# On two conjectures regarding the set of values of Wiener index

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The Wiener index $W(G)$ of a simple connected graph $G$ is defined as the sum of distances over all pairs of vertices in a graph. Since it was one of the first indices introduced in literature, it was extensively studied. One of the proposed problems was the inverse Wiener index problem, i.e. for a given value $w$ the problem of finding a graph $G$ for which $W(G) = w$, where $G$ can be a graph on any number of vertices. That problem was first solved computationally for $w$ up to $10^8$ and then it was fully solved in 2006 when two papers were independently published proving that there are only 49 integers that are not the value of Wiener index for any graph.

Recently, a related Wiened index interval problem was introduced in [1] where the question is what values can Wiener index have on a class of graphs with a given number of vertices and what is the largest interval of consecutive integers among those values. In the same paper some strong results are given on the class $\mathcal{G}_n$ of all simple graphs on $n$ vertices and the following two conjectures are made for the class $\mathcal{T}_n$ of all trees on $n$ vertices ( $W[\mathcal{T}_n]$ denotes the set of all values of Wiener index for a tree from $\mathcal{T}_n$, while $W^{int}[\mathcal{T}_n]$ denotes the largest interval of consecutive integers in $W[\mathcal{T}_n]$.)

**Conjecture 1.** *The cardinality of* $W[\mathcal{T}_n]$ *equals* $\frac{1}{6}n^3 + \Theta(n^2)$.

**Conjecture 2.** *The cardinality of* $W^{int}[\mathcal{T}_n]$ *equals* $\Theta(n^3)$.

We present the proof of these conjectures which is the strongest possible in terms of the highest power of $n$.

## References

[1] M. Krnc and R. Škrekovski, On Wiener Inverse Interval Problem, MATCH Commun. Math. Comput. Chem. **75** (2016), 71–80.