Transmission eigenvalues, discreetness and existence

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The interior transmission problem, which arises in inverse inverse scattering theory, is a boundary value problem compounded of two partial differential equations of second order defined in a bounded domain that corresponds to the scatterer. The boundary value problem is not elliptic in the sense of Agmon-Douglas-Nirenberg so the classical theory of PDE does not provide a direct answer for its solvability. Its homogeneous version is referred to as the transmission eigenvalue problem, which is nonlinear and non self-adjoint eigenvalue problem. In this paper the focus is to prove descretness and existence of real transmission eigenvalues of the following problem

$$\begin{aligned} \Delta w + k^2 n w = 0 \qquad & \text{in} \quad \Omega, \\ \Delta v + k^2 v = 0 \qquad & \text{in} \quad \Omega, \\ w - v = -\eta \frac{\partial v}{\partial \nu} \qquad & \text{on} \quad \partial\Omega, \\ \frac{\partial w}{\partial \nu} = \frac{\partial v}{\partial \nu} \qquad & \text{on} \quad \partial\Omega. \end{aligned}$$

This transmission eigenvalue problem, which appears in the analysis of inverse scattering problem for an inhomogeneous media with thin coating, has not been studied in the existing literature. It presents additional difficulties due to more complicated boundary conditions. Establishing the discreetness of transmission eigenvalues is important in order to prove the solvability of the interior transmission problem, since the latter satisfies the Fredholm Alternative. On the other hand the existence of real transmission eigenvalues, which is much harder question due to non-selfadjointness, is important for solving the inverse scattering problem since they provide information on refractive index n of the scattering media

References

[1] F. Cakoni, D. Colton and H. Haddar, Inverse Scattering Theory and Transmission Eigenvalues, CBMS Series, SIAM Publications **88**, Philadelphia, PA, 2016.

[2] F. Cakoni, D. Gintides and H. Haddar, The existence of an infinite discrete set of transmission eigenvalues. SIAM J. Math. Anal. **42** (2010), 237–255.

[3] F. Cakoni, D. Colton and D. Gintides, The interior transmission eigenvalue problem, SIAM J. Math. Anal. **42** (2010), 2912–2921.