# Error bounds for Kronrod extension of generalizations of Micchelli-Rivlin quadrature formula for analytic functions

Rada M. Mutavdžić[1], Aleksandar V. Pejčev[1], and Miodrag M. Spalević[1]

[1]Department of Mathematics, Faculty of Mechanical Engineering, University of Belgrade, rmutavdzic@mas.bg.ac.rs, apejcev@mas.bg.ac.rs, mspalevic@mas.bg.ac.rs

We consider Kronrod extension of generalizations of the well known Micchelli-Rivlin quadrature formula, with the highest algebraic degree of precision, for the Fourier-Chebyshev coefficients. For analytic functions the remainder term of these quadrature formulas can be represented as a contour integral with a complex kernel. We study the kernel, on elliptic contours with foci at the points $\mp 1$ and a sum of semi-axes $\rho > 1$, for the quoted quadrature formulas. Starting from the explicit expression of the kernel, we determine the locations on the ellipses where maximum modulus of the kernel is attained. So we derive effective $L^\infty$-error bounds for these quadrature formulas. Complex-variable methods are used to obtain expansions of the error in these quadrature formulas over the interval $[-1, 1]$. Finally, effective $L^1$-error bounds are also derived for these quadrature formulas. Numerical examples which illustrate the calculation of these error bounds are included.

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