newton type iterative methods for solving non-linear equations. In particular, the focus will be on Aitken type methods, in which the interpolation nodes are controlled by variant of Newton type methods or by a general method of order p. The order of such methods can be increased to as high as desired and in limiting case, the efficiency tends to 2. We shall discuss the case of simple as well as multiple roots. Moreover, numerical examples will be provided in support of the theoretical result.

On generalized averaged Gaussian formulas

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Recently, we proposed a new $(2\ell+1)$-point quadrature rule $\widehat{G}_{2\ell+1}$, referred to as a generalized averaged Gaussian quadrature rule. This rule has $2\ell+1$ nodes and the nodes of the corresponding Gauss rule G_ℓ with ℓ nodes form a subset. This is similar to the situation for the $(2\ell+1)$-point Gauss-Kronrod rule $H_{2\ell+1}$ associated with G_ℓ. An attractive feature of the $\widehat{G}_{2\ell+1}$ is that it exists also when the $H_{2\ell+1}$ does not. The numerical construction, on the basis of recently proposed effective numerical procedures, of $\widehat{G}_{2\ell+1}$ is simpler than the construction of $H_{2\ell+1}$. A survey of these formulas and their applications will be presented.

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On the performances of GCRF model based on digraph networks

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In many real world applications, results obtained using (unstructured) regression can achieve higher regression accuracy if correlation between the outputs of unstructured predictors is incorporated, provided they have some internal structure. Structured regression model such as Gaussian conditional random fields (GCRF) allows the utilization of unstructured predictors as feature functions, modeling of non-linear relationships between inputs and outputs, and symmetric correlation of outputs among themselves. We examine learning task problem defined by GCRF model allowing asymmetric relationships between output variables. In this setting, we show that the constrained optimization problem of the model based on the class of digraph networks with the given sums on arc weights between any two outputs is feasible only in the case when the arcs have the same weights, that is, when the underlying network is undirected. The effectiveness of DirGCRF is shown by reporting the improvement in accuracy with respect to unstructured models in the experiments conducted on different types of synthetic random networks.

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Convergence of the difference scheme for solving parabolic interface problem with delta function

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One interesting class of parabolic problems model processes in heat-conduction media with concentrated capacity in which the heat capacity coefficient contains a Dirac delta function. Such problems are nonstandard and the classical tools of the theory of finite difference schemes are difficult to apply to their convergence analysis. In the present paper a finite-difference scheme, approximating the two-dimensional initial-boundary value problem for the heat equation with concentrated capacity and time dependent coefficients of the space derivatives, is derived. Abstract operator method is developed for analyzing this problem. Convergence in special discrete $W_2^{1,1/2}$ anisotropic Sobolev norm is proved.

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SDFEM for an elliptic singularly perturbed problem with two parameters

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A singularly perturbed problem with two small parameters in two dimensions is investigated. Using its discretization by a streamline-diffusion finite element method with piecewise bilinear elements on a Shishkin mesh, we analyze the superconvergence property of the method and suggest the choice of stabilization parameters to attain optimal error estimate in the corresponding streamline-diffusion norm. Numerical tests confirm our theoretical results.

Error estimates for certain cubature rules

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We estimate the error of selected cubature formulae constructed by the product of Gauss quadrature rules. The cases of multiple and (hyper-)surface integrals over n-dimensional cube, simplex, sphere and ball are considered (see [16]). The error estimates are obtained as the absolute value of the difference between cubature formula constructed by the product of Gauss quadrature rules and cubature formula constructed by the product of corresponding Gauss-Kronrod or corresponding generalized averaged Gaussian quadrature rules. Generaziled averaged Gaussian quadrature rule $\widehat{G}_{2l+1}$ is $(2l+1)$-point quadrature formula. It has $2l+1$ nodes and the nodes of the corresponding Gauss rule G_l with l nodes form a subset, similar to the situation for the $(2l+1)$-point Gauss-Kronrod rule H_{2l+1} associated with G_l. The advantages of $\widehat{G}_{2l+1}$ are that it exists also when H_{2l+1} does not, and that the numerical construction of $\widehat{G}_{2l+1}$, based on recently proposed effective numerical procedure (see [24]), is simpler than the construction of H_{2l+1}.

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FIELD 5 – HISTORY AND TEACHING OF MATHEMATICS AND INFORMATICS

The universality of Mihailo Petrović

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This talk is prepared on the occasion of the 150th anniversary of Mihailo Petrović birth.

The public in Serbia because of the nickname Alas of Mihailo Petrović knows that he had something with fishing. In addition, through the TV series and film "The hat of Professor Kosta Vujic" many know that Mihailo Petrović was a brilliant mathematician and a fantastic violinist.

Three above mentioned points (mathematics, music, fishing) are enough to see the universal talent of Mihailo Petrović. If this is added to the other contributions of Mihailo Petrović to Serbian science and culture at the end of the XIX and in the first half of the XX century, it is clear that the legendary Mika Alas is one of the most important figures of the history of the Serbian nation.

The aim of this talk is to point out the biography of Mihailo Petrović in the shortest terms, and to highlight that he was dealing successfully with many theoretical (Differential equations, Analysis, Algebra, Mathematical phenomenology etc.) and applied sciences (Computer machines, Mechanics, Physics, Astronomy etc.) as well as with other, for Serbia, with other useful jobs (Cryptography, Inventions, Ethnology, History, Literature, Fisheries etc.).

Bridge of Mathematics

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We present an annual event 'Bridge of Mathematics' held in Vrnjačka Banja, the center of tourism in Serbia. Manifestation got his name because the bridge dedicated to maths in Banja. Math educators from Serbia and neighboring countries are coming each May to share their experiences and take part in various type of activities, such as workshops, popular and public talks, discussions, etc. A lot of students from Serbia and region are attracted by participation in a creative and popular quiz of the same name and its four brainy and entertaining games: Math Scattergories, Geometrical Challenge, The Project and Math Bouquet Traditionally, the final round of the quiz is held during festival.

On dependence between variables in mathematical problems

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This talk is prepared on the occasion of the 150th anniversary of Mihailo Petrović birth.

In Journal "Mathematical journal for high school students"[§] published in Belgrade in the 1930s, the article under title "On dependence between variables in mathematical problems" was published by Mihailo Petrović.

On simple examples from elementary mathematics, Mihailo Petrovic explains and illustrates the fact that among known and unknown variables in mathematical problems there are dependencies that define domains in which problems are given and indicate when these problems are impossible.

Authors are going to illustrate the ideas of Mihailo Petrović by two examples.

1. Determine catheti a and b of right–angled triangle, if $a + b = 8$ and hypotenuse c is equal to 5.

2. Determine volume of regular triangle–based pyramid if the side edge of pyramid is equal to 3, and the area of the cross–section of the pyramid and the plane, containing one basic edge of the pyramid and which is normal on the opposite side edge, is equal to 14.[¶]

[§]Owner and editor of this journal was Prof. Dr. Jovan Karamata, and the members of editorial board were mathematicians from the whole Kingdom of Yugoslavia

[¶]This problem was one of the problems for entrance exam for Mathematical Gymnasium in 2017

Implementation of common quality framework in professional - pedagogical supervision/inspection in mathematics teaching and recommendations for improving practice based on Hattie's *Visible learning* - theoretical concept

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Establishing a common quality framework enables a relevant discussion between the actors of the educational process based on evidence and recommendations for improving teaching and learning. The findings of professional - pedagogical supervision/inspection provide data that can be used for the expertise in teaching and learning mathematics. The theoretical concept of John Hattie's visible learning supports the quality of the learning process in general and can be useful for the successful practice in mathematics teaching by applying the recommendations obtained from extensive research in the field of education and educational processes.

Applying graphs of functions of a single variable in solving equations and inequalities

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In Serbia teaching mathematics in elementary and secondary schools is predominately based on solving problems in all the mathematical branches. The traditional models of teaching usually consist of introducing the new math terminology, defining the introduced terms, describing the charasteristics of the defined terms, and, finally, solving the problems. The focus in teaching mathematics is on the problem itself and

the procedure of solving it. The commonly used terminology is: Solve the equation, or Solve the inequality. The goal is achieved when, by applying the right solving procedure, you come to the correct solution of the problem.

In mathematics workbooks there are a lot of problems with good solutions. However, we are aware that in other countries different approaches are applied in schools, and their experience is different from ours. In Austria and some other countries, mathematics handbooks are used instead of math textbooks and workbooks. Their materials for young learners are full of complex problems ranging from calculating the perimeter and area of various figures to applying trigonometry for the purpose of the orientation at the open sea. During the process of solving the problems students have to apply most of their already acquired knowledge in these areas. It is accepted that the process of solving problemsis only one phase of the whole procedure, but not the goal. The analysis of the methods of the solving process, as well as the analysis of the kinds and types of the solutions have to be considered as the important parts, phases of the procedure. These analyses have a great influence on developing the studentscritical thinking and creativity which are considered important characteristics of teaching and learning.

The acquried knowledge and skills of drawing various categories of a single variable funcion: linear, quadratic, exponential, logarithmic, trigonometric etc. can be used for efficient and successful solving the problems related to the determination of the types and number of the solutions of the equations and inequalities. This approach is superior to the traditional algebric one because it enables us to visualise the process. Also, the determination of different types of functions enables the analysis of the types and number of solutions. In the traditional algebric approach the analysis is done after reaching the solution of the problem. This new approach can enable our students to use their acquired competences in different mathematical areas, and transfer them into functional competences in solving various math problems.

This paper contains a lot of examples which will illustrate the fact that the new approach can be successfully applied in solving some difficult problems.

Pedagogical patterns for assessing learners′ knowledge and skills

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An educator′s pedagogical qualification is an important factor for the successful planning, organization and conduct of quality education. Many teachers are specialists in a particular field but lack pedagogical knowledge and skills. They are not trained to teach. They teach intuitively, learn from their mistakes, draw ideas and inspiration from their colleagues and students. Sharing pedagogical experience and promoting good practices in a suitably compact way is a topical issue. The pedagogical patterns describe in an abstract way pedagogical problems and situations, as well as possible approaches to their solution. They can be used repeatedly, in different ways, in different contexts, in teaching different subject areas. The article describes four pedagogical patterns for assessing learners′ knowledge and skills. They offer different assessment approaches, possible problems along with their respective solutions, and consequences of their use. They use different techniques for self-assessment, collegial evaluation, 360-degree feedback and performance evaluation.

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Mihailo Petrović in ICMI

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The International Commission on Mathematical Instruction (ICMI) was founded in 1908 at the International Congress of Mathematicians in Rome, with the aims of improving teaching standards around the world. Mihailo Petrović was a delegate of Serbia in ICMI, since its founding. That was the first registered Serbian activity in international associations dealing with the teaching of mathematics. In this talk, we discuss Petrović's commitment to education development.

`http://alas.matf.bg.ac.rs/~websites/digitalnilegatmpalas/`

Intuition about probability in elementary school children

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The paper deals with initial development of stochastic reasoning in elementary school children. After short overview of researches that deal with this topic, we focus attention on our study of children's intuition about probability. The research was carried out on the territory of three cities in Serbia. The survey involved 1347 students from grades 2nd to 8th. A questionnaire contained the tasks about problems with throwing a dice or coin. Taking into account that concept of probability still is not integrated in elementary school mathematics in Serbia, presented results can give some insights on what can be a starting point for student's learning about probability.

The reputable class "Let's oscillate"

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Reputable classes are structured in their basic scheme and organization in the usual way, but at a higher and more quality level. Nothing is completely new and unknown in those classes but it has been prepared and realized in a sample way so it can be used as an example. The reputable class "Let's oscillate" is intended to link three subjects in the horizontal correlation: physics, mathematics and computing and informatics . It was realized with the aim to determine the level of knowledge and control of concepts in the field of oscillatory motion (physics), as well as the application of acquired knowledge from trigonometry and the use of ICT. Motivating pupils to use the mobile phone as an accessory in solving the KAHOOT quiz and using the GEO GEBRA web tools in teaching was useful in assessing how much the new approach contributes to the continuous acquisition of knowledge, and how many students are motivated by this type of teaching to study more. This type of class is designed to promote understanding of mechanisms of social grouping, prevention of discrimination and the recognition of the importance of good verbal and non-verbal communication (precise expression and understanding of the spoken).

Using problem teaching system toward new discoveries

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The project that is presented in this paper is based on the usage of the problem teaching, applied in classes of talented students for mathematics and computer science, at The First High School of Kragujevac. The students have created the class presentation containing the results of their hypothesis and research in collaborative way, between each other and a teacher. The final product is a presentation of results in the first number of School Electronic Paper. "Mathematics everywhere around us", "Parabola, everywhere around us" and "Archimedes spiral of recursive equations" are topics of students research, in the first, second and third class, respectively. The project has the aim to involve a few generation of students, in the year 2017 up to 2020. The main project effects are: students put their efforts to go deeper in learning, establishing knowledge, making new knowledge using new tools and developed skills; they become more independent in creative manner, making better knowledge applicability by transferring knowledge; they solve problems in their own way using heuristic approach. Students Jasmina Vulovic and Andjela Radojevic have demonstrated results of their research by a short video. The project results have been published in School Electronic Paper, available at The First High School of Kragujevac website. The students can make connections between teaching curriculum and real life situations but also can visualize their conclusions. Various IKT tools were used in this research.

Assessment of self-regulated learning and mathematics knowledge in pre-service teachers

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Problem solving skills and mathematical reasoning have important role in contemporary teaching of mathematics, and are considered as significant competences for 21st century. Self-regulated learning appears as a powerful construct in educational theories, and can be considered as a strategy, when applied in education, helps in achieving better learning performances. Recent researches indicate that high level of self-regulated learning skills, enhance problem solving skills and mathematical reasoning and by itself represents important competences which pre-service teachers need in their future work.

In order to enhance these competences of pre-service teachers, we will assess self-regulated learning and mathematics knowledge. The aim of the research presented in this paper is to analyze correlation between pre-service teachers' self-regulation skills and mathematical problem-solving abilities. Research was performed with a sample of 112 pre-service teachers at Faculty of Education in Sombor. For assessment of self-regulated learning skills SRUM [3] questionnaire was used, and mathematical knowledge test was used for testing problem solving abilities. The hypothesis that developed self-regulated learning skills help students, pre-service teachers, in solving mathematical problems is tested. The results of statistical analysis show the level of motivation, self-regulated strategies and problem solving skills in pre-service teachers, correlation between specific components of self-regulation and mathematics achievements, as well as dependence of these components on students' academics achievement. Pedagogic implications of this research include the need for developing pre-service teachers' self-regulation abilities because it contributes to better understanding, solving and interpreting mathematical problem tasks.

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Analysis of students mathematical problem-solving success in relation to gender and age

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Individual differences in achievement and ability for doing math between genders are most likely much smaller than the differences within gender. At elementary and secondary levels, male and female students score similarly on many tests, and girls get relatively good grades in math classes. However, some gender differences in math attitudes and skills appear during elementary school, and finally, boys are much more likely than girls to pursue careers in some fields which are related with mathematics, such as programming, computer science and engineering. There exists a stereotype that girls don't like math and that they are not as good at it as boys. Many empirical studies report small gender differences between the abilities of math problem-solving of boys and girls. In most of them, those differences are in favor of boys.

This research is conducted on the sample of secondary school students who took part in mathematical competition *Kangaroo without borders* in Serbia and it aimed to challenge and examine the differentiation of secondary school students problem solving success according to gender.

On the influence of software application for visualization in teaching double and triple integrals

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This article presents the influence of software application on examining double and triple integrals. The research was conducted with two groups of second year students from the Faculty of Engineering, University of Kragujevac. The experimental group was taught by the teacher with the help of materials made in *Wolfram Mathematica* with the focus on the visualization of multivariate functions for the purpose of determining double and triple integrals. In the control group, materials were presented by teacher without the use of computer. Both the groups were tested after the lectures. The students from the experimental group showed significantly better theoretical, practical and visual knowledge. Additionally, students from the first group were highly interested in this way of learning.

Student's opinion on the role of teachers in choosing a method of work

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The main aim of this announcement is to express our opinion on the connection of the role of teacher with the types of methods they use in the classroom. As subject teachers, we have carried out of the monitoring of student's opinions about the content of teaching materials, the style of teaching, methods of work and the role of teachers in the classroom. Sixty lectures have been held in two classes of gifted students for mathematics in the First Grammar School of Kragujevac, in the first and second grades with a total of 36 students, within the courses of Analysis with Algebra and Geometry. The students have also performed a self-assessment of the level of achievement (within each teaching unit and within the subject) on the barometer.

The application of computers in teaching mathematics shown on the example of the following lesson: Calculating the surface of a flat figure - application of the definite integral

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The possibility to activate students motivation as well as their ability to work independently is the greatest advantage of using computers in education. The subject of this paper is using computers in teaching mathematics shown on the example of the lesson Calculating the surface of a flat figure – the application of definite integral. The research was conducted on the first-year students of Traffic Engineering during the school years 2012-2013 and 2015-2016. Two groups of students were formed in each school year with twenty students per group. The purpose of this research was to master the same school program applying two different methods; the first was the traditional method and the second was using computers, or more precisely, the GeoGebra software. The aim of the testing conducted after teaching the above mentioned unit was to show the effect of using computers in teaching, and the aim of the survey conducted among the students was to show their opinion about, and attitude towards, using computers in education.

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Assessment of learning in mathematics in lower grades of primary school in Serbia

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Assessment is a component of every teaching process in schools and has an important influence on academic achievements, as well as fulfillment of students' potential. We are interested in which ways teachers in lower grades of primary schools assess achievement in learning mathematics of their pupils. Do they use assessment *for* learning and formative assessment, if they support and develop mathematical thinking and reasoning in young students, and how can assessment of mathematics in general be improved?

This research is focused on finding out if teachers observe *The rule book for an assessment*, a bylaw document which legally regulates certain segments of assessment in primary school [4]. A sample of 302 teachers from primary schools in the Republic of Serbia participated in this research, and data were collected through a questionnaire constructed for the purpose of this research.

The results show that more than 90% teachers observe the bylaws of assessment, but there are segments of assessment, important for teaching and learning mathematics, which are not in compliance with *The rule book for an assessment*. The analysis of the results show that significant number of teachers (28, 74%) omit the first phase of assessment which refers to the estimation of students' previous knowledge, which is an important component of the lesson planning; that the wide range of teachers (86, 42%) give short written tests to their students, but they do not use collected results as assessment for learning, i.e. in order to check if the aim of the particular lesson is achieved or if students mastered particular parts of program contents, as it is provided by *The rule book for an assessment*. The results also show that techniques of the final assessment vary depending on teachers, and that many teachers carry out oral examinations by way of having students do exercises on the blackboard, and it is not clear if teachers insist on the suitable oral expression of mathematical terms, and if they support pupils' mathematical reasoning.

Considering the observation of the bylaws of assessment, there were no variations between teachers working in different teaching environment (i.e. rural vs. urban), teachers with different levels of education, and kind and length of the working experience, but a difference was observed between male and female teachers.

To obtain better insight into the process of assessment of learning mathematics, it is necessary to conduct a research which would include analysis of tests used by teachers in order to assess the achievements of their students in mathematics, and a more detailed examination of other ways that teachers use in assessment in mathematical classes.

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Mihailo Petrović and his contribution to the development of mathematics in Serbia

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This talk is prepared on the occasion of the 150th anniversary of Mihailo Petrović birth.

It is well known that Mihailo Petrović, among other things, very successfully dealt not only with mathematics as a science, but also by raising young mathematical scientific researchers in Serbia. His contributions was very important in development of mathematical teaching, too.

The aim of this talk is to emphasize the work of Mihailo Petrović:

- in mathematics as a science;
- on the development of teaching courses and writing textbooks;
- on founding a mathematical seminar;
- on raising young mathematical scientific researchers.

Carl Friedrich Gauss and the formula for the sum of the first n natural numbers

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It is likely that rare are those who haven't heard the story from Gauss' (1777-1855) youth: one day his primary school teacher asked the pupils to calculate the sum of the first 100 natural numbers. Gauss quickly announced the result was 5050, which is correct. How did he manage to do that? What was the exact task, to find sum of the first hundred natural numbers, or some other consecutive hundred numbers, as some sources cite? Did Gauss find the formula by himself during the class, or perhaps he knew it all along? The aim of this paper is to search for answers to posed questions.

Significant achievements in a wide range of interests of Mihailo Petrović Alas

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Mihajlo Petrović Alas was an acknowledged European scientist, but at the same time a very modest person. He lived a simple life and loved nature, but he also managed to achieve outstanding scientific results. In addition to mathematics, Professor Petrović worked with dedication and love to improve the level of knowledge and skills in a wide range of interest areas. Among other things, he made a significant contribution to the development of fisheries in Serbia, was a member of scientific expeditions in the polar regions, successful inventor, violinist and a travel writer.

FIELD 6 – COMPUTER SCIENCE

Automated proving of some inequalities involving trigonometric, inverse trigonometric and exponential functions

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Trigonometric, inverse trigonometric and exponential functions play an important role and have applications in engineering and in many areas of pure and applied science. In particular, various inequalities related to the above-mentioned functions have been studied and effectively applied to problems in pure science and many areas of engineering, such as telecommunications, electronics and aeronautics. Estimates, stemming from these inequalities, are particularly useful in approximation theory, Fourier and Harmonic Analysis.

Proving these kinds of inequalities by hand, if possible, is usually complicated. Also, solving these types of problems proves to be an error-prone task involving complex and demanding computations.

Various techniques and provers, both interactive and automated, have been developed in the last decade. However, existing provers are not suitable for direct applications by users not familiar with their design and the underlying formal system.

In this work we present some of the inequalities and automated techniques that we worked on and developed. These techniques involve power-series expansions, some recently developed estimates related to the Taylor polynomials, as well as some applications of existing theorems about analytic functions (see [1-15]).

Our approach and solutions prove to be efficient and easy to follow, while staying mathematically rigorous. Also, in many cases we were

able to estimate the error of approximation, as well as to generate new families of inequalities.

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Software development optimisation theory defined with graphs

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The aim of the software development process is to produce the best possible product with the given resources (money, time). As a part of the development process, quality assurance must also be rationalized. To do so, an abstract space is defined (software testing space), where software product is presented using graph theory. Test graph presents software product with all its functionalities. Test cases in a test graph are connections between vertices and vertices represent unit tests. Test suite and test phase are defined as subgraphs of test graph. The weights in test graph represent the cost and value of implementation for functionality. The first optimization algorithm (A_1), designed as the first step in the optimization of the software testing process, eliminates duplicated test cases. The second algorithm (A_2) alters the quantity of test cases for a given test phase. It is the method of drastically reducing the testing cost while jeopardizing the quality of the product. The third algorithm is a construction of an Optimal Test Phase (OTP), it is A_3 - OTP Construction. This optimization means that a maximum quality, given the resources, is reached. Depending on the circumstances algorithms A_1 and A_2, and A_1 and A_3 can be used together.

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Optimizing Lagrangian particle tracking in parallel environment

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In this paper, we present the adaptive parallel approach to Lagrange particle tracking, where particles possess various lifetime values. Lagrange particle method is very suitable for a distributed programming model because the particle trajectories are entirely independent, but static domain decomposition where each processor is responsible for a certain amount of particles will cause the reduced efficiency of the distributed algorithm. The presented algorithm removes defects of the static domain decomposition and brings a novel approach to the discrete particle tracking. The algorithm introduces the master/slave model with partial trajectory optimization, where a certain number of processors produce partial trajectories and put them to the distributed queue while remaining processors simulate particle motion using generated partial paths. The whole system is adaptive to the total number of processors, while optimal job configuration, partial trajectory length, and the number of producers/consumers, is selected using ML model and evolutionary approach. The paper presents speedup improvement in the use case of Radon progeny behavior in the diffusion chamber, where particles posses exponential distribution of lifetime values and Maxwell speed distribution. The algorithm is implemented in C language, using MPI, and archives speedup close to ideal.

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Methodology of time series quality assessment in concrete dam monitoring systems

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Thermal-seepage-stress analysis represent an essential part of dam health monitoring. The average large dam in Europe is about 50 years old, which puts dam operators in urgent need for upgrading and improvement of dam monitoring and decision-support systems. Reliability of the analysis is primarily dependent on quality of measured data, organized and stored in form of time series. In this paper we present a methodology for the assessment of monitored data related to thermal, seepage and deformation processes. The methodology is based on the consensus of several state of the art statistical and AI outlier detection methods, supplemented by hybrid and original methods. A novel Superimposed Multiple Linear Regression model has shown some advantages over traditional regressions, especially in case of time series with frequent and rapid changes of pattern. Due to different nature of dam monitoring time series, variants of the methodology are proposed for time series with strong seasonal behavior and those strongly dependent on other measurements, such as water level. The proposed approach was validated using case study of large arc dam located in southeastern Europe, where time series related to thermal and seepage processes were assessed.

Extension of a dynamic geometry software with new data types and operations

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Contemporary education supersedes the traditional one (teacher-to-student lessons) with technology-based learning. Dynamic Geometry Software (DGS) play a significant part in devising, creating and demonstrating visually plentiful interactive teaching and learning materials. Dynamic drawings created with DGS are usually specified as expressions written in functional domain-specific languages. As the acceptance of DGS increases, a need for their extensibility also arises, to cover all the fields of application to different teaching subjects and domains.

The extensibility of software for dynamic algebra and geometry is achieved by adding new semantics, represented by new data types and operations. Thereby, it is necessary to address a few potential problems such as compatibility with existing data types, extension of standard operations, property separation of new data types and visualization subsystem extension. Also, the implementation needs to be straightforward and modular. We present our approach to this topic, in a form of a extensibility framework supported with metadata, and provide several practical examples.

We have developed the SLGeometry DGS implemented in C# on the .NET Framework. It has a genericized functional language and the corresponding expression evaluator that act as a framework into which specific semantics is embedded with metadata.

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A simulation based optimization of power production in hydropower systems

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Optimal power production in a hydropower system involves determining the daily operational strategy to be followed to maximize the profit in accordance to the electrical power price trend on the market. At same time, the water consumption has to be minimized and a large set of hydrological conditions and operation guidelines must be obeyed. Our paper presents the framework for solving this complex multi-objective optimization problem using parallel multi-objective genetic algorithm. Due to the time-consuming evaluations which includes running computationally expensive simulation of coupled hydraulic and hydro-energetic model, it may take days and months for the GA to find an acceptable solution. This is unacceptable in circumstances where decisions have to be made on hourly basis. Our solution uses WoBinGO [1] parallel framework for genetic algorithm based optimization which enables speed-up and consequently reduces optimization execution time significantly. Real-world case study with three highly coupled hydropower plants held by two different stakeholders shows that optimization can be done in a reasonable time and that results of the optimization are satisfying.

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Optimization as a service in cloud environment

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Cloud computing provides on-demand access to a large amount of computing resources. This makes it ideal for executing large-scale optimizations using evolutionary algorithms without the need of owning any infrastructure. However, the price of engaging Cloud for computationally expensive evaluations can be high, so it is important to efficiently utilize those resources. In this paper we present OSICE - a comprehensive, cost-effective and easy-to-use Cloud-based optimization service for solving large-scale optimization problems using parallel evolutionary algorithms. OSICE offers machine learning based prediction engine for the estimation of IaaS engagement cost. It provides users with an assessment of the frameworks behavior on the underlying infrastructure in terms of Pareto optimal combinations of a total time required to complete their particular optimization, and a cost of resource consumption during that period. As a proof of concept, this framework has been used to solve real world complex optimization problem from the field of hydroinformatics. Obtained results show that by using proposed prediction engine, significant savings can be achieved both in terms of optimization time and infrastructure cost.

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RNN for solving linear matrix equations

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We are concerned with the solution to the general time-invariant matrix equation $AV(t)B = D$ and the time-varying matrix equation $A(t)V(t)B(t) = D(t)$ by means of gradient based neural network (GNN) model, called the GNNABD model. The resulting matrix generated by the GNNABD model is defined by the choice of the initial state and coincides with the general solution of the matrix equation $AVB = D$. Several particular appearances of this matrix equation and their applications in approximatting various inner and outer inverses are considered. Particularly, two particular cases of the general GNNABD model, globally convergent to the Moore-Penrose inverse and the Drazin inverse are defined and investigated theoretically and numerically. The influence of various nonlinear activation functions on several variants of the GNNABD model are investigated.

Required matrix equations can be solved by the generalized nonlinearly activated GNN model (GGNN model) which is applicable in both time-varying and time-invariant case and possesses the form

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} = \dot{V}(t) = \gamma A^{\mathrm{T}}\mathcal{F}(D - AV(t)B)B^{\mathrm{T}}. \tag{4}$$

The matrix-valued activation function $\mathcal{F}(E)$, $E = (e_{ij})$, is defined as $(f(e_{ij}))$, $i, j = 1, 2, \ldots, n$, where $f(\cdot)$ is a scalar-valued monotonically-increasing odd function.

Theorem 9. *Assume that real matrices $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{p\times q}$ and $D \in \mathbb{R}^{m\times q}$ satisfy*

$$AA^{(1)}DB^{(1)}B = D, \tag{5}$$

for some inner inverses $A^{(1)}$ and $B^{(1)}$. If an odd and monotonically increasing function $f(\cdot)$ is used to define the array activation function $\mathcal{F}(\cdot)$, then the state matrix $V(t) \in \mathbb{R}^{n\times m}$ of the GNNABD model (4) *satisfies $AV(t)B \to D$ when $t \to +\infty$, for an arbitrary initial state matrix $V(0)$.*

Theorem 10. *Assume that the real matrices $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{p\times q}$ and $D \in \mathbb{R}^{m\times q}$ satisfy*

$$AA^{\dagger}DB^{\dagger}B = D. \tag{6}$$

Then the unknown matrix $V(t)$ of the model GNNABD is convergent when $t \to +\infty$ and has the limit value

$$\tilde{V} = A^{\dagger}DB^{\dagger} + V(0) - A^{\dagger}AV(0)BB^{\dagger} \tag{7}$$

for every initial matrix $V(0) \in \mathbb{R}^{n\times p}$.

Some appearances of the general linear matrix equation $AXB = D$ are considered.

Conditions for the existence and representations of $\{2\}$-, $\{1\}$- and $\{1,2\}$-inverses which satisfy certain conditions on ranges and/or null spaces are introduced in [4]. These representations are applicable to complex matrices and involve solutions of certain matrix equations.

Solution $\tilde{V}$ of the matrix equation

$$BV(t)CAB = B$$

defined by the GNABD model

$$\dot{V}(t) = B^{\mathrm{T}}\mathcal{F}(B - BV(t)CAB)(CAB)^{\mathrm{T}} \tag{8}$$

gives $\tilde{V} \in (CAB)\{1\}$. Then $X = B\tilde{V}C$ gives various representations of outer inverses, according to Urguhart formula.

Algorithms arising from the introduced representations are developed. Particularly, these algorithms can be used to compute the Moore-Penrose inverse, the Drazin inverse and the usual matrix inverse. The implementation of introduced algorithms is defined on the set of real matrices and it is based on the Simulink implementation of GNN models for solving the involved matrix equations. In this way, we develop computational procedures which generate various classes of inner and outer generalized inverses on the basis of resolving certain matrix equations. As a consequence, some new relationships between the problem of solving matrix equations and the problem of numerical computation of generalized inverses are established. Theoretical results are applicable to complex matrices and the developed algorithms are applicable to both the time-varying and time-invariant real matrices.

The general computational pattern for commuting generalized inverses is based on the general representation $B(CAB)^{(1)}C$, where the matrices A, B, C satisfy various conditions imposed in the proposed algorithms.

The general computational pattern for computing generalized inverses can be described in two main steps:
(1) Solve appropriate linear matrix equation $BUCAB = B$ with respect to U using GNNABD model.
(2) Compute the matrix product BUC.

GNN models defined in [1, 2, 3, 5] can be derived as modifications of some appearances of the GNNABD model.

The GNNABD model for solving the matrix equation $AA^{\mathrm{T}}VA^{\mathrm{T}}A = A$ is given by

$$\dot{V} = \gamma AA^{\mathrm{T}}\mathcal{F}\left(A - AA^{\mathrm{T}}V(t)A^{\mathrm{T}}A\right)A^{\mathrm{T}}A, \tag{9}$$

and it is called as GNNABD-MP.

Theorem 11. *Let $\tilde{V}(t)$ be a solution of the model* (9)*. Then the matrix $X(t) = A^{\mathrm{T}}\tilde{V}(t)A^{\mathrm{T}}$ converges to the Moore-Penrose inverse $A^{\dagger}$ for every initial matrix $V(0)$.*

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Some optimization methods for non-monotonic reasoning in System P

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System **P**, introduced by Kraus, Lehman and Magidor [4], represents a core of various default systems. Reasoning in System **P** can be modeled by a logic with approximate conditional probabilities [7]. This probabilistic logic enriches classical propositional calculus with binary probabilistic operators which are applied to propositional formulas: $\mathrm{CP}_{\geqslant s}(\alpha, \beta)$, $\mathrm{CP}_{\leqslant s}(\alpha, \beta)$ and $\mathrm{CP}_{\approx s}(\alpha, \beta)$, with the intended meaning that the conditional probability of α given β is "at least s", "at most s" and "approximately s", respectively. It was shown that formulas $\mathrm{CP}_{\approx 1}(\alpha, \beta)$ can be used to model defaults of the form: "if β, then generally α".

Satisfiability problem for a set of defaults can be converted to a satisfiability problem for a probabilistic formula in a logic mention above. That problem can be reduced to a system of linear inequalities, and as such a number of different methods can be used for its solving. The main contributions of this paper are development of methodology for using optimization methods to solve the considered problem and presentation of the obtained results.

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Assessing the effects of muscle disease on force generation using multi-scale muscle model

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Computational muscle model is a common tool used for simulating muscle behavior and investigating its structural and functional properties. To be used for investigating and predicting the influence of various disorders and diseases on muscle functional behavior those models must integrate physical and biochemical properties across multiple length and time scales. They must be expressive in terms of describing intrinsic biophysical processes which can be separately defined for each subcomponent in the complex muscle structure. Single-scale muscle models, both biophysical and phenomenological, cannot comply. To address these demands, multi-scale muscle model must be used.

We present the methodology for multi-scale muscle modeling and its usage in predicting functional behavior of a disordered muscle. We use two-scale model which describes macroscopic muscle mechanics using finite element method, while the material attributes of the muscular tissue comply to the Huxley model, employed at the microscopic scale. To demonstrate the methodology, we use a simplified 2D geometry of a muscle body for simulations of physiological tasks of a muscular organ in health and disease. We simulated *mdx* mouse effect on a healthy muscle model. The *mdx* mouse model is a model of Duchenne muscular dystrophy (DMD) used to study the disease mechanisms and potential treatments. We demonstrate how the disease affects mobility of the muscle and assess its effects on force generation in comparison with healthy muscle behavior.

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POSTERS – ABSTRACTS

On Carleson-type embeddings for Bergman spaces of harmonic functions

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Given a measure μ on a bounded domain $\Omega \subset \mathbb{R}^n$ with C^1 boundary we investigate the following problem: when is a weighted harmonic Bergman space $A^p_\alpha(\Omega)$ continuously embedded in weighted space $L^p(\Omega) = L^p(\mu, \Omega)$? We give a sufficient Carleson type condition for all $\alpha > -1$ and $0 < p < \infty$ which is also necessary for $p > 1 + \frac{\alpha+2}{n-2}$.

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Studuy of watching systems in cubic graphs

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A watching system in a graph G, which is an extension of identifying code, is a finite set $W = \{w_1, w_2, \ldots, w_k\}$ where each w_i is a couple $w_i = (v_i, Z_i)$, where v_i is a vertex and $Z_i \subseteq NG[v_i]$ such that $\{Z_1, \ldots, Z_k\}$ is an identifying system. In the present paper, we determine the watching number of some well-known cubic graphs.

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Constructive development and gamification of the mathematic teaching

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In the presented paper, students of the Mathematics were introduced with models of preparation of planned curriculum which are already established within engineering course. Constructive approach to development is used to challenge students to methodically prepare games for elementary school children based on the official mathematics curriculum. During development of the educational games programming students were introduced with rules of Software engineering and other programming rules and procedures common for engineering studies. Finally their newly developed products were presented to kids, and opinion of the students on this approach and school kids on the resulting material was examined.

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Statistical analysis of the ratio of product of two independent stable Weibull random variables and Gamma random variables

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In the present paper, the exact probability density function and cumulative distribution function of the ratio of product of two independent stable Weibull random variables and Gamma random variables are derived in terms of the Meier-G function. Those type of the products and ratios of distributions appears naturally for solution of problems in engineering, economics, telecommunications etc. Especially important value for the systems modeled by those products is cumulative distribution function which can calculate cases of system failure. For practical applications routine in the Mathematica software has been developed for the evaluation of the Meier-G function. Finally, numerical experiments are carried out to show the accuracy and correctness of the expressions hereby deduced.

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On finite capable groups

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A group H is said to be capable, if there exists another group G, such that $\frac{G}{Z(G)} \cong H$, where $Z(G)$ denotes the center of G. Define:

$$H_1 = \langle x, y, z \mid x^9 = y^3 = 1, z^3 = x^3, yx = x^4y, zx = xyz, zy = yz \rangle,$$
$$H_2 = \langle x, y, z \mid x^{p^2} = y^p = z^p = 1, yx = x^{p+1}y, zx = x^{p+1}yz, zy = x^pyz \rangle,$$
$$H_3 = \langle x, y, z \mid x^9 = y^3 = 1, z^3 = x^{-3}, yx = x^4y, zx = xyz, zy = yz \rangle,$$
$$H_4 = \langle x, y, z \mid x^{p^2} = y^p = z^p = 1, yx = x^{p+1}y, zx = x^{dp+1}yz, zy = x^{dp}yz \rangle,$$

where $p > 3$, $d \not\equiv 0, 1 \pmod{p}$. The aim of this paper is to prove all groups H_i, $1 \leq i \leq 4$, are not capable.

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Construction of some codes based on finite geometries structures

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In this paper we study the construction of these codes from a geometric approach based on points, lines and planes of projective and affine geometries over finite fields and designs. These structures of finite geometry provide a powerful tool for constructing several codes.

We will present some methods of constructing low-density parity control codes based on matrix incidents of finite geometry structures such as designs. We will examine some of techniques of extensions and shortening these codes, optimizing their performance and decoding.

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