On the characteristic rank of vector bundles over oriented Grassmannians

Marko Radovanović[1]

[1]Faculty of Mathematics, University of Belgrade, markor@matf.bg.ac.rs

We study the cohomology algebra of the Grassmann manifold $\widetilde{G}_{k,n}$ of oriented k-dimensional subspaces in $\mathbb{R}^{n+k}$ via the characteristic rank of the canonical vector bundle $\widetilde{\gamma}_{k,n}$ over $\widetilde{G}_{k,n}$ (denoted by charrank($\widetilde{\gamma}_{k,n}$)). Using Gröbner bases for the ideals determining the cohomology algebras of the "unoriented" Grassmannians $G_{k,n}$ we prove that charrank($\widetilde{\gamma}_{k,n}$) increases with k. In addition to that, we calculate the exact value of charrank($\widetilde{\gamma}_{4,n}$), and for $k \geq 5$ we improve a general lower bound for charrank($\widetilde{\gamma}_{k,n}$) obtained by Korbaš. Some corollaries concerning the cup-length of $\widetilde{G}_{4,n}$ will also be given.

Joint work with Branislav Prvulović.