Inequalities involving polar derivative of a polynomial with restricted zeros

Prasanna Kumar

Department of Mathematics, Birla Institute of Technology and Science, Goa, India,
prasannakornaya@rediffmail.com

It is well known that if $P(z)$ is a polynomial of degree n, then $\max_{|z|=1} |P'(z)| \leq n \max_{|z|=1} |P(z)|$. This inequality is known as Bernstein's inequality. The subject of inequalities for polynomials and related classes of functions plays an important and crucial role in obtaining inverse theorems in Approximation Theory. Many a times results related to inverse theorems have depended upon first obtaining the corresponding analogue or generalization of Markov's and Bernstein's inequalities. These inequalities have motivated the researchers for significant new literature in Mathematics. Bernstein's inequality and its generalizations concerning the growth of polynomials have entered into different domains, in different norms. Here we obtain some results concerning the inequalities involving polar derivative of a complex polynomial with restricted zeros. The results presented over here improve upon the earlier results.

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