On the compressed essential graph of a commutative ring

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Let R be a commutative ring. In this paper, we introduce and study the compressed essential graph of R. The compressed essential graph of R is the graph $EG_E(R)$, whose vertices are equivalence classes of zero-divisors of R and two distinct vertices $[x]$ and $[y]$ are adjacent if and only if $\operatorname{ann}(xy)$ is an essential ideal of R. It is shown that for a reduced ring R, $EG_E(R) = \Gamma_E(R)$, $\Gamma_E(R)$ denotes the compressed zero-divisor graph of R, and $V(EG_E(R)) = 2^n - 2$ also it has $c(n,k)$ vertices of degree $2^k - 1$, for all $1 \leq k \leq n-1$, where R is Noetherian and $|\operatorname{Ass}(R)| = n > 2$. Furthermore, it shown that for a non-reduced ring R with $3 < |EG_E(R)| < \infty$, $EG_E(R) = \Gamma_E(R)$ if and only if

(i) $\operatorname{Nil}(R) = \operatorname{ann}_R(Z(R))$;

(ii) if $\operatorname{ann}_R(a)$ is an essential ideal of R, then $a \in \operatorname{Nil}(R)$;

(iii) every non-zero element of $\operatorname{Nil}(R)$ is irreducible in $Z(R)$.

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